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Rational Assessment of Flexural Fatigue Characteristics of Ferrocement for Reliable Design

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Abstract

*The shortcoming of a design method based on a stress-life (*S–N*) plot is pointed out and a new method based on a probability-stress-life (*P–S–N*) plot is proposed. Rectangular stress distribution is shown to be better for predicting steel stress when designing ferrocement against fatigue by using the* P–S–N *relationship of wire tested in the air. The inclusion of runouts in regression analysis is found to lead to more reliable* P–S–N *relationships.*

Keywords: Ferrocement, flexural fatigue, probabilistic design, regression analysis, runouts, stress distributions, steel stress, reliability, cracked section analysis.

INTRODUCTION

The authors have shown[1] that for a conservative design one can use the results of fatigue tests performed on the reinforcement in the air. This, of course, gives the advantage of not having to undertake time- and resource-consuming tests on the composite. A reliable design has to be based on (a) a reliable model for predicting steel stress in ferrocement and (b) a reliable S–N relationship. The authors have shown that the method for predicting steel stress based on a rectangular stress distribution assumption is more reliable, economical and simpler than that based on 'elastic cracked section analysis'. This paper is mainly concerned with a 'reliable S–N relationship'.

Fatigue in corrosive environments is particularly stochastic in nature. The scatter of fatigue test results, even in normal environments, is far more pronounced that that of static ones. Therefore, appropriate statistical and probability analyses are essential in order to present and evaluate the results meaningfully and use them in reliability based designs of infrastructure.[2] All other researchers in ferrocement have described fatigue behaviour of specimens by using S–N plots. It should be noted that a design based on the S–N equation implies that the designer accepts a failure probability of 50%, despite the fact that the fatigue life is highly stochastic.

Weibull,[3] therefore, pointed out that the probability of failure P has to be introduced and interconnected with the two other main quantities S and N. He developed a method for producing P–S–N relationships, the use of these relationships is justified and the effects of inclusion or exclusion of runouts are discussed.

A BRIEF REVIEW ON THE PREDICTION OF PEAK STEEL STRESS

It is generally accepted that the fatigue properties of ferrocement are primarily governed by those of the reinforcement. Most researchers have used elastic cracked section analysis to predict steel stress.[4–6]

Through a theoretical study[1] it can be seen that the peaks of steel stress at cracked sections decrease with the increase in the number of load cycles. The existing elastic cracked section analysis leads to an overestimation of steel stress. The overestimated value is found to approach the theoretical upper boundary. The authors have proposed a simple new method for predicting the value of the theoretical lower boundary of steel stress by the rectangular stress distribution assumption. As shown in Fig. 1, the depth of compressive mortar X is the first calculated by the following equation:

$$M=\sigma_c \times X \times b \times (h - t_s/2 - X/2 - C) \quad (1A)$$

where: M is the moment applied to the section; σ_c is the mortar stress, which is assumed to be equal to $0{\cdot}55 \times$ flexural compressive mortar strength[1,7] according to ACI Committee 215.[8]; b and h are width and depth of the cross-section, respectively; and t_s is the height of the reinforced zone.

After the X-value is determined, the steel stress σ_s can be easily calculated by the balance condition:

$$\sigma_s \times A_s = \sigma_c \times b \times X \quad (1B)$$

where A_s is the area of steel.

Test results of 108 specimens (Fig. 2), cured and stored in a normal environment, showed that this method is more reliable, economical and simpler than that based on elastic cracked section analysis.[1]

BRIEF DESCRIPTION OF TEST SERIES

Tensile fatigue tests on wire tested in the air

One batch of 6·35 × 6·35 × 0·71 mm diameter, galvanized, drawn square weldmesh was used. Pieces of weldmesh, 65 × 14 mm, were used as sources of static and fatigue specimens tested in the air by an INSTRON machine. The specimens were randomly chosen from the different rolls of mesh to ensure that the properties found from these tests were representative values. Both static and fatigue specimens were cut in such a way that only a single wire was left in the middle gauge. Both ends of the mesh were embedded in quick-set epoxy between two 35 × 20 × 1 mm thick steel plates to improve gripping. The elastic modulus, yield and ultimate wire strength were 139 000, 345 and 403 MPa, respectively, determined by static tests. For the 27 fatigue specimens, the maximum stress varied from 48 to 75% of the ultimate stress. The minimum cyclic stress stayed at 12·5% for all the tests. Tests were terminated at 2000 000 cycles. They were carried out at a frequency of 20 Hz.

