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Approximate Strength of Lightweight Aggregate Using Micromechanics Method

Chung-Chia Yang* and Ran Huang†
*Institute of Materials Engineering and †Department of Harbor and River Engineering, National Taiwan Ocean University, Keelung, Taiwan, Republic of China

This study presents a method for estimating the strength of lightweight aggregate. Cylindrical specimens with various aggregate volume ratios (volume of coarse aggregate/total aggregate volume) were cast and tested. Micromechanics method was applied by considering a perfect bond between mortar and aggregate. The approximate aggregate strengths determined from the concrete strength, component properties, and the volume ratio of aggregate are between 15 and 30 MPa. Both matrix strength and composite strength are much higher than the lightweight aggregate strength.
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KEY WORDS: Compressive strength, Lightweight aggregate, Lightweight concrete, Micromechanics, Volume ratio

In normal weight concrete, the aggregate strength is not a factor affecting concrete strength because the aggregate is much stronger than the matrix and the transition zone [1]. However, as the strength of lightweight aggregate usually is lower than those of the other two phases, the failure of concrete then is controlled by aggregate fracture.

The strength of natural coarse aggregate can be obtained empirically by testing the bedrock; however, it is not easy to obtain the strength of artificial lightweight aggregate in the same way. The strength of lightweight aggregate can be evaluated from indirect tests such as the Los Angles abrasion test (ASTM C 131), hardness test, and aggregate crushing value test (ACV) (BS 812). Nilsen et al. [2] suggested that the specific energy dissipation due to a stress cycle can be used to assess the quality of lightweight aggregates. Chang and Su [3] proposed a method to estimate the compressive strength of an aggregate particle by using the theory of granular mechanics.

Researchers have studied the influence of aggregate properties on high strength and lightweight concretes [4–7] and demonstrated that the strength of concrete is controlled by the weakest component. The strength of lightweight concrete is limited by the strength of the coarse aggregate if the fracture surface initially passes through the coarse aggregates.

The inhomogeneity stress disturbance can be simulated by an inclusion-induced eigenstress when the eigenstrain is chosen properly [8]. Such equivalency was derived from the equivalent inclusion method. Mori and Tanaka [9] have applied the concept of average field to analyze the macroscopic properties of composite materials. The average field in a body contained inclusions with eigenstrain. In addition, the shape effect of dispersoids introduced in Eshelby's method [10] was applied to evaluate the properties of composite materials. Yang and Huang [11] have proposed a model to predict the compressive strength of cement based composite by considering concrete as a two-phase material.

In this study, three types of artificial coarse aggregates were made of cement and fly ash with different combinations through a cold-pelletizing process. The compressive strength of lightweight concrete was obtained using a universal testing machine. A numerical approach was made by using a micromechanics method to evaluate the compressive strength of lightweight aggregate based on the tested composite strength. By taking a perfect bond between aggregate and matrix into account, the average stress field in aggregate was calculated. For lightweight concrete, the weakest component in concrete is lightweight aggregate, and the compressive strength of lightweight concrete then is controlled by the strength of coarse aggregate. When the lightweight concrete reaches its ultimate strength, the average stress in aggregate is the strength of the aggregate.

Experimental Program

In the experimental program, lightweight concretes were made of different artificial aggregates and matri-

Address correspondence to: Dr. C.C. Yang, Institute of Materials Engineering, National Taiwan Ocean University, 2 Pei-Ning Road, Keelung, Taiwan 202, Republic of China.
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TABLE 1. Physical properties of aggregate

Type of Aggregate	Unit Weight (SSD; g/cm^3)	Unit Weight (OD; g/cm^3)	Water Absorption (30 min; %)
Type A	1.62	1.22	33.39
Type B	1.66	1.28	29.97
Type C	1.73	1.43	21.05

ces. The aggregates were produced from Portland cement and fly ash in various combinations. Mortar was considered as a matrix with various fine aggregate volume ratios. The specimens were cast and cured in the laboratory.

