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On the Characterization of Flexural Toughness in Fiber Reinforced Concretes

Vellore S. Gopalaratnam*

Department of Civil Engineering, University of Missouri–Columbia, Columbia, Missouri 65201, USA

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Ravindra Gettu

Structural Technology Laboratory, Universitat Politècnica de Catalunya, ETSECCPB, Barcelona 08034, Spain

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Abstract

The article comprises two parts. The first part presents a summary of the available methods of characterizing the flexural toughness of fiber reinforced concretes (FRC), with a review of most of the toughness standards and guidelines from standards institutions and other professional agencies in North America, Europe and Japan. Also reviewed are other significant proposals available in the published literature. The second part of the article includes a discussion of the merits and drawbacks of these measures. Other related issues discussed include: the fundamental significance, problems with regard to experimental measurements and the potential for practical design implementation of a toughness measure.

Keywords: Ductility, energy absorption capacity, fiber reinforced concrete, first-crack strength, flexural strength, fracture energy, fracture toughness, residual strength, testing, toughness index.

1 INTRODUCTION

The ability of fiber reinforced concrete (FRC) composites to absorb energy has long been recognized as one of the most important benefits of the incorporation of fibers in plain concrete. Improvements in the fracture, impact and fatigue performance of FRC are among the consequent benefits.

Many tests have been developed to directly characterize the energy absorption capacity of such composites in simple loading configurations such as compression, flexure and tension. The flexural test is the most popular because it stimulates more realistically the conditions in many practical situations and is simpler to conduct than the tension test. The results allow toughness characterization through one or more of the following: absolute energy absorption, dimensionless indices related to energy absorption capacity, equivalent flexural strengths as prescribed post-cracking deflections or other parameters that describe the post-cracking response of the composite. Although intended to characterize the material behavior, results from these tests are usually affected by the specimen size and geometry. Nevertheless, these tests have potential engineering uses as evident by the spurt, in recent years, of standards and recommended specifications for the flexural toughness testing of FRC.[1–14] A critical review of these standards and other significant proposals[15–23] available in the published literature is undertaken here to identify possible improvements in toughness characterization procedures and use.

Alternately, there are other types of tests that have been used to measure the fracture proper-

*Paper prepared while a Visiting Professor, Universitat Politècnica de Catalunya, Barcelona, Spain.

ties, impact resistance and fatigue resistance of FRC. To varying degrees these non-standard tests, particularly the notched beam test discussed in Section 2.4, allow the measurement of properties of a more fundamental nature that are related to the ability of the composite to absorb energy. An exhaustive discussion of these tests is beyond the scope of this article. However, the basis for some of these tests can be established from the discussions presented here as they provide fundamental relevance to the engineering aspects of flexural toughness measurements. Therefore, reference is made to these tests wherever appropriate.

2 FLEXURAL TOUGHNESS CHARACTERIZATION

The characterization of flexural toughness of FRC is often accomplished using (unnotched) beam specimens tested in four-point loading, Fig. 1(a). Lesser used, but perhaps more suitable to characterize the fracture parameters of the composite, is the three-point bending test on center-notched beams, Fig. 1(b). More recently, toughness characterization using more 'application-specific' plate-bending tests have been recommended,[9] Fig. 1(c). The discussions in this paper will deal primarily with the beam tests since they produce a predominantly uniaxial stress-state and are hence more amenable to the understanding of the basic mechanics of composite behavior. The plate test, Fig. 1(c), which may facilitate easier practical implementation of toughness results in selected applications like 'slab-on-grade' and 'tunnel lining' invoke biaxial bending and other structural effects, and hence will be discussed only in a limited sense in Section 2.3.

2.1 General features of the four-point flexural test set-up

Most of the tests reviewed here use a four-point flexural configuration. Specimen sizes vary slightly (Table 1) but a common size that is often recommended is a beam of cross-section 150 × 150 mm and a length of 600 mm, loaded at third-points using an outer span of 450 mm (Fig. 1(a)). A geometrically proportionate specimen with a 100 × 100 mm cross-section is recommended for composites with shorter fibers (fiber length, l_f < 30 mm). Beams with even smaller depths are often recommended for