Flexural fatigue tests on ferrocement specimens

The mix proportions in the mortar were 1 : 2·5 : 0·5 for OPC : sand : water, respectively. The ferrocement specimen size was 350 × 125 × 30 mm thick. Six layers of weldmesh (chosen from the different rolls of weldmesh, randomly) were used as reinforcement to give a nominal percentage of reinforcement of 1·20%. No skeletal steel was used and 5 mm spaces were fastened onto the first layer of reinforcement to obtain a nominal cover of 5 mm.

To simulate a sewerage environment a sulphuric solution of pH value 5·5 was chosen and put into a glass fibre tank for storing specimens at room temperature.[7] In order to ensure that the ferrocement specimens were subjected to severe corrosive conditions they were preloaded before placing into the storage tank. The specimens were predeflected to a level[7] which corresponded to an average load, equivalent to the maximum allowable steel stress (nominal) of 207 MPa, as defined

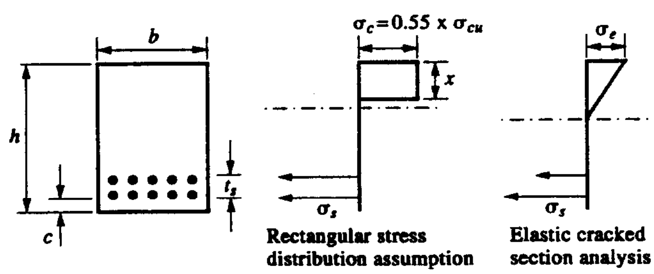

Fig. 1. Model for predicting steel stress.

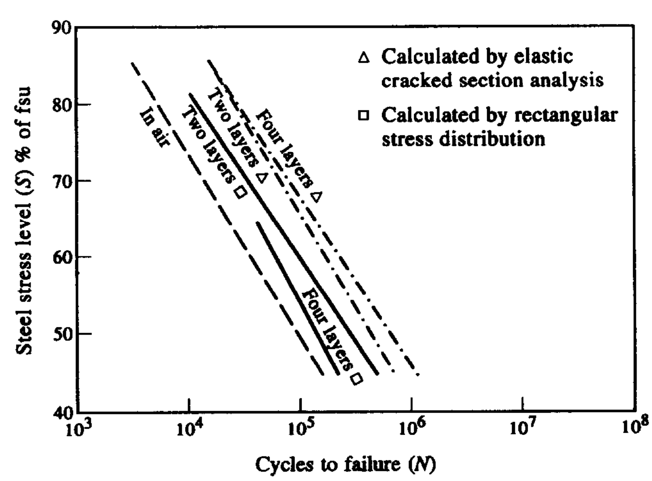

Fig. 2. P–S–N relationships. Ungalvanised weldmesh (1·6 mm diameter) in air and in ferrocement.

by ACI.[8] This was done not only to simulate crack width but also to simulate conditions of real-life structures where the chemical attack and the stress corrosion occur concurrently.

Thirty-five fatigue and nine static specimens were tested. They were divided into two groups (Table 1), one for each of the following conditions:

(1) 28 days of normal curing with no preloading (Group CN)
(2) 28 days of normal curing plus 36 months in a sulphuric environment (Group CR36M)

Each group of specimens was composed of three static and 15 fatigue specimens. For 15 fatigue specimens in Group CN, five specimens were tested for each maximum steel stress level (nominal) of 63·8, 57·8 and 54·7% of the ultimate steel strength, calculated by using the relatively reliable rectangular stress distribution method.[7] Noticing that six out of 15 specimens of Group CN were runouts, for 15 fatigue specimens in Group CR36M, five specimens were tested for each of the higher maximum steel stress levels (nominal) of 75 and 67%; three specimens were tested under a nominal maximum stress level of 65% and two specimens were subjected to a nominal maximum stress level of 60%. In addition, to act as a control, eight specimens (three for static test and five for fatigue test) were cured in a nominal curing environment (no preloading) up to the same age as the group of CR36M specimens in the sulphuric environment before being tested in fatigue. These fatigue control specimens are named CN36M. The minimum stress level (nominal) of 12·5% was used for all tests.