Coarse Aggregate (Inclusion)

Three types of artificial coarse aggregates denoted by A, B, and C were selected. The spherical aggregates with diameters between 0.5 and 1.0 cm were made of cement and fly ash through a cold-pelletizing process. Water was the wetting agent used as the coagulant so that the wet mixture would be pelletized by rolling a tilted pan. Type A, B, and C aggregates were made with fly ash/cement ratios of 0.1, 0.15, and 0.2 (by weight), respectively. The physical properties of the aggregates are shown in Table 1.

Mortar (Matrix)

Mortar specimens were made of type I cement and natural sand. The cubic specimens (50 × 50 × 50 mm) were cast and cured in the laboratory. The mix proportions of the mortar are summarized in Table 2. All of the mixtures had a water/cement ratio of 0.30, and the superplasticizer was adjusted to keep the same flow. To study the effect of the volume ratio of fine aggregate on the elastic modulus of mortar, six fine aggregate volume ratios V_s (volume of fine aggregate/volume of mortar, a/t = 0.0, 0.1, 0.2, 0.3, 0.4, and 0.5) were selected in the mix design. For determining the elastic moduli of the mortars, two axial electric strain gages (gage length = 10 mm) were mounted on opposite sides of the specimen to measure the compressive strains. The compressive test was conducted using a 100-ton universal testing machine according to the specification of ASTM C87. The load was applied at a rate within the range of 0.14 to 0.34 MPa/s. Continuous measurements were recorded to obtain the stress/strain curves, and the secant moduli were determined from the stress/strain curves.

Concrete (Composite)

Lightweight concrete specimens were cast using type I cement, superplasticizer, water, natural sand, and lightweight coarse aggregate with a water/cement ratio of 0.3. Four coarse aggregate volume ratios (volume of coarse aggregate/volume of total aggregate, C_a/T_a = 0.3, 0.4, 0.5, and 0.6) were considered in the mix proportions. To keep a constant water/cement ratio, the lightweight coarse aggregate was immersed in water for at least 30 minutes before mixing, and then the surface was dried with towels. The lightweight concrete mix design is given in Table 3. Notation for the specimens is such that the first letter indicates the type of aggregate (A, B, and C), and the second number represents the volume ratio of coarse aggregate. Mortar and concrete cylinders (ϕ 100 × 200 mm) were cast and cured. At the age of 28 days, the elastic moduli and compressive strengths of the specimens were measured according to the specification of ASTM C 469-81 and ASTM C 39-81, respectively.

TABLE 2. Mix design and volume ratio of fine aggregate

Mix No.	Water (kg/m^3)	Cement (kg/m^3)	Sand (kg/m^3)	SP (kg/m^3)	Volume Ratio (volume of sand/volume of mortar; %)
M0	449.1	1603.9	0	32.1	0
M1	403.5	1441.1	261.8	28.8	10
M2	357.9	1278.2	523.6	25.6	20
M3	312.3	1115.4	785.4	22.3	30
M4	266.7	952.6	1047.2	19.1	40
M5	221.1	789.7	1309.0	15.8	50

Theoretical Background

The relationship between the applied stress $\underset{\sim}{\sigma}^o$ and the properties of the constituent materials of the composite is determined using a micromechanics method. Because the strength of lightweight aggregate is usually the lowest component in lightweight concrete, the failure is controlled by aggregate fracture. In this study, lightweight concrete was considered as a composite and the perfect bond was assumed between mortar and aggregate. The average stresses in the inhomogeneities, $\underset{\sim}{\sigma}^o + \langle\underset{\sim}{\sigma}\rangle_\Omega$, and in the matrix, $\underset{\sim}{\sigma}^0 + \langle\underset{\sim}{\sigma}\rangle_M$, are derived based on the equivalent inclusion method [8]. From the viewpoint of the average stress fields, the strength of the lightweight concrete composite is controlled by the lightweight aggregate so that the fracture propagates when the average stress fields in the lightweight aggregate reach its ultimate strength.