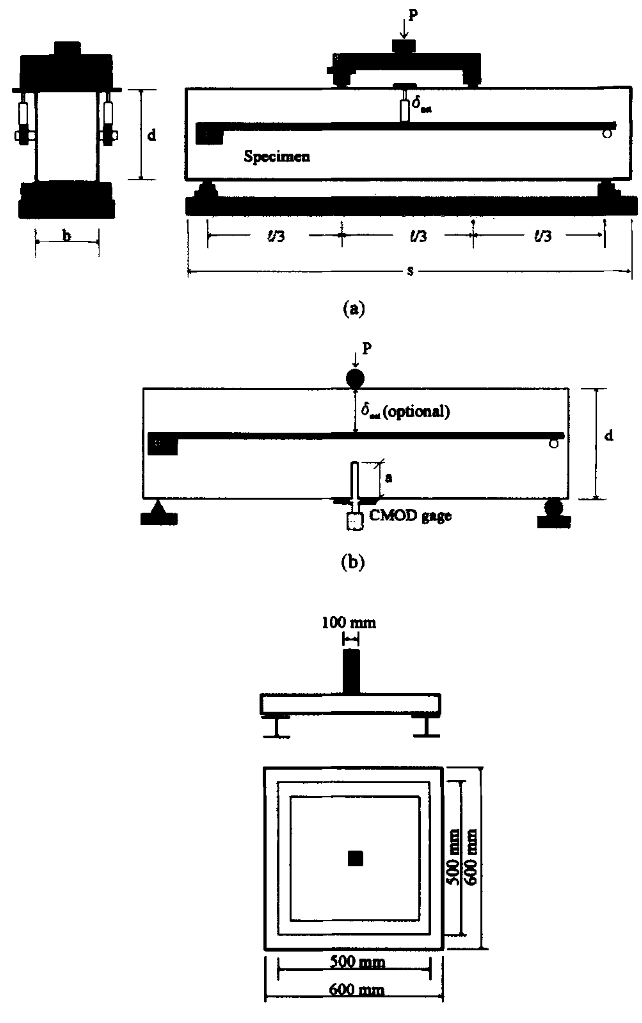

Fig. 1. General test configurations used for flexural toughness testing: (a) four-point bending, (b) notched-beam test, and (c) plate test.

sprayed applications.[9,13,14] Measurement of the net-deflection of the specimen at midspan is prescribed by most standards. However, the measurement of net-deflections at the load-points is preferable and more useful. Deflection measurements are to be made using electrical transducers. However, ASTM C 1018[4] also permits the use of mechanical dial gages. Load is applied through fixtures that allow for rotational freedom as illustrated in Fig. 1(a). Total load is measured using a load-transducer. Tests are to be conducted using a stiff testing machine at a prescribed specimen deflection rate (normally a permissible *range* of deflection rates is specified). When conventional hydraulic testing machines are used, it is necessary to control the machine manually (or through servo-control) to obtain the specified rate. Screw-driven machines allow for loading at pre-

Table 1. A summary of the various test specifications for measurement of toughness of fiber reinforced concrete

Reference[a]	*Specimen and loading*	*Specimen dimensions*[b]	*Parameters monitored*[c]	*Deflection rate and test limit*[d]	*Toughness measures*	*Other quantities defined and remarks*
ACI Guideline ACI 544 (Old) [1]	Beam Third-point loading	$b = d = 100$ mm $l = 300$ mm $s = 350$ mm	Total load Net deflection at midspan	0·05–0·10 mm/min $\delta_l > 1{\cdot}9$ mm	Ratio of energy absorbed up to deflection of 1·9 mm to that adsorbed at δ_f	First crack strength
ACI Guideline ACI 544 [1]	Beam Third-point loading	$b = d = 100$ mm $l = 300$ mm $s = 350$ mm	Total load Net deflection at midspan	0·05–0·10 mm/min Test until complete fracture	Ratio of energy absorbed for fracture of FRC beam to that for a plain concrete beam	
Norma Española UNE 83-510-89 [2]	Beam Third-point loading	$d/b < 1{\cdot}5$ $l = 3d$ $s > 3d + 50$ mm	Total load Deflection at midspan	$l/1000 - l/3000$/min $\delta_l > 1{\cdot}1$ ($l/150$) and $\delta_l > 1{\cdot}1$ ($15{\cdot}5\delta_f$)	Energy adsorbed until the deflection of $l/150$ Ratio of energy adsorbed up to $15{\cdot}5\delta_f$ to that up to δ_f	First crack strength
Normalisation Française[h] P 18-409 [3]	Beam Third-point loading	$b = d = 140$ mm $l = 420$ mm $s = 560$ mm	Total load Net deflection at midspan (avg.)	$0{\cdot}25 \pm 0{\cdot}03$ mm/min $\delta_l = 3$ mm	Ratio of load at deflection δ to load P_f (δ = 0·7, 1·4, 2·8 mm)	P_f determined according to Fig. 5
ASTM standard C 1018-92 [4]	Beam Third-point loading	100 × 100 × 350 mm preferred Variations permitted	Total load Net deflection at midspan, or Net deflection at load-points	0·05–0·10 mm/min $\delta_f > 5{\cdot}5\ \delta_f$ Higher limits as required	Ratio of energy absorbed up to $n\delta_f$ to that absorbed up to δ_f (n = 3 and 5·5) Higher n as required Residual strength factor(s)	First-crack strength First-crack deflection Energy absorbed up to δ_f See Section 2.2.4 for definition of Residual strength factors
CUR[e] Recommendation Aanbeveling 35 [5]	Beam Third-point loading Reference concrete[f]	$b = d = 150$ mm $l = 450$ mm $s = 600$ mm	Total load Net deflection at midspan	$0{\cdot}2 \pm 0{\cdot}04$ mm/min $\delta_l = 3$ mm	Energy absorbed up to deflection δ (δ = 1·5, 3 mm) Equivalent flexural strength up to deflection δ Ratio of equivalent flexural strength up to deflection δ to the first crack strength	First crack strength First crack load determined according to Fig. 5
DBV[g,h] Recommendation [6–8]	Beam Third-point loading	$b = d = 150$ mm $l = 600$ mm $s = 700$ mm	Total load Net deflection at midspan (avg.)	0·2 mm/min $\delta_l = 3{\cdot}5$ mm	Equivalent flexural load-carrying capacity until deflection $\delta_{limit} = (G_{fibers}/\delta)$ Equivalent flexural strength until deflection δ_{limit}	First-crack strength See Fig. 3 for definition of G_{fibers}, δ_{limit} and δ First crack load determined according to Fig. 5
EFNARC[i,j] Specification Draft [9]	Beam Third-point loading	$b = 125$ mm $d = 75$ mm $l = 450$ mm $s = 600$ mm	Total load Net deflection at midspan	$0{\cdot}25 \pm 0{\cdot}05$ mm/min $\delta_l = 5$ mm	Residual strength at deflections of 1 mm and 3 mm ASTM C 1018 parameters I_{10}, I_{20}, I_{50} and R_{10-20}, R_{20-50}	Toughness classification based on residual strength (Section 2.2.4) First crack strength Flexural strength
	Plate (Fig. 1(c)) all edges simply supported Center punch load	600 × 600 mm 100 mm thick Punch dimensions 100 × 100 mm	Load Deflection at the center of the plate	1·5 mm/min $\delta_l = 25$ mm	Energy absorption capacity until deflection of 25 mm	First-crack and maximum load Energy deformation curve Toughness classification based on energy absorption (Section 2.3)
Norme Belge NBN B 15-238 [10]	Beam Third-point loading	$b = d = 150$ mm $l = 3d$ $4d \leq s \leq 5d$	Total load Net deflection at midspan	$0{\cdot}07 \pm 0{\cdot}04$ mm/min for $\delta \leq 0{\cdot}5$ mm $0{\cdot}5 \pm 0{\cdot}2$ mm/min for $\delta > 0{\cdot}5$ mm $\delta_l > l/150$	Ratio of load at a deflection of l/n to the first-crack load (n = 600, 450, 300, 150) Energy absorbed up to deflection l/n (n = 300, 150) Equivalent flexural strength up to deflection l/n (n = 300, 150)	First crack strength Flexural strength