Tests were carried out at a frequency of 5 Hz under constant cyclic load by a TESTWELL Servo-Controlled System. A continuous spray of

Table 1. Fatigue test results of ferrocement in a sulphuric environment

Specimen group	*Cover thickness (mm)*	*Maximum steel stress level (%)* Nominal	Recalculated	Adjusted	*Failure cycle*
CN	5·5	54·7	57·2	57·1	2000000
	6·4	54·7	60·2	60·1	2000000
	6·5	54·7	60·3	60·2	2000000
	6·6	54·7	60·5	60·4	779660
	7·0	54·7	61·9	61·7	2000000
	6·2	57·8	62·6	62·5	2000000
	6·3	57·8	62·8	62·7	2000000
	6·6	57·8	64·1	64·0	581650
	7·5	57·8	67·3	67·1	147980
	8·5	57·8	71·2	70·9	101630
	6·1	63·8	68·6	68·5	70410
	6·7	63·8	70·9	70·7	37540
	6·8	63·8	71·2	71·0	121320
	7·3	63·8	73·4	73·2	13370
	8·4	63·8	78·5	78·2	8250
CN36M	5·5	67·0	68·6	68·7	411530
	6·5	67·0	71·2	71·1	104380
	6·5	67·0	72·6	72·5	42390
	7·5	67·0	77·2	76·9	31300
	9·0	67·0	85·4	85·0	7100
CR36M.2	5·5	60·0	61·6	61·5	2000000
	6·5	60·0	65·3	65·1	2000000
	6·0	65·0	68·8	68·7	47670
	7·0	65·0	73·1	72·8	105260
	7·0	65·0	73·1	72·8	4400
	5·0	67·0	67·0	67·0	323080
	6·5	67·0	72·9	72·8	32380
	7·0	67·0	75·2	75·0	33730
	8·5	67·0	82·9	82·5	23880
	9·0	67·0	85·9	85·4	6400
	5·0	75·0	75·0	75·0	373590
	8·0	75·0	90·8	90·6	1770
	8·0	75·0	90·8	90·6	4710
	8·5	75·0	94·2	94·0	2070
	9·0	75·0	98·0	97·9	1110

pH = 5·5 sulphuric acid solution was applied onto the specimens during the fatigue tests. Failure was defined as fracture of the outermost reinforcement. The specimens which did not fail after 2000 000 cycles were tested to failure under static loading.

After the specimens were broken either due to fatigue or static failure (of run-out), the cover to outermost reinforcement and the specimen thickness were measured. The reinforcement exposed at the fracture surface was examined for evidence of rust.

THE PROPOSED METHOD FOR PRODUCING *P–S–N* PLOTS

Equal steel stress levels for each individual specimen in every sub-group: one of the basic requirements for using Weibull's *P–S–N* method

Weibull[3] developed a model for producing *P–S–N* plots of tensile metal specimens. One of the basic requirements for using this method is *to arrange a sub-group of test data for a given stress level* in ascending order of value of fatigue life.

Adjusted steel stress levels for each individual ferrocement specimen in every sub-group: a necessary approach to a reliable estimate of the steel stress level

Although the test specimens are usually carefully prepared and fabricated, they always vary slightly in the overall thickness and the cover to reinforcement. Because the thickness of ferrocement is thin the influence of reinforcement location on the steel stress level is significant. A calculated result shows that the steel stress of a 30 mm thick specimen with an actual cover of 9 mm is 23% higher than that with a nominal cover of 5 mm for a given moment. As a result, the maximum and minimum steel stresses were recalculated with the measured thickness and reinforcement cover by using a rectangular stress distribution assumption.[7]

Because the actual back-calculated maximum and minimum stress levels vary for each individual specimen the authors suggest that the calculated maximum stress should be adjusted to give an equivalent minimum stress of 12·5% by using the modified Goodman diagram. The following equation is used:

$$S = f_{su} - (f_{su} - 0{\cdot}125 f_{su})/(f_{su} - \sigma_{min}) \times (f_{su} - \sigma_{max}) \quad (2)$$

where: S is the adjusted maximum steel stress, f_{su} is the ultimate strength of steel and σ_{max} and σ_{min} are maximum and minimum steel stress, respectively.

For CR36M specimens the adjusted steel stress is 22·9% higher than the nominal stress level in one case and the average difference is 9·78%; for CN specimens the average difference is 7·12%. These figures reflect the importance of doing the back-calculations and adjustment of stress levels. Because *adjusted steel stress levels vary for each of the specimens in a sub-group*, Weibull's method for producing *P–S–N* plots is not applicable to ferrocement in flexure. The authors have, therefore, adapted Weibull's method for the *P–S–N* relationship.