Considering the existence of spherical inhomogeneities, $\Omega = \sum_{i=1}^{N} \Omega_i$, with elastic moduli $\underset{\sim}{C}^*$ and a volume fraction V_f randomly embedded in an infinite matrix with the stiffness $\underset{\sim}{C}$, the stress disturbance in the applied compressive stress, $\underset{\sim}{\sigma}^o$, due to inhomogeneities can be simulated by the eigenstress caused by the

TABLE 3. Mix proportions

Mix No.	Water (kg/m^3)	Cement (kg/m^3)	SP (kg/m^3)	Fine Aggregate (kg/m^3)	Coarse Aggregate (kg/m^3)	Volume Ratio
A3					291.6	
B3	178.43	626.06	9.39	1095.8	298.8	0.18
C3					311.4	
A4					388.8	
B4	178.43	626.06	9.39	939.2	399.4	0.24
C4					415.2	
A5					486.0	
B5	178.43	626.06	9.39	782.7	498.0	0.30
C5					519.0	
A6					583.2	
B6	178.43	626.06	9.39	626.2	597.6	0.36
C6					622.8	

fictitious misfit strain (Figure 1). In this study, the fictitious misfit strain (eigenstrain), $\underset{\sim}{\epsilon}^*$, was introduced to simulate the inhomogeneity effect. By use of the equivalent inclusion method and the Mori-Tanaka theory [9], the total stress in the inhomogeneity can be represented as follows:

$$\underset{\sim}{\sigma}^o + \langle \underset{\sim}{\sigma} \rangle_\Omega = \underset{\sim}{C}\{\underset{\sim}{C}^{-1}(\underset{\sim}{\sigma}^o + \langle \underset{\sim}{\sigma} \rangle_M) + \langle \underset{\sim}{\Delta\gamma} \rangle - \langle \underset{\sim}{\epsilon}^* \rangle\} \qquad (1)$$

or

$$\underset{\sim}{\sigma}^o + \langle \underset{\sim}{\sigma} \rangle_\Omega = \underset{\sim}{C}^*\{\underset{\sim}{C}^{-1}(\underset{\sim}{\sigma}^o + \langle \underset{\sim}{\sigma} \rangle_M) + \langle \underset{\sim}{\Delta\gamma}\}, \qquad (2)$$

where $\langle \underset{\sim}{\Delta\gamma} \rangle$ is the average disturbance strain caused by eigenstrain $\langle \underset{\sim}{\epsilon}^* \rangle$ in a single inhomogeneity.

Using the condition of the average of the stress disturbance is zero [8], the average eigenstrain $\langle \underset{\sim}{\epsilon}^* \rangle$ is obtained by solving eq 1:

$$\langle \underset{\sim}{\epsilon}^* \rangle = \alpha^{-1}(\underset{\sim}{C} - \underset{\sim}{C}^*)\underset{\sim}{C}^{-1}\underset{\sim}{\sigma}^o \qquad (3)$$

with $\alpha = (1 - V_f)(\underset{\sim}{C}^* - \underset{\sim}{C})\underset{\sim}{S} - V_f(\underset{\sim}{C} - \underset{\sim}{C}^*) + \underset{\sim}{C}$ and the average total stresses in the inhomogeneities are obtained as:

$$\underset{\sim}{\sigma}^o + \langle \underset{\sim}{\sigma} \rangle_\Omega = [(1 - V_f)\underset{\sim}{C}(\underset{\sim}{S} - \underset{\sim}{I})\alpha^{-1} \times (\underset{\sim}{C} - \underset{\sim}{C}^*)\underset{\sim}{C}^{-1} + \underset{\sim}{I}]\underset{\sim}{\sigma}^o, \qquad (4)$$

where $\underset{\sim}{S}$ is the Eshelby tensor for a single inclusion that solely exists in an infinite homogeneous medium. The Eshelby tensor is a function of the geometry of the inclusion and Poisson's ratio of the matrix (see Appendix).

