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Effect of Fibers on the Punching Shear Strength of Slab-Column Connections

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Abstract

Twelve sets of small scale fiber reinforced concrete (FRC) slab-column connections were tested to investigate the effect of fiber reinforcement on the punching shear resistance of flat slabs. The parameters studied included type, content and aspect ratio of fibers, and span-depth ratio of the slab.

The results indicate that the use of hooked steel fibers improves substantially the ductility of shear failure of slab-column connections, modifies their failure mode, favorably, from pure punching to flexural, and leads to a significant increase in their ultimate shear capacity. The increase in ultimate shear resistance varied almost linearly with the steel fiber content. While polypropylene fibers increase the ductility of shear failure, they are not as effective as steel fibers in increasing the punching shear resistance.

Based on evaluation of the current test results and other experimental data reported in the technical literature, a design equation is proposed to predict the ultimate punching shear strength of slab-column connections containing deformed steel fiber reinforcement.

Key words: columns, connections, ductility, fiber reinforced concrete, flat slabs, punching shear, reinforced concrete, slabs.

NOTATION

b_0	Perimeter of critical shear surface
d	Depth to center of reinforcing steel
d_f	Diameter of fiber
f'_c	Cylindrical concrete compressive strength
L	Length of fibers
P_u	Measured ultimate punching load
V_c	Ultimate punching shear load of conventional slabs
V_{cf}	Ultimate punching shear load of fiber slabs
V_f	Volume fraction of fibers
$V_f L/d_f$	Fiber reinforcing index
ΔP_u	Incremental increase in punching shear load

INTRODUCTION

The design of flat-slab systems such as flat slabs, flat-plate slabs, or mat foundation, is normally governed by their resistance to punching shear at the slab-column connections. Punching shear failure is generally very brittle, it occurs without warning, and therefore should be avoided. Indeed, the general design philosophy of North American building codes[1,2] is that whenever overloaded, flexural members should be able to develop a yield mechanism such as to fail in a ductile flexural mode before failing in shear.

Fiber reinforcement had long been known to enhance the material and mechanical properties of concrete by controlling crack propagation.[3-6] This had inspired some investigators to study the possibility of improving the punching shear resistance of flat slabs by using steel fiber reinforcement in the vicinity of the slab-column connections. A number of experimental studies undertaken in this regard[7-10] have shown that using moderate densities of mainly crimped and corrugated steel fiber reinforcement (between 0·5 and 1·2% by volume fraction) leads to a significant improvement in the ductility of shear failure and energy absorption properties; increases the ultimate punching shear strength by up to 40%; and enhances substantially the load-carrying capacity and behavior of the slabs under service loads.

Although the use of fiber reinforcement had been recognized as being a 'viable alternative' and an 'economical solution' for increasing the punching shear capacity of slab–column connections, experimental studies in this area are still relatively limited. Also, while the means by which fiber reinforcement increases the punching shear capacity of slabs had been described recently by Alexander and Simmonds,[9] no direct relationship is currently available to estimate the punching shear strength of concrete slabs as a function of fiber parameters.

This paper presents the results of an experimental study to evaluate the effect of fiber reinforcement on the punching shear strength and behavior of slab–column connections. Two types of fibers were used, namely, hooked steel and polypropylene fibers. The experimental results are discussed and available data on the ultimate punching shear resistance of slab–column connections reinforced with different types of steel fibers are compared and analysed. Based on the analysis, a simple design equation is proposed to predict the increase in ultimate punching shear strength that may be expected due to the presence of 'deformed' steel fibers.

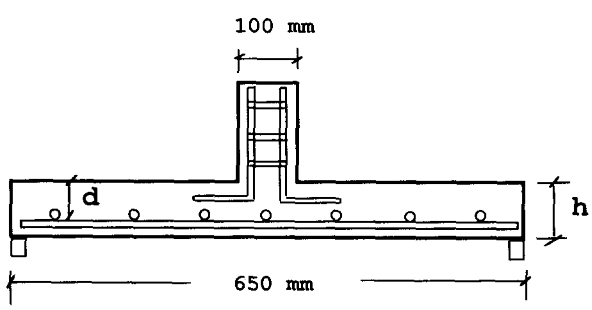

Fig. 1. Typical dimensions and reinforcement details of the specimens.

EXPERIMENTAL PROGRAM

Test specimen and materials

The test specimens consisted of a 650 × 650 mm (25·6 × 25·6 in) small scale isolated slab–column connection. The specimens were simply supported over 40 mm (1·6 in) wide supports along all four edges, thus, allowing the corners to lift during load application. The load was applied axially through a 100 × 100 mm (4 × 4 in) square column stub cast monolithically at the center of the slab. Two slab thicknesses were studied, namely, 55 mm (2·2 in) and 75 mm (3 in). Considering the simple supports as lines of contraflexure, the selected slab thicknesses correspond to span–depth ratios of the prototype slab of about 26 and 18, respectively.