The proposed method for the *P–S–N* relationship for ferrocement in flexure

The proposed method for producing *P–S–N* plots is composed of three steps and is introduced through the following example. For this illustration the adjusted stress levels and numbers of load cycles at failure of one group of specimens are tabulated in Table 2.

Step one: establishing the S–N plot family

For this group of specimens, the slope of the *S–N* curve was established by using the least square method with stress as the dependent variable (because the variation coefficients of the equations using stress as the dependent variable are smaller than those using life as the dependent variable[6]). With this slope, one line for each point on the *S–N* graph is plotted (the line through the point). The intercepts of this set of parallel lines, with a line parallel to the *N*-axis at an arbitrarily chosen value of the maximum stress level (75%) of ultimate steel stress, are arranged in ascending order. Figure 3(a) shows how these lines were established for a group of specimens.

Step two: establishing the P–N plot

The probability of failure of each intercept is found by the empirical equation, as shown in Fig. 3(a). These probability values and corresponding values of fatigue life are plotted in the *P–N* coordinate system for creating the *P–N* regression equation, as shown in Fig. 3(b). This relationship is assumed to be linear because it furnished the most convenient and best fit to the data obtained so far.

Step three: establishing the P–S–N plot

The *P–S–N* relationship could the be determined with the points chosen from the *P–N* regression

Table 2. Fatigue test results of one group of ferrocement

Specimen No.	*Maximum steel stress level (%)*			*Failure cycle*
	Nominal	*Recalculated*	*Adjusted (S)*	
1	60·0	63·0	62·7	1349180
2	65·0	66·4	66·3	115060
3	65·0	66·4	66·3	202010
4	65·0	70·2	69·8	129380
5	70·0	70·0	70·0	97670
6	70·0	70·0	70·0	460160
7	70·0	74·2	74·1	337130
8	80·0	80·0	80·0	49920
9	80·0	86·9	86·7	61680
10	80·0	86·9	86·7	30630
11	80·0	95·3	95·2	4140

line (Fig. 3(b)) and the predetermined slope for the *S–N* curve. The 5, 50 and 95% probabilities of failure lines for the group of specimens are shown in the *P–S–N* plot (Fig. 3(c)). It should be noted that in using this method, the slopes for all probability lines are assumed to be the same. In reality this may not be strictly true.

P–S–N RELATIONSHIPS FOR WIRES IN AIR

The *P–S–N* regression equations and plots are presented in Table 3 and Fig. 4, respectively.

P–S–N RELATIONSHIPS FOR THE COMPOSITE

Table 1 shows the fatigue life for all ferrocement specimens together with their nominal, recalculated and adjusted maximum stress levels. In the table, steel stress is expressed as a percentage of ultimate strength. The adjusted maximum stress level *S* is used to set up *P–S–N* relationships. The regression *P–S–N* equations of ferrocement are presented in Table 3. The corresponding *P–S–N* lines are shown in Figs 5 and 6. Calculations using *P–S–N* equations for CRM36M specimens (with runouts) show that for a given steel stress level of 60% the fatigue lives based on failure probabilities of 5 and 50% are 284940 and 3068211, respectively. This results shows the importance of the establishment of *P–S–N* relationships.

It can be seen that the scatter of data is more pronounced for preloaded specimens (CR36M) than control specimens (CN). This may be because the storage, in a corrosive environment,