In lightweight concrete, the strength of concrete is controlled by the strength of the coarse aggregate. In eq 4, when the applied compressive stress, $\underset{\sim}{\sigma}^o$, reaches the strength of lightweight concrete $\underset{\sim}{f_c'}$, the strength of lightweight aggregate can be obtained as:

$$\underset{\sim}{f_a} = [(1 - V_f)\underset{\sim}{C}(\underset{\sim}{S} - \underset{\sim}{I})\alpha^{-1}(\underset{\sim}{C} - \underset{\sim}{C}^*)\underset{\sim}{C}^{-1} + \underset{\sim}{I}]\underset{\sim}{f_c'}. \qquad (5)$$

The relationship between the volume fraction (V_f) and the composite strength (f_c') can be obtained for a speci-

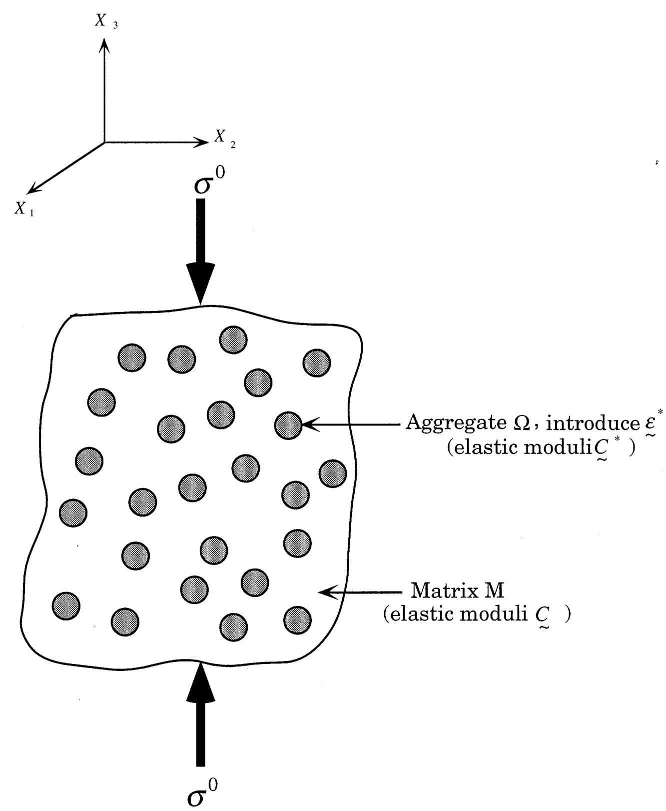

FIGURE 1. Spherical inclusions embedded in an isotropic infinite body under uniaxial compressive stress.

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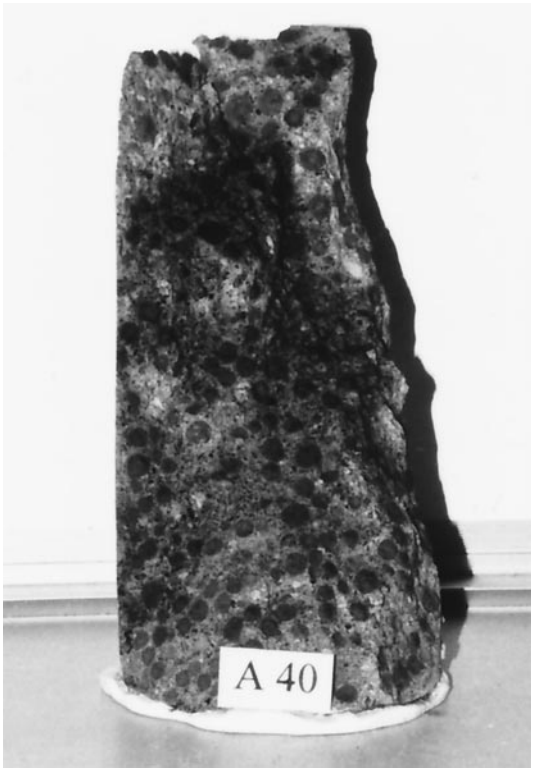

FIGURE 2. Fractured surfaces of the lightweight aggregate concrete.

fied aggregate strength using eq 5. For a given V_f and f'_c, the aggregate strength can be estimated from a series of f'_c vs. V_f curves.