Typical dimensions of the specimens are shown in Fig. 1. Input parameters, reinforcement details and specimen designation are summarized in Table 1. The specimens were grouped, as shown in Table 1, into two series (A and B), depending on their thickness. The ordinary reinforcement ratio and fiber parameters (type volume fraction)

Table 1. Summary of test variables

Series	*Slab*	*Fiber reinforcement*			*Concrete compressive strength (MPa)*	*No. of slabs tested*
		Fiber type	*Fiber volume fraction (%)*	*Fiber aspect ratio*		
A	A1	–	0·0	–	29·6	2
$h = 55$ mm	A2	Steel	0·45	100	30·0	2
$d = 39$ mm	A3	Steel	0·8	100	31·4	2
Ordinary	A4	Steel	1·0	60	24·6	2
reinf. = 5 (10 mm)	A5	Steel	2·0	60	20·0	2
deformed bars	A6	Polypr.	1·0	0·5 in long	32·2	2
B	B1	–	0·0	–	31·4	2
$h = 75$ mm	B2	Steel	0·45	100	31·4	2
$d = 55$ mm	B3	Steel	0·8	100	31·8	2
Ordinary	B4	Steel	1·0	60	29·1	2
reinf. = 7(10 mm)	B5	Steel	2·0	60	29·2	2
deformed bars	B6	Polypr.	1·0	0·5 in long	34·1	2

of the slabs in Series A were similar to those in Series B. Two identical specimens were tested for each input variable to reduce the scatter in the results.

The concrete mix was designed to achieve a 28 day compressive strength of 27·6 MPa (4000 psi). Ordinary Portland cement was used with washed sand and graded crushed limestone aggregate having 10 mm (1/2 in) maximum size. The cement:sand:aggregate proportions by weight were 0·33:0·5:1. No additives or superplasticisers were used. The concrete compressive strength of each slab was determined using standard 152 × 305 mm (6 × 12 in) cylinders taken before adding fibers. The compressive strengths f'_c for the various specimens at the time of testing are given in Table 1.

The fiber reinforcement used consisted of loose 30/50 hooked steel fibers (length = 30 mm, diameter = 0·5 mm), collated 50/50 hooked steel fibers and 12·5 mm (1/2 in) long monofilament polypropylene fibers. The densities of the steel fibers by volume fraction of concrete used were 1 and 2% for the 30/50 fibers and 0·45 and 0·8% for the 50/50 fibers. These densities correspond to fiber contents of 80 kg m^{-3} (135 lb yd^{-3}), 160 kg m^{-3} (270 lb yd^{-3}), 35 kg m^{-3} (60 lb yd^{-3}) and 64 kg m^{-3} (108 lb yd^{-3}), respectively. The polypropylene fibers were added in 1% by volume fraction, which corresponds to a fiber content of 8·8 kg m^{-3} (15 lb yd^{-3}).

Grade 60 deformed 10 mm (No. 3) bars with an actual yield strength of 501 MPa (72·7 ksi) were used as the ordinary reinforcement. The average effective depth of the reinforcing steel was 39 mm (1·54 in) for the slabs in Series A and 55 mm (2·2 in) for the slabs in Series B, respectively. A relatively high reinforcement ratio equal to 1·12% was selected for all specimens so that they develop punching shear failure before attaining their nominal flexural resistance.

Test procedure

The specimens were mounted on four separate concrete pedestals and loaded centrally through the column stub using MTS testing machine. A stroke control test at a slab central deflection rate of about 0·5 mm min^{-1} was selected for loading. Central deflection of the slabs were measured using a linear variable differential transducer (LVDT). Measurements were recorded automatically using a computerized data acquisition system without interrupting the load application. The entire load–deflection response was also monitored using an X–Y recorder. All specimens were loaded beyond the peak load to determine the influence of fibers on the ductility of shear failure.

Measurements of the angle at which the shear cracks propagated away from the face of the column were also taken at the end of each test. Crack patterns and failure modes of each specimen were inspected carefully.

DISCUSSION OF TEST RESULTS

Load–deflection plots for the specimens in Series A and Series B are shown in Figs 2 and 3, respectively. Failure modes and observed ultimate punching shear resistance for the various specimens are summarized in Table 2 along with the ACI Building Code predictions. The results given represent the average of two specimens tested and are normalized to $b_0 d\sqrt{f'_c}$ where d is the average effective depth of the ordinary reinforcing steel and b_0 is the perimeter of the critical shear failure surface taken at a distance $d/2$ away from the column face.