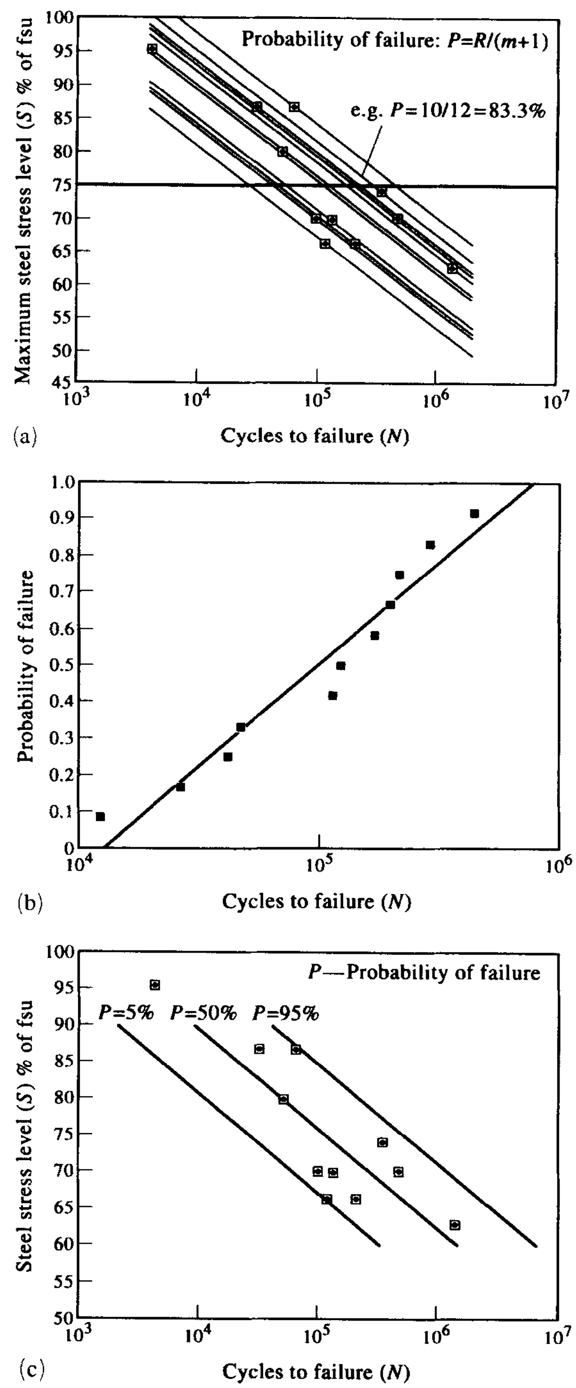

Fig. 3. (a) Establishment of *P–S–N* relationship; (b) *P–N* relationship for specimens; (c) *P–S–N* relationship for specimens.

of preloaded specimens contributed to an additional factor to the scatter of fatigue testing.

COMPARISON OF *P–S–N* RELATIONSHIPS BETWEEN FERROCEMENT COMPOSITES AND WIRE SPECIMENS

The *P–S–N* relationships of wire and air and in ferrocement with different curing conditions are

Table 3. Regression P–S–N equations[†]

Specimen	$P=5\%$	$P=50\%$
$WIRE_r$	$S=95{\cdot}100-8{\cdot}021\ \mathrm{Lg}\ N$	$S=104{\cdot}560-8{\cdot}021\ \mathrm{Lg}\ N$
$WIRE_n$	$S=87{\cdot}464-6{\cdot}241\ \mathrm{Lg}\ N$	$S=96{\cdot}360-6{\cdot}24\ \mathrm{Lg}\ N$
CN_r	$S=99{\cdot}160-6{\cdot}641\ \mathrm{Lg}\ N$	$S=102{\cdot}275-6{\cdot}641\ \mathrm{Lg}\ N$
CN_n	$S=102{\cdot}336-7{\cdot}311\ \mathrm{Lg}\ N$	$S=105{\cdot}385-7{\cdot}311\ \mathrm{Lg}\ N$
$CR36M_r$	$S=109{\cdot}911-9{\cdot}151\ \mathrm{Lg}\ N$	$S=119{\cdot}355-9{\cdot}151\ \mathrm{Lg}\ N$
$CR36M_n$	$S=112{\cdot}747-10{\cdot}031\ \mathrm{Lg}\ N$	$S=122{\cdot}707-10{\cdot}031\ \mathrm{Lg}\ N$

[†] P, S and N: probability of failure, adjusted steel stress level and failure cycle, respectively.
[‡] Subscripts r and n: with and without runouts, respectively.