Results and Discussion

In this study, concrete was considered as a composite material in which coarse aggregate were embedded in a matrix of hardened mortar. For lightweight concrete, the weakest component is usually lightweight aggregate and the fracture surfaces passed through the coarse aggregate as well as the matrix (Figure 2). The aggregate strength controls the ultimate strength of the lightweight concrete. When the elastic properties of concrete, the volume ratio of aggregate, and the compressive strength of concrete are given, the aggregate strength can be determined from eq 5.

Elastic Modulus of Matrix

The elastic moduli of mortar with six fine aggregate volume ratios V_s (a/t = 0.0, 0.1, 0.2, 0.3, 0.4, and 0.5) were obtained from the test. Figure 3 shows the relationship between volume ratio of fine aggregate and elastic modulus of mortar, E_m. It appears that the mortar elastic modulus increases with increase in aggregate volume fraction. The best-fitted curve in Fig. 3 is obtained and expressed as:

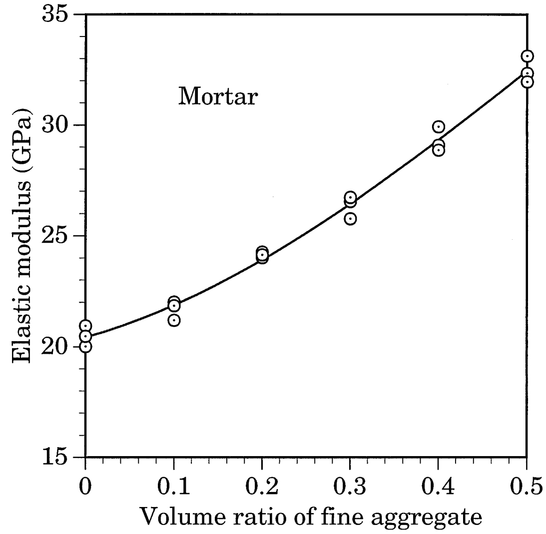

FIGURE 3. Elastic modulus vs. volume ratio curve.

$$E_m = -19.725V_s^3 + 35.904V_s^2 + 11.027V_s + 20.428, \quad (6)$$

where E_m is in GPa. The elastic moduli of the mortar with various volume ratios of fine aggregate were computed and are presented in Table 4.

Elastic Modulus of Inclusion

According to the Mori-Tanaka method, the elastic moduli of coarse aggregate were derived from the tested

TABLE 4. Properties of mortar, aggregate, and concrete

Mix No.	Elastic Modulus (GPa) Mortar (Tested)	Concrete* (Tested)	Aggregate (eq 7)	Strength (MPa) Concrete† (Tested)
A3	29.33	23.02	5.80	41.37
A4	28.13	20.60	6.11	37.31
A5	26.44	18.21	6.32	34.62
A6	24.87	15.80	5.85	31.51
			(Average = 6.01)	
B3	29.33	23.79	7.82	44.17
B4	28.13	21.53	8.08	41.40
B5	26.44	19.01	7.79	38.25
B6	24.87	17.22	8.19	35.73
			(Average = 7.97)	
C3	29.33	24.66	10.30	49.89
C4	28.13	22.58	10.52	47.05
C5	26.44	20.32	10.37	45.16
C6	24.87	18.65	10.73	41.67
			(Average = 10.48)	

*Average of five specimens.
†Average of seven specimens.