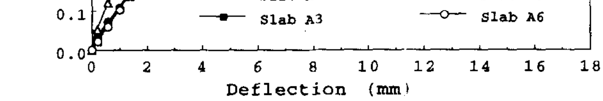

Fig. 2. Normalized load–deflection behavior of specimens in Series A.

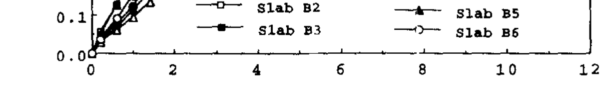

Fig. 3. Normalized load–deflection behavior of specimens in Series B.

Table 2. Summary of test results

Series	Slab	Failure mode	Normalized ultimate shear strength $P_u/(b_0 d\sqrt{f'_c})$ (SI)		
			Test[†]	Calcul.[†] (ACI)	Test/Calc.
A	A1	Punching	0·53	0·33	1·61
$h = 55$ mm	A2	Punching	0·57	0·33	1·73
$d = 39$ mm	A3	Flexural	0·64	0·33	1·94
Ordinary	A4	Flex-punch	0·64	0·33	1·94
reinf. = 5 (10 mm)	A5	Flexural	0·64	0·33	1·94
deformed bars	A6	Punching	0·53	0·33	1·61
B	B1	Punching	0·52	0·33	1·58
$h = 75$ mm	B2	Punching	0·60	0·33	1·82
$d = 55$ mm	B3	Punching	0·61	0·33	1·85
Ordinary	B4	Punching	0·64	0·33	1·94
reinf. = 7 (10 mm)	B5	Punching	0·79	0·33	2·39
deformed bars	B6	Punching	0·60	0·33	1·82

†To convert to SI units, multiply by 12·03.

All test specimens, except Specimens A3, A4 and A5 in Series A, showed clear evidence of punching shear failure. Shear failure was characterized by cracks forming around the perimeter of the column followed by the column stub punching through the slab. Representative crack patterns for the slabs in Series A and B at failure loads are shown in Figs 4 and 5.

The results summarized in Table 2 clearly demonstrate that the ultimate punching shear strength of slab-column connections generally increases with increasing volume fraction of steel fibers. Adding hooked steel fibers in 1 and 2% by volume fraction increased the average ultimate punching resistance in comparison with the conventional slabs by about 22 and 36%, respectively. The corresponding increase was not very sensitive to the span-depth ratio of slabs (compare results of Series A and B).

For the range used in this test, the fiber aspect ratio (ratio of length-diameter of fibers) does not seem to influence much the punching shear strength of the slabs. For instance, Specimens A3 and A4 in Series A, as well as Specimens B3 and B4 in Series B, which have approximately equal fiber densities but much different fiber aspect ratios, mobilized almost equal ultimate shear resistance and their failure mode was approximately similar.

Slabs without fibers exhibited a nearly square failure surface. However, as the steel fiber volume fraction increased from 0·45 to 2%, the failure surface changed to a nearly circular shape (Figs 4 and 5). Furthermore, the presence of fibers resulted in stretching the failed surface on the tension face of the slab away from the column face, thereby increased their punching shear resistance. The distance from the outer edge of the failed surface to the column face varied betwen $2{\cdot}30 \times h$ and $2{\cdot}82 \times h$ (h = slab thickness) for slabs containing fiber reinforcement in both Series A and B as compared to $1{\cdot}82 \times h$ for the conventional slabs.

Comparing the load-deflection behavior of slabs with fibers to those without in both Series A and B, as shown in Figs 2 and 3 it can be observed that the descent in the load-deflection response beyond the ultimate load is far more gradual for the FRC slabs as opposed to the sudden and significant loss in the shear resistance encountered in the conventional slabs (Specimens A1 and B1). In general, the ductility of shear failure increased as the volume fraction of steel fibers increased.

The presence of relatively moderate to large hooked steel fiber contents modified the failure mode, favorably, from pure punching to either pure flexural mode (Specimens A3 and A5) or to mixed punching-flexural mode (Specimen A4). This observation is evident from the crack pattern of the corresponding Specimens presented in Fig. 4, which shows flexural yield lines extending radially from each corner of the column stub towards the corners of the slab to form a yield-line mechanism. It is also clear from the load-deflection response of Specimens A3, A4 and A5, shown in Fig. 2, which is typical of a ductile reinforced concrete structural member undergoing flexural failure. Because of their relatively small span-depth ratio and the associated large shear deformation, none of the slabs in Series B experienced a change in the mode of failure as encountered in the slabs of Series A.