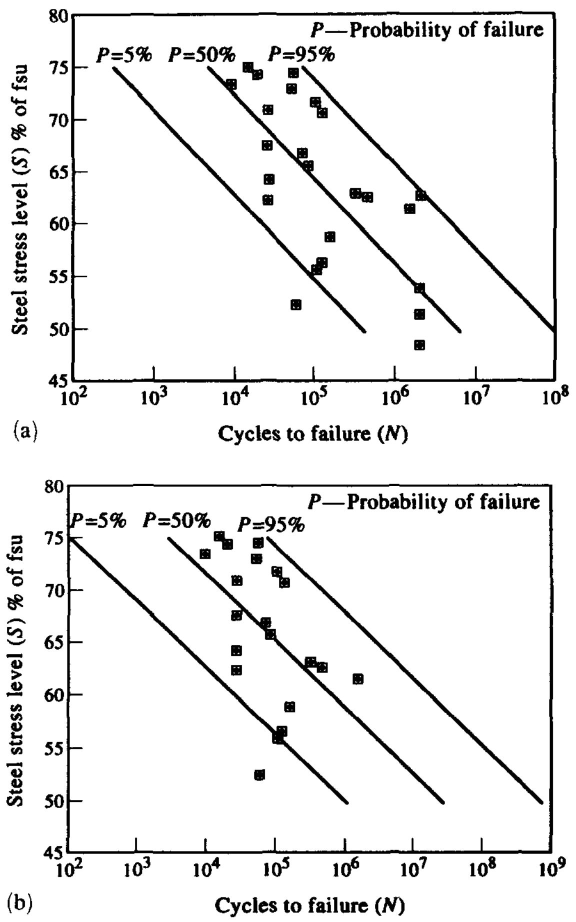

Fig. 4. (a) P–S–N relationship for wire specimens (with runouts); (b) P–S–N relationship for wire specimens (without runouts).

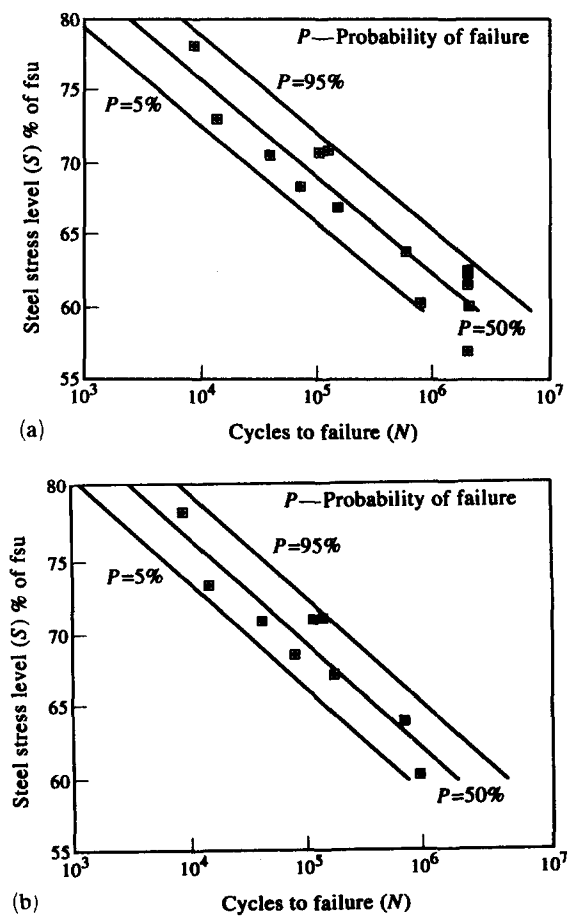

Fig. 5. (a) P–S–N relationship for CN specimens (with runouts); (b) P–S–N relationship for CN specimens (without runouts).

shown in Figs 7(a) and 7(b). The fatigue life of wires in the air is very significantly lower than those embedded in ferrocement specimens (either including or excluding runouts) even though steel stress is underestimated by the rectangular stress distribution assumption. When comparing the P–S–N relationships for ferrocement control specimens CN (Fig. 5) with the wire specimens (Fig. 4), it can be seen that there is a larger scatter of data for single-stand wire tested in air. The following factors may be responsible.

Stress distribution and failure location

The response of single wire to tensile fatigue loading is a rectangular stress distribution along the length of the wire (Fig. 8). Failure may take place at any section between two transverse wires.