Table 1. *contd.*

Reference[a]	*Specimen and loading*	*Specimen dimensions*[b]	*Parameters monitored*[c]	*Deflection rate and test limit*[d]	*Toughness measures*	*Other quantities defined and remarks*
Japan Concrete Institute Standard JCI-SF4 [12]	Beam Third-point loading	$b = d = 100$ mm for $l_f < 40$ mm $b = d = 150$ mm for $l_f > 40$ mm $l = 3d + 80$ mm	Total load Net deflection at midspan, or Net deflection at load-points	$l/1500 - l/3000$/min	Energy absorbed up to deflection of $l/150$ Equivalent flexural strength up to deflection of $l/150$	Flexural strength Load ratio P^*_{max}/P_{max} P^*_{max} = Maximum load on reloading after unloading at $0{\cdot}9\ P_{max}$ in the post-peak region P_{max} = Maximum load
Norwegian Concrete Association[h,j] NCA Pub. No. 7 [13]	Beam Third-point loading	$b = 125$ mm $d = 75$ mm $l = 450$ mm $s = 550$ mm	Total load Net deflection at midspan	$0{\cdot}25 \pm 0{\cdot}05$ mm/min $\delta_l = 5$ mm	Residual flexural strength at deflections of 1 mm and 3 mm	Toughness classification based on residual flexural strength (Section 2.2.4) Flexural strength
RILEM Draft Recommendation [14]	Beam Third-point loading	Specimen — Test A $s \geq l + 25$ mm $b > 50$ mm, $b/d = 4$–7 $d \leq 25$ mm, $l/d \geq 20$ Specimen — Test B $s \geq l + 25$ mm $b > 50$ mm, $b/d = 4$–7 $d \leq 25$ mm, $l/d \geq 6{\cdot}67$	Total load Load-point deflection	1·5–8 mm/min cross-head speed Smaller of: $P_l = 0{\cdot}4\ P_{max}$ post-peak or $\delta_l = s/10$	Energy absorbed up to limit specified in the previous column	Maximum load

[a]Numbers in square brackets correspond to reference numbers used in the reference list.
[b]Depth = d, thickness = b, span = l, overall length = s.
[c]Continuous record of load–deflection response required in all standards except ASTM (use of dial gage and discrete load-deflection monitoring is permitted).
[d]Deflection limit for test = δ_l. Load limit for test = P_l.
[e]CUR — Civieltechnisch Centrum Uitvoering Research en Regelgeving (The Netherlands).
[f]Plain concrete with cube compressive strength of 40 MPa.
[g]DBV — Deutschen Beton-Vereins (German Concrete Association).
[h]Steel fiber reinforced concrete.
[i]EFNARC — European Federation of National Association of Specialist Contractors and Materials Suppliers for the Construction Industry.
[j]Specification for sprayed concrete.

scribed constant crosshead-displacement rates. Load–deflection response is recorded continuously using an X–Y plotter. If mechanical transducers are used, manual plotting is acceptable.[4]

2.2 Interpretation of results from the four-point flexural tests

While specimen geometries and test procedures in most standards are comparable, how the results from these tests are used to compute toughness, what they really represent and how these results can be interpreted differ significantly. Rather than describe each standard or test proposal separately in this section, it is appropriate to review general features that are common to most tests. Further, even when specific differences in these standards are discussed, they are categorized using the basic concepts adopted in defining toughness measures rather than by the test standards themselves.