As seen in Table 2, adding polypropylene fibers in as large as 1% by volume fraction increased the ultimate punching shear resistance of the conventional slab of Series B (Specimen B6) only by about 15% and had practically no effect on the shear resistance of the slab in Series A (Specimen A6). However, the ductility of punching failure as seen from the descending branch of the load-deflection response of Specimens A6 and B6 given in Figs 2 and 3, respectively, increased due to the presence of polypropylene fibers. Therefore, while the use of relatively large densities of polypropylene fibers improves the ductility of shear failure of concrete slabs, their ultimate punching shear resistance is not likely to increase. More experimental data are needed, however, to support this observation.

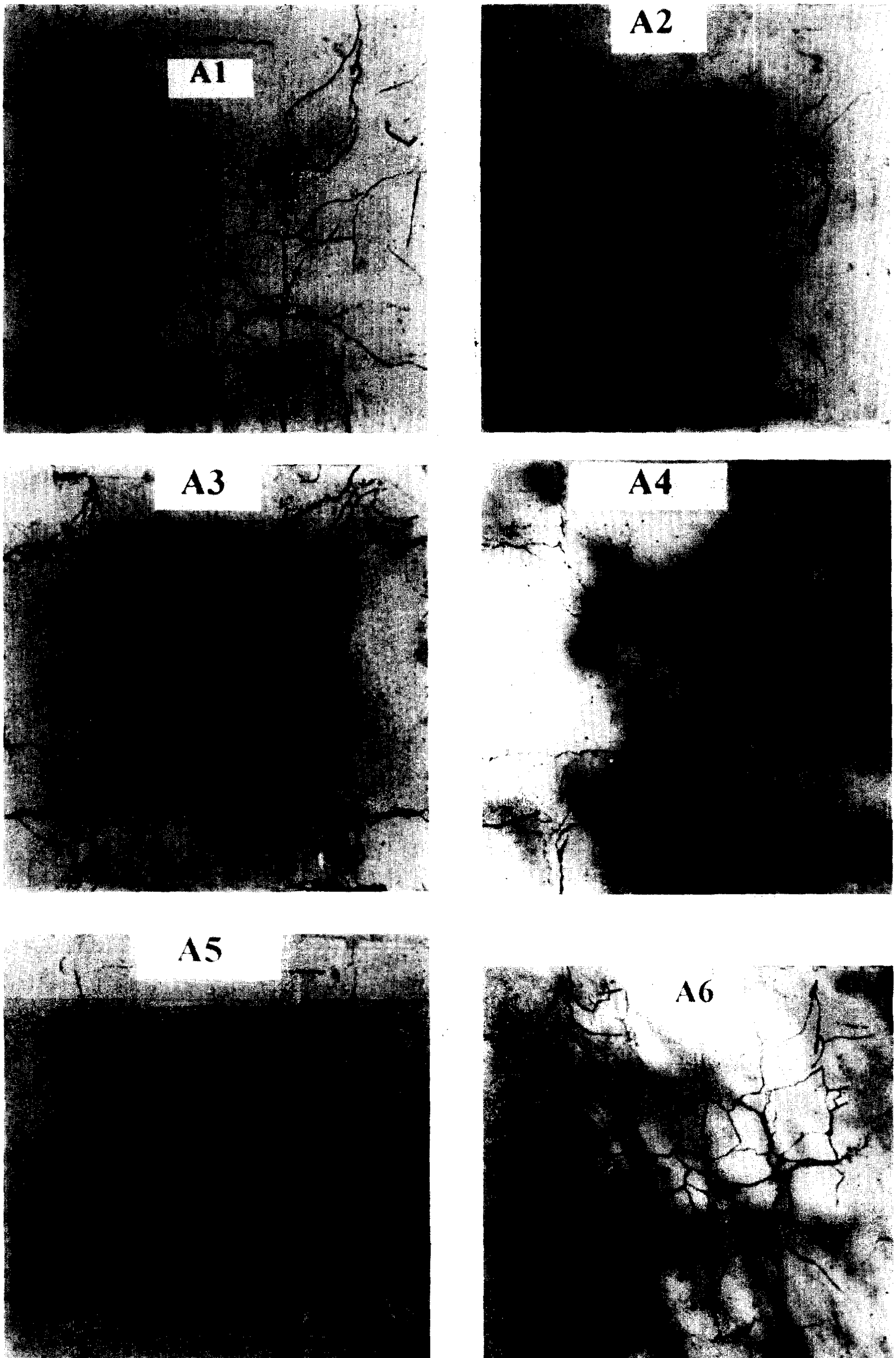

Fig. 4. Crack pattern at the tension face of the specimens in Series A.