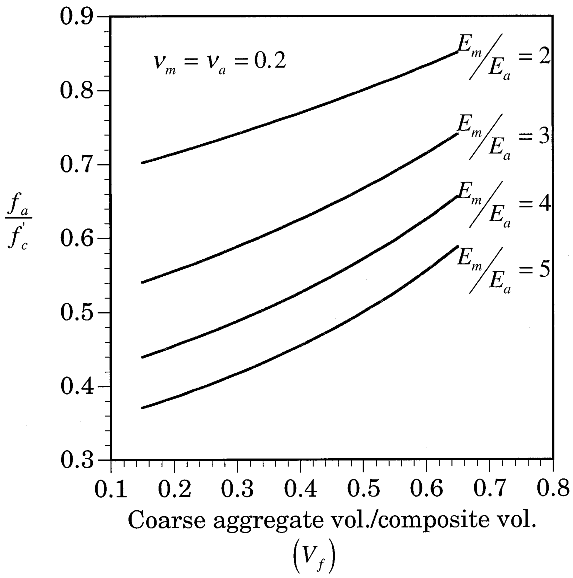

FIGURE 4. A dimensionless parameter, f_a/f_c', vs. aggregate volume ratio.

elastic moduli of composite and matrix using the following equation:

$$\underset{\sim}{C} = \{\underset{\sim}{C}^{-1} + V_f[\{(1 - V_f)(\underset{\sim}{C}^{*} - \underset{\sim}{C})\underset{\sim}{S} - V_f(\underset{\sim}{C} - \underset{\sim}{C}^{*}) + \underset{\sim}{C}\}^{-1}]^{-1}(\underset{\sim}{C} - \underset{\sim}{C}^{*})\underset{\sim}{C}^{-1}\}, \qquad (7)$$

where $\overline{\underset{\sim}{C}}$ is the elastic moduli tensor of the composite. The Poisson's ratio of the matrix and composite was assumed to be 0.2. The computed elastic moduli are between 6.0 and 10.5 MPa for the three types of lightweight aggregates shown in Table 4. The moduli of mortar and concrete are dependent on the volume ratio and are two to five times that of aggregate moduli (E_m/E_a = 2 to 5).

Strength of Inclusion

Using eq 5, the relationships between f_a/f_c' (lightweight aggregate strength/composite strength) and the inclusion volume ratio were found for various E_m/E_a ratios as shown in Figure 4 ($E_m/E_a \geq 2$). For a given aggregate strength and E_m/E_a ratio, the composite strength decreases as the volume ratio increases. For a given volume ratio, the f_a/f_c' ratio increases with a decrease in E_m/E_a ratio. It appears that both volume ratio and elastic modulus of aggregate significantly affect the composite strength for a given matrix. In other words, the concrete strength increases with an increase in E_m/E_a ratio for a given f_a. Furthermore, the aggregate strength can be obtained from eq 5 if the concrete strength, f_c', is known. The elastic moduli and strength

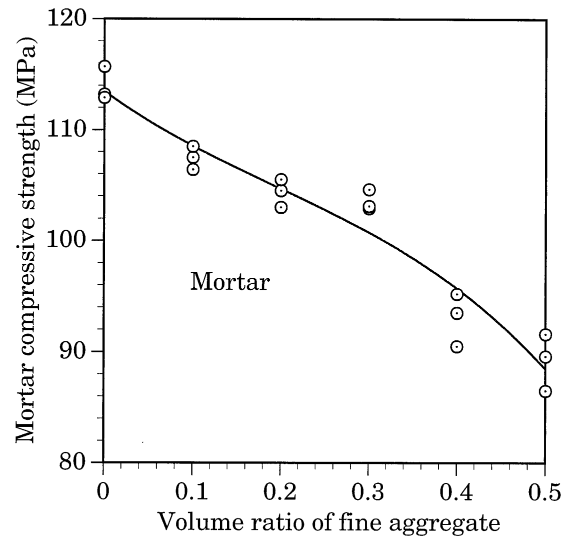

FIGURE 5. Compressive strength vs. volume ratio curve.

of concrete were obtained by experiment as tabulated in Table 4, and mortar strength vs. volume ratio curve is displayed in Figure 5. The mortar strength is between 88 and 115 MPa.