2.2.1 Energy-based dimensionless indices

The ACI 544 toughness index based on Henegar's[15] proposal was the first energy-based dimensionless index used to characterize the performance of FRC. It constituted the first major effort in recognizing that energy absorption (also related to ductility or brittleness) may be important in addition to strength (particularly true for FRC and high strength concretes). The ACI 544 toughness index (Fig. 2) is defined as the ratio of the area under the load–deflection curve up to a midpoint deflection of 1·9 mm (0·075 in) to the area under the same curve up to first-crack deflection, δ_f. Among the problems with this approach are that the first-crack deflection, as discussed later, is difficult to determine reliably, and that the choice of the fixed deflection limit of 1·9 mm is arbitrary. Realistically, deflection limits should be based on serviceability requirements and hence be 'application-specific'.

The ASTM C 1018[4] essentially uses the same basic idea. The significant difference is that the limiting deflections are prescribed as multiples of the first-crack deflection. Toughness indices I_5, I_{10} and I_{20} are computed at deflection limits of 3 δ_f, 5·5 δ_f, and 10·5 δ_f, Fig. 2. The limiting deflections have been chosen to provide reference toughness index values of 5, 10 and 20, respectively, for an elastic-plastic material. Barr and Liu[16] proposed a similar dimensionless

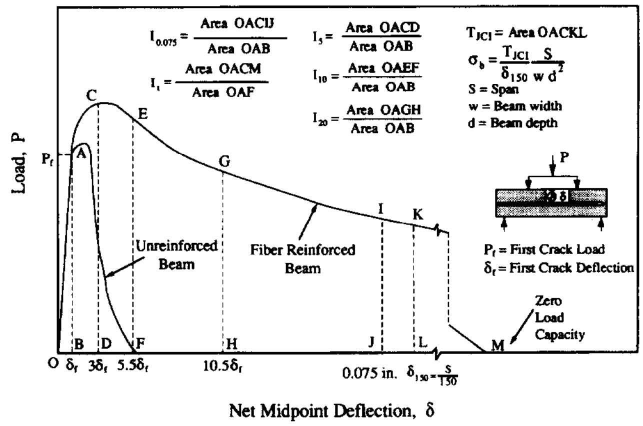

Fig. 2. Toughness definitions from ACI Committee 544 guidelines,[1] ASTM C 1018 Test Standard,[4] and JCI SF4 Test Standard.[12]

toughness index based on the ratio between the area under the load–deflection curve up to 2 δ_f and four-times the area under the load–deflection curve up to δ_f. This type of index provides an upper-bound value of 1 (for post-cracking strain-hardening modulus approaching the initial elastic modulus) and a lower-bound value of 0·25 (for elastic ideally brittle materials). The index proposed by Barr and co-workers[16–18] was developed so as to be applicable for general notched and unnotched specimen geometries (e.g., eccentric compression, compact tension, four-point bending) although no direct correlation between toughness indices measured in the different geometries can be readily made. The ASTM C 1018 indices,[4] and the index defined by Barr and Liu[16] rely on the first-crack even more than the ACI 544 toughness index since the limiting deflections are multiples of the first-crack deflections.

The Spanish standard[2] requires computation of a dimensionless index equivalent to I_{30} of the ASTM C 1018 (computed at a deflection limit of 15·5 δ_f) perhaps recognizing that at the smaller limiting deflections such an index is not a sensitive toughness measure. It also requires reporting of the first-crack strength and the energy adsorption capacity, as in the Japanese standard.[12]

Wang and Backer[21] have also proposed the use of an energy-based dimensionless index to characterize toughness. The index has been defined as the ratio of the area under the load deflection curve up to a prescribed compliance ($20C_0$ used as an example, where C_0 is the initial compliance) to the energy absorbed at first-crack. This definition is akin to indirectly limiting deflection but at varying values for materials exhibiting different post-cracking behavior. The limiting deflection is greater for materials with higher post-crack stiffness (relative to the pre-crack stiffness). Using several different types of post-cracking responses, they have demonstrated that their index provides a better representation of the energy absorption capacity than the ASTM C1018 toughness indices. The level of sensitivity reported for this type of index and the potential for practical implementation of compliance-based limits make this approach an attractive option deserving further investigation. It should, however, be noted that since compliance is a specimen size-specific parameter, toughness for different FRC composites based on compliance limits should be compared only while using identical specimen sizes.