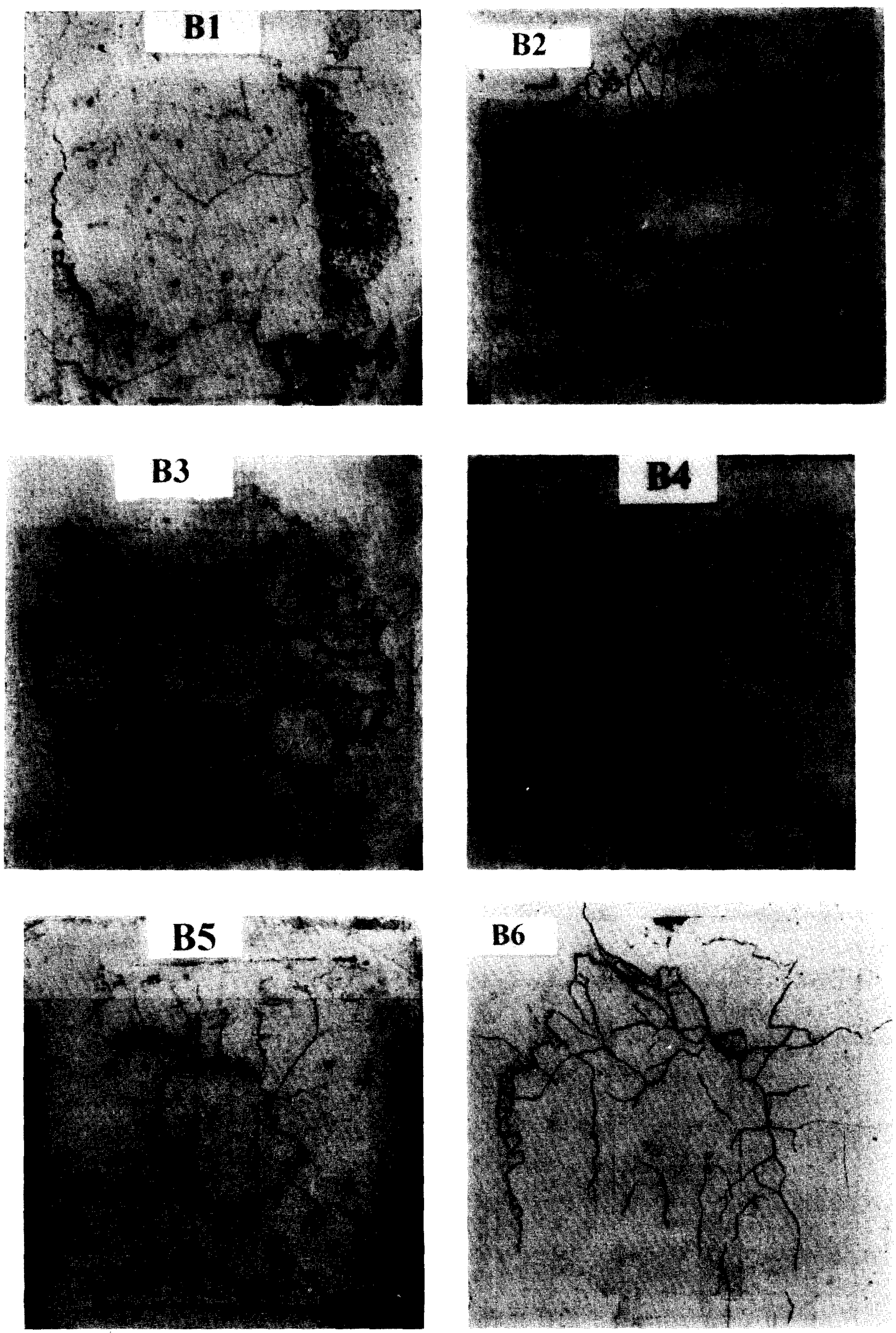

Fig. 5. Crack pattern at the tension face of the specimens in Series B.

ANALYSIS OF TEST RESULTS

The influence of hooked steel fibers on the ultimate punching shear strength of the slab–column connections tested in the current investigation is best illustrated by plotting the normalized ultimate punching load $P_u/(b_0 d\sqrt{f'_c})$ for all the specimens tested as a function of two simple fiber parameters: volume fraction of fibers V_f and fiber reinforcing index $V_f L/d_f$. The results are shown in Figs 6 and 7, respectively. The fiber volume fraction and fiber reinforcing index are commonly used parameters to describe the effect of fiber reinforcement on the structural and material properties of fiber reinforced concrete.

It is interesting to observe, in Figs 6 and 7, that the normalized ultimate punching shear capacity varies almost linearly with volume fraction of fibers and fiber reinforcing index. Nevertheless, the linear trend is seen to be more consistent with volume fraction of fibers as compared to fiber reinforcing index.