By assuming a Poisson's ratio of 0.2, the strength of coarse aggregate was computed from the elastic moduli of the components and the strength of concrete. Figures 6, 7, and 8 show the relationship between coarse aggregate volume ratio and the compressive strength of concretes. The corresponding computed results also are illustrated in the figures. The curve correlates the coarse aggregate volume ratio with the compressive strength of concrete. For type A aggregate, the concrete strengths

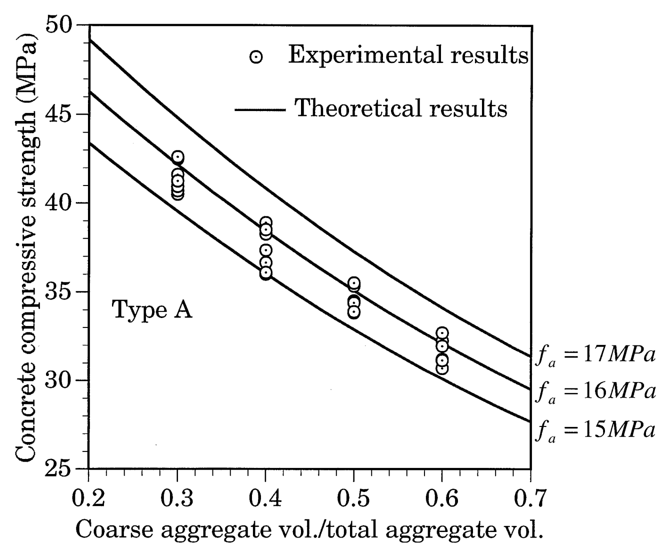

FIGURE 6. Compressive strength vs. volume ratio curves for type A aggregate.

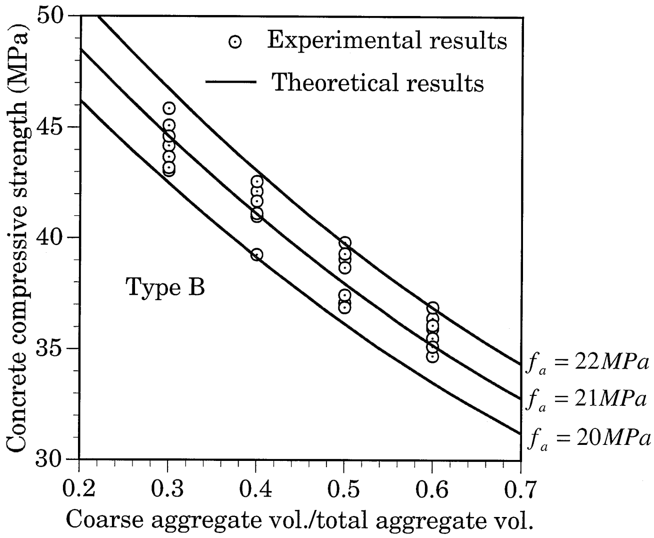

FIGURE 7. Compressive strength vs. volume ratio curves for type B aggregate.

are within the curves that were computed from eq 5 by assuming aggregate strength to be 15 and 17 MPa. By using the same analytical procedure and comparing the experimental data with the computed results, the estimated strengths of type B and C aggregates are 20 to 22 MPa and 28 to 30 MPa, respectively. It also appears that the lightweight concrete strength decreases as the coarse aggregate volume ratio increases. The concrete compressive strength is less than the matrix strength but higher than the aggregate strength. An increase in aggregate strength or a decrease in aggregate volume ratio can significantly improve the composite strength.