In a more recent revision, ACI 544[1] additionally recommends an alternate toughness index, I_t, that is defined as the ratio of the energy absorption capacity of an FRC beam to that of its unreinforced counterpart (Fig. 2). The definition provides a fundamental measure of the effectiveness of fiber incorporation until complete failure. The need to test a companion plain concrete beam and the need to test the FRC beam up to complete fracture make the practical implementation of the index I_t difficult.

2.2.2 *Energy absorption capacity*

The Japanese Concrete Institute (JCI) toughness definition,[12] T_{JCI}, is computed for a standard size beam as the area under the load deflection curve up to a limiting deflection of $l/150$ (Fig. 2). Similar indices have also been proposed in the Belgian,[10] Dutch,[5] German,[6–8], RILEM[14] and Spanish[2] test specifications. Gopalaratnam *et al.*[19,20] have observed that the energy absorption capacity, defined as energy absorbed per unit cross-sectional area of the beam specimen computed at any deflection limit, has in addition to the sensitivity desired for toughness characterization, the potential for correlation to more fundamental fracture parameters for the material.

One limitation cited for toughness measures such as energy absorption capacity is that it is specimen-size specific.[19,20,23] However, for a prescribed specimen size, this value still serves as a reference that is realistically more sensitive to fiber type and reinforcing parameters than the dimensionless energy-based indices of the ASTM-type.[19,20] In principle, even if the energy based dimensionless indices at small displacements do not exhibit size dependent behavior, the strength and ductility of brittle cementitious composites are inherently size dependent.[26,31–33] As a result, none of the toughness measures discussed here and available to date can realistically claim to be truly size-independent. The other limitation of the JCI toughness definition is that the limiting deflection is large and does not reflect a useful level of serviceability for many applications. The Belgian, Dutch and German specifications have partially overcome this limitation by requiring energy absorption computations also at smaller deflection limits (Table 1).

Noteworthy is the fact that the German recommendations[6–8] specify energy absorption capacity that explicitly reflects the benefits of fiber incorporation. The basic idea has some similarities to the ACI 544 index I_t. Energy absorption capacity of a standard size FRC beam is computed up to specified deflection limits by subtracting the idealized contribution of an unreinforced concrete beam of identical size, Fig. 3. The limitations of the ACI index, I_t, namely the need to test the plain concrete and FRC beams up to complete fracture, are eliminated. Energy absorption capacity of the standard size plain concrete beam is idealized as $P_f\ (\delta_f + l/2000)/2$ (Fig. 3). This idealization can be improved to reflect the true fracture energy of the plain concrete matrix.

2.2.3 *Strength-based dimensionless indices*

It is convenient to readily implement strength-based design in the post-cracking range until specific energy-based design procedures become available. Given this fact, many standards have chosen to either exclusively[3] or additionally[10] define strength-based dimensionless indices for the post-cracking regime (equivalent to load-based indices, given the normalization procedure and the elastic analysis typically used to compute stress in the post-cracking regime). The Belgian specification, for example, uses in addition to a toughness definition based on absolute energy, dimensionless load ratios to characterize the shape of the load–deflection response in the post-cracking regime, Fig. 4. Deflection limits are specified as functions of the span (l/n, where $n = 600$, 450, 300 and 150). The French standard exclusively uses load ratios

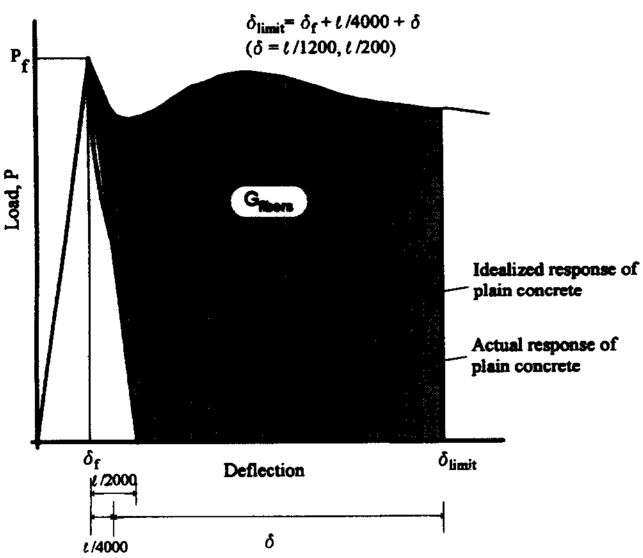

Fig. 3. Toughness measures in German recommendations[6–8] for conventional and sprayed steel fiber reinforced. Note that the idealized toughening due to plain concrete is excluded.