Stress distribution of wire in mortar at the lower load cycle n_1 and higher load cycle n_2 is shown in Fig. 8.[1] The peak steel stress at cracked section decreases with the increase of load cycles. The bond failure zone of the steel–mortar interface increases with the increase in number of load

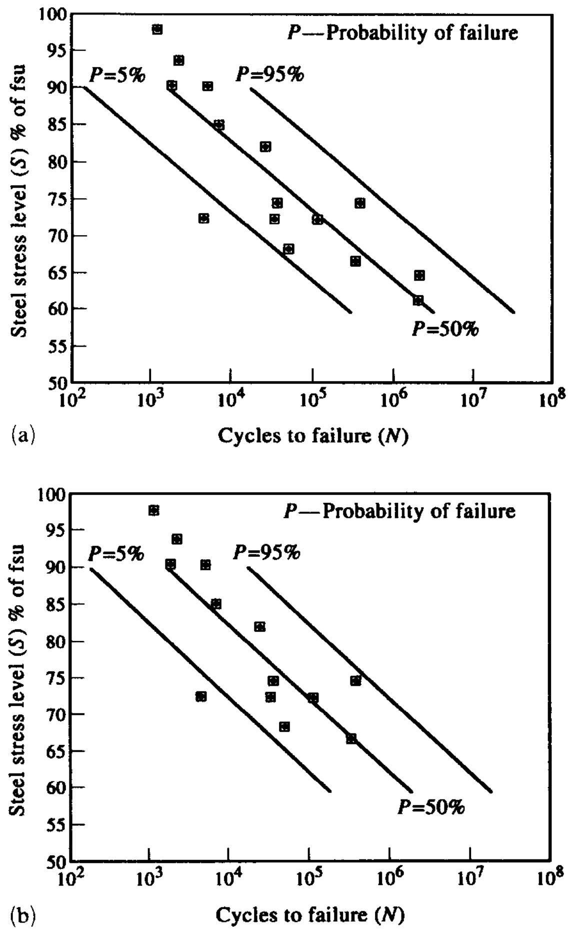

Fig. 6. (a) P–S–N relationship for CR36M specimens (with runouts); (b) P–S–N relationship for CR36M specimens (without runouts).

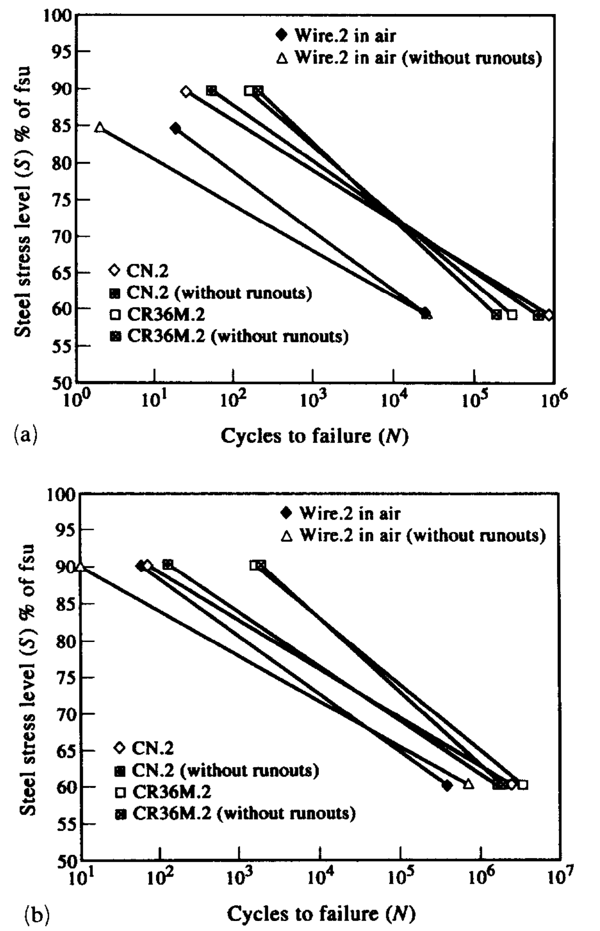

Fig. 7. (a) P–S–N relationship of wire in air and in ferrocement (P= 5%); (b) P–S–N relationship of wire in air and in ferrocement (P= 50%).

cycles. Only the wire sections within the bond failure zone are subjected to maximum stress. Failure generally appears within the bond failure zone. This results in a lower failure probability than wire tested in air, as verified by other test results obtained at Leeds, UK.[5,6] The CR36M ferrocement specimens have a longer fatigue endurance than CN control specimens (Figs 7(a) and 7(b)). One of the main reasons may be that the corrosion only developed in a limited thickness in the tensile zone of mortar so that the bond between mortar and steel continued to increase with the increase of storing age.[7] Corrosion effects, if any, were overtaken by improvement in bond due to longer storage.