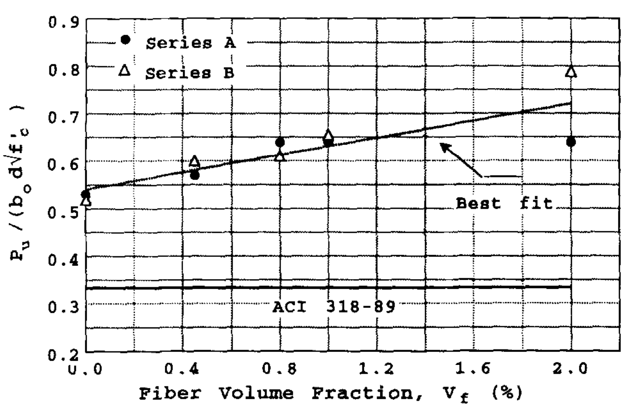

Fig. 6. Variation of normalized ultimate punching shear strength with volume fraction of steel fibers.

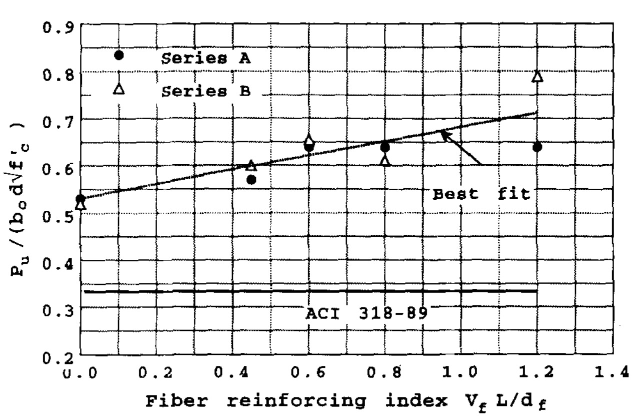

Fig. 7. Variation of normalized ultimate punching shear strength with steel fiber reinforcing index.

The experimental results obtained in the current study can be described using the following best fit linear equation with a correlation factor equal to 0·74 (see Fig. 6):

$$P_u(\mathrm{N}) = (0{\cdot}54 + 0{\cdot}09\,V_f)\,b_0 d\sqrt{f'_c} \qquad (1)$$

where V_f is in %, b_0 and d are in mm and f'_c is in N mm^{-2}.

Equation (1) is limited to the type of steel fibers and slab geometry used in current investigation. In an attempt to obtain a more general representation of the effect of fiber reinforcement on the ultimate punching shear strength of slab–column connections, the current results were analysed along with test data reported by Swamy and Ali,[7] Theodorakopoulous and Swamy,[8,10] using lightweight concrete, and Alexander and Simmonds.[9]

In the above mentioned studies, the specimens tested were full scale slab–column connections with span–depth ratios of the prototype slab equal to 32 (Refs 7, 8, 10) and 40 (Ref. 9). Several variables were studied in addition to fiber reinforcement, including concrete cover (Ref. 9), distribution of ordinary and fiber reinforcement, area of ordinary tension and compression reinforcement, type of concrete (normal weight and lightweight), concrete compressive strength, and size of column or loaded area (Refs. 7, 8 and 10). A summary of relevant information related to these experiments is provided in Table 3. More details on the experimental programs and test variables are found in the aforementioned studies.

In order to bypass the influence of several parameters studied by different investigators as mentioned above and reflect only the effect of fiber reinforcement, comparison was made of the measured increase in the normalized ultimate punching shear strengths $[P_u/(b_0 d\sqrt{f'_c})]$ of the specimens containing steel fibers relative to those of identical specimens tested without fibers (control specimens). The corresponding normalized incremental increase, termed $\Delta P_u/(b_0 d\sqrt{f'_c}$, is summarized for the various specimens in column 8 of Table 3 and plotted for the combined experimental results as functions of V_f, as shown in Fig. 8. Notice that only specimens containing deformed steel fibers were considered in the comparison since experimental data using plain steel fibers is very limited. The effect of plain steel fibers on the punching shear strength of slabs was reported by Swamy and Ali,[7] using the results of one specimen, to be inferior to crimped or hooked fibers.