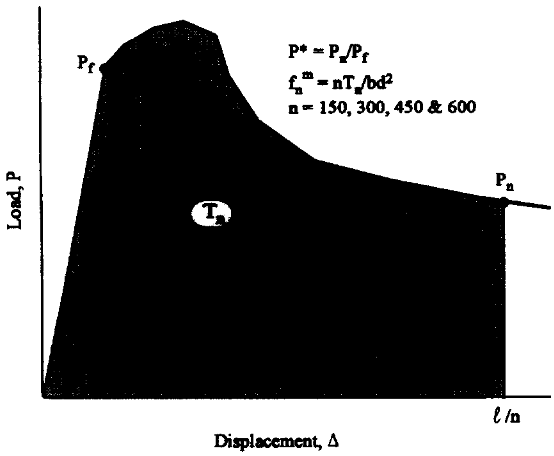

Fig. 4. Load (strength) ratios, absolute energy absorption capacity and equivalent flexural strength used as toughness measures in the Belgian standard.[10]

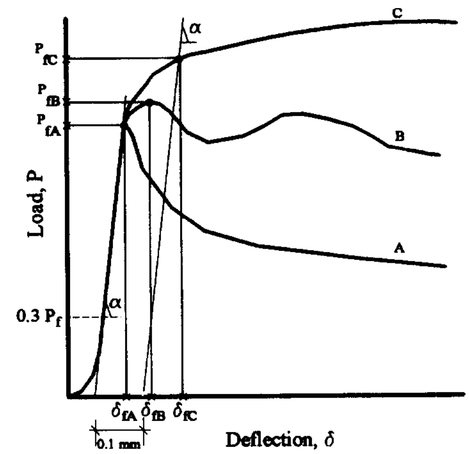

Fig. 5. First-crack definition is more objective in most European standards than the American and Janapese standards. The above definition is from the German recommendation.[6] Offset deflections of 0·05 or 0·1 mm have been typically used in other European standards.

at prescribed deflections (0·7 mm, 1·4 mm and 2·8 mm) for a standard beam size (span = 420 mm, Table 1) to characterize post-cracking ductility. In both the Belgian and French standards, the loads are normalized with respect to the load at first-crack. It should be noted that the load at first-crack is objectively defined in both these standards, much like the German recommendation, Fig. 5.

2.2.4 Residual strength measures

In addition to the energy-based dimensionless indices, the ASTM C 1018[4] requires the computation of dimensionless residual strength indices: $R_{5,10}$ and $R_{10,20}$ are computed as $20(I_{10}-I_5)$ and $10(I_{20}-I_{10})$, respectively. The residual strength indices are intended to represent the average strength retained between 3 δ_f and 5·5 δ_f, and between 5·5 δ_f and 10·5 δ_f, respectively, relative to the first-crack strength. Hence, in addition to toughness indices I_5, I_{10} and I_{20}, it is necessary to know the first-crack strength. Problems with the ASTM C 1018 toughness indices (i.e., the inability to identify FRC materials with significantly different post-cracking responses[19–21,23,27]) coupled with the lack of objectivity in identifying the first-crack strength[23] make the residual strength factors unreliable. Also, unlike the equivalent flexural strength defined at specified deflection limit(s) (see Section 2.2.5), it is difficult to practically use residual strength indices that are specified between two deflection limits.

The Norwegian Concrete Association (NB)[13] in its technical specification and guidelines for sprayed concrete bases its toughness classification directly on post-cracking strength at prescribed deflection limits measured on specified specimen size (depth 75 mm, width 125 mm, outer span 450 mm). Class 2 corresponds to strengths greater than 2 and 1·5 MPa at deflection limits of 1 and 3 mm, respectively. Class 3 corresponds to strengths greater than 3·5 and 3·0 MPa at deflection limits of 1 and 3 mm, respectively. Class 0 is reserved for unreinforced matrix, and Class 1 is used when fiber type and fiber volume fraction are explicitly specified. The EFNARC recommendation[9] uses toughness classification identical to that proposed by the Norwegian Concrete Association. This type of approach to characterize toughness may be unsuitable for general purpose use.

2.2.5 *Equivalent flexural strength*

In addition to the energy-based toughness measure T_{JCI}, the Japanese standard[12] recommends the use of an equivalent flexural strength σ_b (also called the 'flexural toughness factor') which is expressed as $T_{JCI}l/(\delta_{limit}bd^2)$. Similar equivalent flexural strengths have been specified in the Dutch and the Belgian standards[5,10,11] at two different deflection limits. The Belgian standard uses deflection limits of $l/300$ and $l/150$ while the Dutch standard specifies deflection limits of 1·5 and 3 mm (for the standard beam presented in Section 2.1, where $l = 450$ mm, the deflection limits in the two standards are identical). Additionally, the Dutch standard also requires reporting of the equivalent flexural strength ratio defined as equivalent flexural strength at the prescribed deflection limit divided by that computed at first-crack (Table 1).

The German recommendations,[6–8] which use the energy absorption capacities at two prescribed deflection limits (see Fig. 3 and Section 2.2.2 for details), also require the computation of the equivalent flexural strength at these two deflection limits in a manner somewhat similar to the JCI method.