Corrosion of materials

Mortar corrosion on its own has an insignificant influence on fatigue life. Calculations show that for a given moment, even if the flexural compressive strength decreases by 20%, the calculated steel stress increases only by about 2%. This

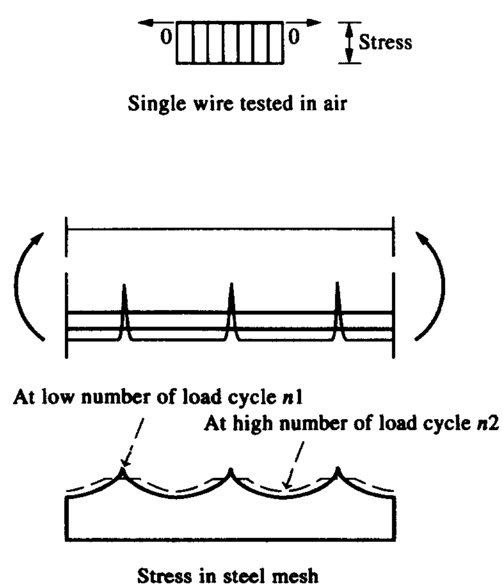

Fig. 8. Steel stress distribution at different load cycles.

Table 4. Mean values of the results of the test series for crushed gravel (water storage 28 days) (CG(wet))

Norm. compressive stress σ_l/f_C *(%)*	*Notched tensile strength* σ_{2max} *(N mm^{-2})*	*Spec. fracture energy* G_f *(N m^{-1})*	*Norm. spec. fracture energy* G_f/G_{f0} *(%)*	*Number of specimens (n)*
0	3·64	105·4	100·0	3
20	3·50	87·8	83·3	3
30	4·00	106·0	100·7	3
40	3·83	100·4	95·2	3
50	3·64	98·4	93·3	3
60	3·96	102·2	97·0	3
70	3·67	111·5	105·8	3
80	2·49	119·4	113·3	3

is because the change in compressive strength leads only to small change in the lever arm at the cracked section.[7]

Steel corrosion has a significant influence on fatigue life. Test results[9] in an accelerated marine environment for up to one and a half years (8640 cycles of 70°C hot-dry and 40°C warm-immersed exposure) showed that the frequency of red rust on wire at the broken section was 20–100% for all five specimens with actual covers of 5–7 mm. Severe corrosion of wire in uncracked parts was also observed where the cover was stroked and removed. The corrosion of steel resulted in a great loss of fatigue life. The fatigue endurance of specimens in this accelerated marine environment is not only significantly lower than that of specimens in normal or sulphuric enviroments but also lower than that of wire tested in air. This can be explained on three accounts. Firstly, the reduction of cross-section of steel due to corrosion led to actual higher steel stress than the predicted value. Secondly, the S–N relationship of corroded wire actually followed a corrosion fatigue mechanism. Thirdly, debonding of mortar and steel due to corrosion increased the total volume of the steel which is subjected to the maximum level of stress in the composite.

Based on the above and other large experimental studies conducted at Leeds, UK[5,6] it can be seen that the use of the fatigue behaviour of wire tested in air to predict fatigue behaviour of ferrocement will lead to a conservative design unless steel corrosion takes place.

DISCUSSION OF THE PLOTS WITH SPECIAL REFERENCE TO RUNOUTS

For each group, the P–S–N relationships, including and excluding runouts, are compared in Figs 7(a) and 7(b). It can be seen that the result obtained from one case is different from the other. Therefore, one needs to choose between the two cases for a sensible use of P–S–N relationships.

When referring to P–S–N curves of CN and CR36M, including runouts in Figs 7(A) and 7(B), it can be inferred that those lines would move towards the right if the test did not stop at load cycle 2 000 000 for runouts. In this case the curve excluding runout leads to an unnecessarily conservative design. However, some test results[6,7] do show that P–S–N plots excluding runouts locate on the right side of the plot including runouts. In this case the curve excluding runouts leads to a nonconservative design. The authors suggest that the curve including runouts should be used for a reliable design.

CONCLUSIONS

The use of S–N relationships for predicting fatigue behaviour leads to an unreliable design, however, the proposed method for producing P–S–N relationships, incorporating the probability aspects, is more appropriate. Also, the inclusion of runouts in the regression analysis results in a reliable P–S–N relationship.

Finally, a rectangular stress distribution is relatively more reliable and economical for predicting steel stress when designing ferrocement against fatigue by using the P–S–N relationship of wire tested in the air.

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