Table 3. Summary of combined experimental results

Reference	*Type of concrete*	*Specimen designation*	*Type of steel fiber*	*Fiber aspect ratio*	*% fiber volume fraction*	*Mode of failure*	*Normalized increase in ultimate 'punching' load*
Swamy & Ali[7]	Normal weight	S-2	Crimped	100	0·6	Punching	0·074
		S-3	Crimped	100	0·9	Punching	0·105
		S-4	Crimped	100	1·2	Punching	0·134
		S-11	Crimped	100	0·9	Punching	0·065
		S-12	Hooked	100	0·9	Punching	0·044
		S-18	Crimped	100	1·37	Punching	0·070
		S-9	Crimped	100	0·9	Flexural	0·078
		S-10	Crimped	100	0·9	Flexural	0·116
		S-16	Crimped	100	0·9	Flexural	0·132
Theodorakopoulos & Swamy[8]	Light-weight	FS-2	Crimped	100	0·5	Punching	0·094
		FS-3	Crimped	100	1·0	Punching	0·122
Alexander & & Simmonds[9]	Normal weight	P11F31	Corrugated	0·5 in long	0·4	Flex-punch	0·054
		P11F66	Corrugated	0·5 in long	0·84	Flex-punch	0·078
		P38F34	Corrugated	0·5 in long	0·43	Flex-punch	0·053
		P38F69	Corrugated	0·5 in long	0·88	Flex-punch	0·080
Theodorakopoulos & Swamy[10]	Light-weight	FS-2	Crimped	100	0·5	Punching	0·094
		FS-3	Crimped	100	1·0	Punching	0·123
		FS-5	Crimped	100	1·0	Punching	0·089
		FS-9	Crimped	100	1·0	Punching	0·140
		FS-11	Crimped	100	1·0	Flexural	0·110
		FS-12	Japanese	60	1·0	Flex-punch	0·070
		FS-13	Hooked	100	1·0	Flex-punch	0·120
		FS-14	Paddle	70	1·0	Flex-punch	0·110
		FS-15	Crimped	90	1·0	Punching	0·130
		FS-16	Paddle	70	1·0	Punching	0·140
		FS-17	Paddle	70	1·0	Flexural	0·150
		FS-18	Paddle	70	1·0	Punching	0·150
Current results	Normal weight	A2	Hooked	100	0·45	Punching	0·040
		A3	Hooked	100	0·8	Flexural	0·110
		A4	Hooked	60	1·0	Flex-punch	0·110
		A5	Hooked	60	2·0	Flexural	0·110
		B2	Hooked	100	0·45	Punching	0·080
		B3	Hooked	100	0·8	Punching	0·090
		B4	Hooked	60	1·0	Punching	0·120
		B5	Hooked	60	2·0	Punching	0·270

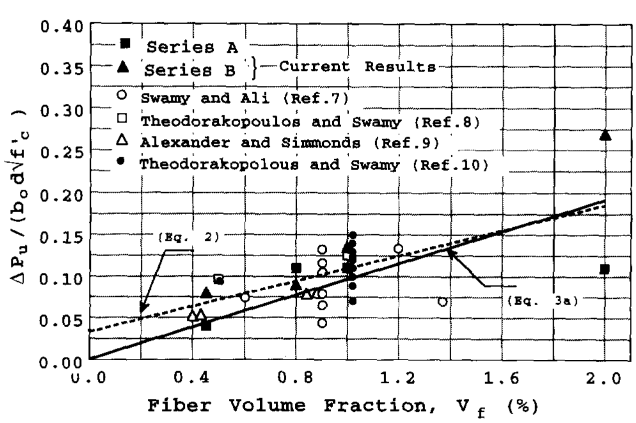

Fig. 8. Variation of normalized ultimate punching shear strength for the combined experimental results with volume fraction of steel fibers.

Table 3 and Fig. 8 clearly illustrate that the $\Delta P_u/(b_0 d\sqrt{f'_c})$ results vary almost linearly with the steel fiber content as suggested earlier in eqn (1). A similar plot of $\Delta P_u/(b_0 d\sqrt{f'_c})$ of the combined experimental results in function of fiber reinforcing index $V_f L/d_f$ resulted in a comparatively large scatter and much less defined trend to that shown in Fig. 8.

It should be emphasized that the scatter shown in Fig. 8 is relatively small despite the simplicity of the fiber parameter V_f used and the large number of parameters (fiber and non-fiber) explored by the different investigators. It is evident from Table 3 and Fig. 8 that, expressed incrementally as proposed, the variation of $\Delta P_u/(b_0 d\sqrt{f'_c})$ results with fiber volume fraction V_f is not very sensitive to variations in the type of concrete materials (normal weight, lightweight aggregates), specimen size or span to depth ratio of slabs and area of ordinary reinforcement provided. Also, it is practically independent of the type or geometry of the deformed steel fibers used. For instance, the average value of $\Delta P_u/(b_0 d\sqrt{f'_c})$ of the experimental results of Theodorakopoulos and Swamy,[10] in which a variety of deformed steel fibers were used in 1% by volume fraction (see Table 3), is about 0·1165 with a standard deviation equal only to 0·024. The corresponding average value is almost equal to that for hooked steel fibers measured in the current investigation.