Trottier and Banthia[23] recommend using the equivalent flexural strength concept of the JCI method with some modifications that are reported to characterize post-peak toughening better. Use of different deflection limits similar to the Belgian standard ($\delta_{limit} = l/n$, n taking different values in the range 3000–150) makes the approach more general than the JCI method in that it can also be applied at small deflection limits. The more significant departure, however, involves the use of only the post-peak (post-'first peak', in FRC exhibiting two or more peaks[23]) energy absorption in their mean strength computations. Equivalent mean post-cracking strength is computed in their proposal as $\{(E_{total} - E_{pre\text{-}peak})l\}/\{(l/n - \delta_{peak})bd^2\}$, where E_{total} is the secant stiffness at a deflection of l/n, $E_{pre\text{-}peak}$ is the pre-peak stiffness, δ_{peak} is the deflection of the peak, b is the specimen thickness and d is the specimen depth. Their proposal neither establishes the fundamental basis nor the motivation for excluding the pre-peak contribution to energy absorption. The arbitrary exclusion of the toughening in the pre-peak regime is inappropriate and misleading. This becomes obvious in the case of composites with high fiber contents or relatively long fibers which exhibit significant strengthening and one distinct peak. The effect of the pre-peak or volumetric contribution, as well as the basis for linking engineering applications of toughness and fracture mechanics in such composites, are discussed in Section 3. Also, it is possible that some commonly used composites may not reach peak load capacities at the specified limiting deflection (e.g., sprayed GFRC composites). Toughness proposals for FRC materials should, in general adequately characterize these types of commonly observed composite behavior in addition to the type of composites studied by the authors.

The equivalent flexural strength approach offers perhaps one of the best ways to practically implement[23,24,29] toughness-based design. However, further refinement of this general concept will be needed to address some of the issues discussed later in Section 3.

2.2.6 *Deflection-based dimensionless indices*

Ward and Li[22] have proposed a method of toughness characterization, where two dimensionless deflection-based indices (T_{max} and T_{50}) are used in conjunction with an energy-based index (T_{10}). T_{max} has been defined as the ratio of the deflection at the peak load to that at the first-crack load. This is analogous to ductility ratios used for reinforced concrete. T_{50} has been defined as the ratio of deflection at 50% of the peak load (post-peak regime) to the deflection at the same load in the pre-peak regime. This parameter is reported to be analogous to Hillerborg's dimensionless parameter l_{ch}/d[28] used to characterize pseudo-ductility in concrete fracture. T_{10} has been defined as the area under the load–deflection curve until the load drops to 10% of the peak load divided by the cross-sectional area of the beam. This parameter approximately measures the total energy absorption capacity of the FRC beam. For FRC systems that exhibit elastic-plastic or elastic-hardening load–deflection response, it may not be practically convenient or possible to determine T_{10} or T_{50}.

Ward and Li have reported results from a limited calibration effort in relating T_{max} to the ratio of the tensile strength in bending to the tensile strength from direct tension (f_f/f_t) and T_{10} to this ratio, as well as to the beam depth d and fiber length l_f. Procedures to use these parameters for design purposes are not obvious, and as the authors themselves suggest, the approach requires further development.

2.3 Plate test for toughness characterization

EFNARC[9] recommends the use of the plate test to characterize the energy absorption capacity of FRC for sprayed applications (Fig. 1(c)). A 600 × 600 mm plate (100 mm thick) is simply supported along all four edges using a 500 × 500 mm span. Load is applied through a 100 × 100 mm punch at a rate of 1·5 mm/min. A plot of the load versus central deflection is used to compute the energy absorbed until a deflection of 25 mm. The performance of the slabs are classified in toughness Class a, b or c, for energy absorption capacities of 500, 700 and 1000 J (Nm), respectively.

Other than the simply supported slab described above, results from field tests have been reported for slabs supported on the ground.[25] Experimentally determined first-crack loads in these tests are reported to be 3–5 times those computed using elastic theory and the Westergaard solution. Although this difference has been attributed to the toughness of FRC,[25] the complex triaxial state of stress under the punch perhaps leads to the larger load carrying capacity recorded. Moens and Nemegeer[24] report that the post-cracking characteristics observed from tests on FRC ground-supported slabs are superior to the post-cracking characteristics recorded in four-point bending tests of identical materials. This is also likely to be due to the triaxial stress state in the plate tests. Interesting illustrative examples of how the equivalent flexural strength concept can be used for the design of industrial floors[24] have been reported. The design approach will benefit from analytical or empirical correlation between the equivalent flexural strength and results from the plate tests.

2.4 Notched beam test for toughness characterization

Typically, one would like to conduct direct tension tests for the determination of fracture properties of brittle tension-weak materials like FRC. However, the tension test is difficult to conduct because of problems associated with the gripping of FRC specimens and in ensuring the stability of the test. Even if uniform tensile strains along the cross-section can be achieved in the elastic regime, unsymmetric cracking inherent to concrete heterogeneity introduces strain gradients in the deformation range of interest for FRC composites. Like the tension test, failure in the notched beam occurs due to the localization of deformations at one critical section. Unlike the tension test, this test is relatively easy to conduct, and is reproducible. RILEM[35,36] has recommended the notched beam test for determining the fracture properties of plain concrete.