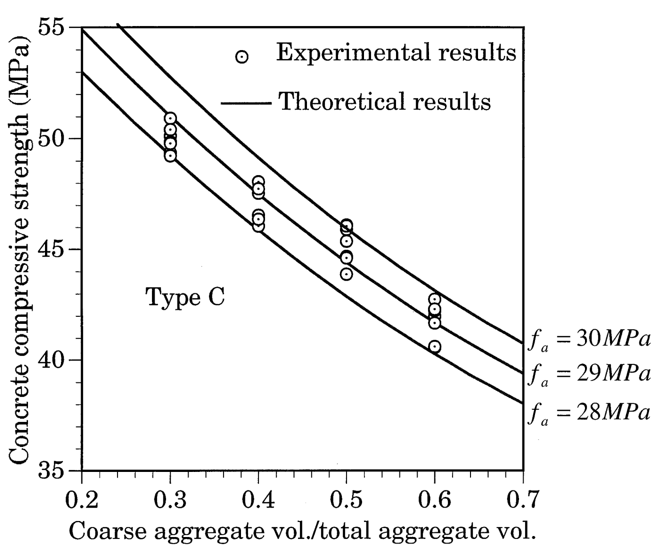

FIGURE 8. Compressive strength vs. volume ratio curves for type C aggregate.

Conclusions

A numerical approach was proposed for estimating the compressive strength of lightweight coarse aggregate and predicting the trend of strength variation for different volume fractions of aggregates using a micromechanics method. Based on the theoretical prediction and experimental results, the estimated strengths of type A, B, and C lightweight aggregates are about 15 to 17 MPa, 20 to 22 MPa, and 28 to 30 MPa, respectively. The lightweight aggregate effect on the concrete compressive strength is dependent on the E_m/E_a ratio, aggregate volume ratio, and aggregate strength. This study is an initiation to apply micromechanics to approximating the aggregate strength. More extensive and more refined researches need to be done to verify the interfacial effect and to obtain the absolute value of the compressive strength of lightweight aggregate.

Appendix

The Eshelby's tensor $\underset{\sim}{S}$ for sphere inclusion is computed as [8]:

$$S_{1111} = S_{2222} = S_{3333} = \frac{7 - 5\nu}{15(1 - \nu)}$$

$$S_{1122} = S_{2233} = S_{3311} = S_{1133} = S_{2211}$$
$$= S_{3322} = \frac{5\nu - 1}{15(1 - \nu)}$$

$$S_{1212} = S_{2323} = S_{3131} = \frac{4 - 5\nu}{15(1 - \nu)}.$$

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References

1. Mehta P.K.; Monteiro P.J.M. *Concrete: Structure, Properties, and Materials*; Prentice Hall: New Jersey, 1993.
2. Nilsen A.U.; Monteiro P.J.M.; Gjørv O.E. *Cem. Concr. Res.* **1994,** *24*, 1423–1427.
3. Chang T.P.; Su N.K. *J. Mater. ACI* **1996,** *93*, 3–9.
4. Swamy R.N.; Lambert G.H. *J. Int. Cem. Com. Light Concr.* **1983,** *5*, 263–275.
5. Mayfield B.; Louati M. Mag. Concr. Res. **1990,** *42*, 29–36.
6. Aitcin P.C.; Mehta P.K. *J. Mater. ACI* **1990,** *87*, 103–107.
7. Zhou F.P.; Lydon F.D.; Barr B.I.G. *Cem. Concr. Res.* **1995,** *25*, 177–186.
8. Mura T. *Micromechanics of Defects in Solids*; Martinus Nijhoff Publishers: Boston, 1987.
9. Mori T.; Tanaka K. *Acta Metall.* **1973,** *21*, 571–584.
10. Eshelby J.D. *Proc. Roy. Soc.* **1957,** *A241*, 376–396.
11. Yang C.C.; Huang R. *Cem. Concr. Res.* **1996,** *26*, 1567–1577.