PREDICTION OF PUNCHING SHEAR STRENGTH

The experimental results depicted in Fig. 8 can be described using the following best fit linear equation (correlation factor = 0·41):

$$\Delta P_u(\mathrm{N}) = (0{\cdot}033 + 0{\cdot}075\,V_f)\,b_0 d\sqrt{f'_c} \quad \text{(SI units)} \qquad (2)$$

in which b_0 and d are in mm and f'_c in N mm^{-2}.

A linear relationship to the results shown in Fig. 8 with zero y-intercept (as it should be) and with a reduction factor $\phi = 0{\cdot}9$ leads to the following reasonably safe prediction equation (see Fig. 8):

$$\Delta P_u(\mathrm{N}) = (0{\cdot}096\,V_f)\,b_0 d\sqrt{f'_c} \quad \text{(SI units)} \qquad (3a)$$

Equation (3a) takes the following form when expressed in Imperial units:

$$\Delta P_u(\mathrm{lb}) = (1{\cdot}15\,V_f)\,b_0 d\sqrt{f'_c} \qquad (3b)$$

where b_0 and d are in inches and f'_c is in psi. Equation (3) is limited to a volume fraction of fibers less than 2% and to the type of steel fibers summarized in Table 3 (mainly crimped, hooked, corrugated and paddle).

For design purposes, the absolute punching shear capacity of steel FRC slabs can be obtained by adding the ultimate shear resistance of conventional slabs (without fibers) and the incremental increase in shear strength due to the presence of fibers calculated using eqn (3). For instance, using the ACI Building Code approach, which is based in part on the work of Moe,[11] the ultimate punching shear resistance V_c for conventional reinforced concrete slab–column connections made with normal weight concrete can be expressed as:

$$V_c(\mathrm{lb}) = \zeta b_0 d\sqrt{f'_c} \qquad (4)$$

where (using imperial units): $\zeta = 2 + 4/\beta_c$, $\alpha_s d/b_0 + 2$ or 4·0, whichever is smallest; β_c = ratio of long side–short side of column; $\alpha_s = 40$ for interior columns, 30 for edge columns and 20 for corner columns. Adding eqn (3b) to eqn (4) leads to:

$$V_{cf}(\mathrm{lb}) = (\zeta + 1{\cdot}15\,V_f)\,b_0 d\sqrt{f'_c} \qquad (5)$$

in which V_{cf} is the ultimate punching shear strength of steel fiber concrete slab. For square interior columns ($\zeta = 4$), eqn (5) predicts an increase in the ultimate punching shear capacity over conventional slabs equal to 15 and 30% for volume fraction of fibers equal to 0·5 and 1%, respectively.

CONCLUSIONS

The following observations and conclusions can be derived based on the results of this study:

(1) Steel fibers increase the ultimate punching shear resistance of slabs. Adding hooked steel fibers in up to 2% by volume fraction increased the ultimate shear resistance of the slabs by about 36%.

(2) The increase in punching shear load due to the presence of steel fibers is controlled primarily by the volume fraction of steel fiber used and is independent of the length or aspect ratio of fibers.

(3) The presence of steel fibers in slabs not only leads to a considerable improvement in the ductility of shear failure but also may modify the failure mode from pure punching to a pure flexural or to a combined shear–flexural mode.

(4) Steel fibers decreased the angle of shear failure plane of the slabs and hence pushed the failure surface away from the column face. This resulted in increasing their punching shear resistance. The distance from the outer edge of the cracked surface to the column face varied in the current experiment between $2{\cdot}30 \times h$ and $2{\cdot}82 \times h$ (h = slab thickness), in steel fiber slabs, as compared to $1{\cdot}82 \times h$ in conventional slabs.

(5) Despite improving ductility and energy absorption of the slabs in the post-failure range, polypropylene fibers were relatively inferior to hooked steel fibers in increasing the punching shear resistance. More research is needed, however, to support this observation.

(6) From comparison and evaluation of available experimental data, the type of concrete materials (normal weight, lightweight aggregate), the area of ordinary reinforcement, concrete cover and the geometry of deformed fiber, have practically no signifi-

cant effect on the incremental increase in punching shear resistance due to the presence of steel fibers.

Based on the results of this experimental study, and other data reported in the technical literature, the following simple design equation is suggested to predict the increase in ultimate punching shear capacity ΔP_u of concrete slabs due to the addition of deformed steel fibers (crimped, hooked, corrugated, paddle):

$$\Delta P_u(\mathrm{lb}) = (1{\cdot}15\, V_f)\, b_0 d \sqrt{f'_c})$$

where V_f is the % volume fraction of fibers used, b_0 is the perimeter of the critical shear failure surface, d is the effective depth of the ordinary reinforcing steel and f'_c is the cylindrical compressive strength of concrete.

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