Gopalaratnam *et al.*[19,20] proposed the use of a notched beam tested under servo-controlled conditions to characterize toughness of FRC. The closed-loop test is controlled by the crack mouth opening displacement (CMOD) as in test procedures for determining fracture parameters of concrete.[36] Toughness was characterized[19,20] in terms of net-deflections adopting the ASTM C 1018 procedure. It was observed that the toughness indices thus obtained were only as sensitive as the indices from unnotched beams but exhibited much less scatter.

The four-point bending configuration[19,20,30] has been used in notched beam tests so that results could be compared to the conventional toughness test on unnotched beams. However, a mid-point loading configuration is obviously more appropriate for notched beam specimens (Fig. 1(c)).[31–33] This configuration (Fig. 1(b)) has numerous advantages not the least of which is the guarantee of stability throughout the test even for unreinforced and high strength concretes with low fiber content.[31–34] Unlike the unnotched specimen, deformation in the notched mid-point loaded specimen is always localized at the notch-plane and the rest of the beam does not undergo significant inelastic deformations. This minimizes the energy dissipated over the entire volume of the specimen and, therefore, all the energy absorbed can be directly attributed to fracture along the notch plane (i.e., planar energy dissipation). Consequently, the energy dissipated in these tests can be directly correlated to material response (except for the structural size effect normally associated with the fracture of concretes[26,31–34]) and its fundamental fracture parameters. Other types of notched specimens with similar localization characteristics have also been used by Barr and co-workers.[16–18]

Among the other advantages of the notched beam test is the possibility of toughness characterization of FRC in terms of CMOD measurements, which are not subject to errors of the kind observed for deflection measurements (Fig. 1(b), Section 3.2.1). Moreover, they can be readily related to crack-width limits and, consequently, to application-specific levels of serviceability.

The load–CMOD response of a standard beam tested using mid-point loading can be directly used to characterize toughness.[20] In principle, all the toughness measures discussed for the load–deflection response in Section 2.2 can be applied to the load–CMOD response as well. However, Gopalaratnam *et al.*[19,20] showed that the sensitivity of CMOD-based toughness indices analogous to ASTM C 1018 indices (using CMOD instead of deflection) is unaltered though the scatter is lower than for the net deflection from the same test.

In recent work, Bryars *et al.*[32,33] tested mid-point loaded notched beams of an 85 MPa-strength silica fume concrete reinforced with straight steel fibers (volume fraction = 0·5%; length = 6, 13 mm; diameter = 0·15 mm). Their toughness index was defined as the ratio between the area under the load–CMOD curve until a prescribed multiple (m; m = 3–10) of a reference CMOD (= δ_{cmod}) and the area until δ_{cmod}. Using the first-peak or bend-over-point value as δ_{cmod}, they found that this index increased with fiber length and was more sensitive for larger values of m. They also concluded from tests on three sizes of beams that the CMOD-based index was practically independent of specimen size. The effect of specimen size on the toughness and other fracture parameters are presently being investigated for more efficient fiber reinforcement in a continuing study by the authors.

3 TOUGHNESS CHARACTERIZATION — ISSUES OF SIGNIFICANCE

Several issues that need to be considered with regard to making toughness characterization procedures robust, fundamentally significant, sensitive to the important material parameters and practical to implement are discussed here. Along with the larger issues not specifically addressed earlier, this section also includes the elaboration of some of the important observations made in Section 2.

3.1 Fundamental issues related to toughness characterization

3.1.1 Shear and its influence on failure mode and deflections

The general test geometry described in Section 2.1 is common to most of the test standards discussed in this article. The test configuration has a shear-span–depth ratio of 1, and a span–depth ratio of 3. The extremely small shear-span–depth ratio causes unusually high shear stresses much like those in deep beams and walls. If results from tests on reinforced concrete deep beams are any guide, a significant and unintended geometry-dependent influence (changes in the failure mode itself) may be implicit in the results from this configuration. Also this state of stress is unrepresentative of many of the more common applications of FRC (tunnel lining, slab-on-grade, etc.). In case four-point loading is used, shallower beams (such as the one recommended for shotcrete by RILEM[14], NB[13] and EFNARC[9]) with larger shear-span–depth ratios should be considered. It should be noted that for the common test configuration described in Section 2.1, the additional deformation due to shear, for typical FRC composites is of the order of 25%.

3.1.2 Volumetric versus planar energy dissipation mechanisms

Since the failure of FRC in many practical applications occurs primarily due to localized tensile failure, it is convenient to consider a phenomenological model[19] for fracture in tension to understand the contributions of various energy dissipating mechanisms. In principle, this same approach is also valid for the flexural failure of FRC beams[28]. Figure 6(a) shows a FRC panel subjected to direct tension. Figure 6(b) shows a schematic plot of the load–deformation response from a stable tension test conducted under deformation-controlled conditions. The diagram has a distinct elastic region where the load is proportional to the deformation. Following this is a zone of pre-peak nonlinearity, which prior to localization can be attributed to volumetric energy absorption. Axial strain, as a result, is gage-length independent in this region.

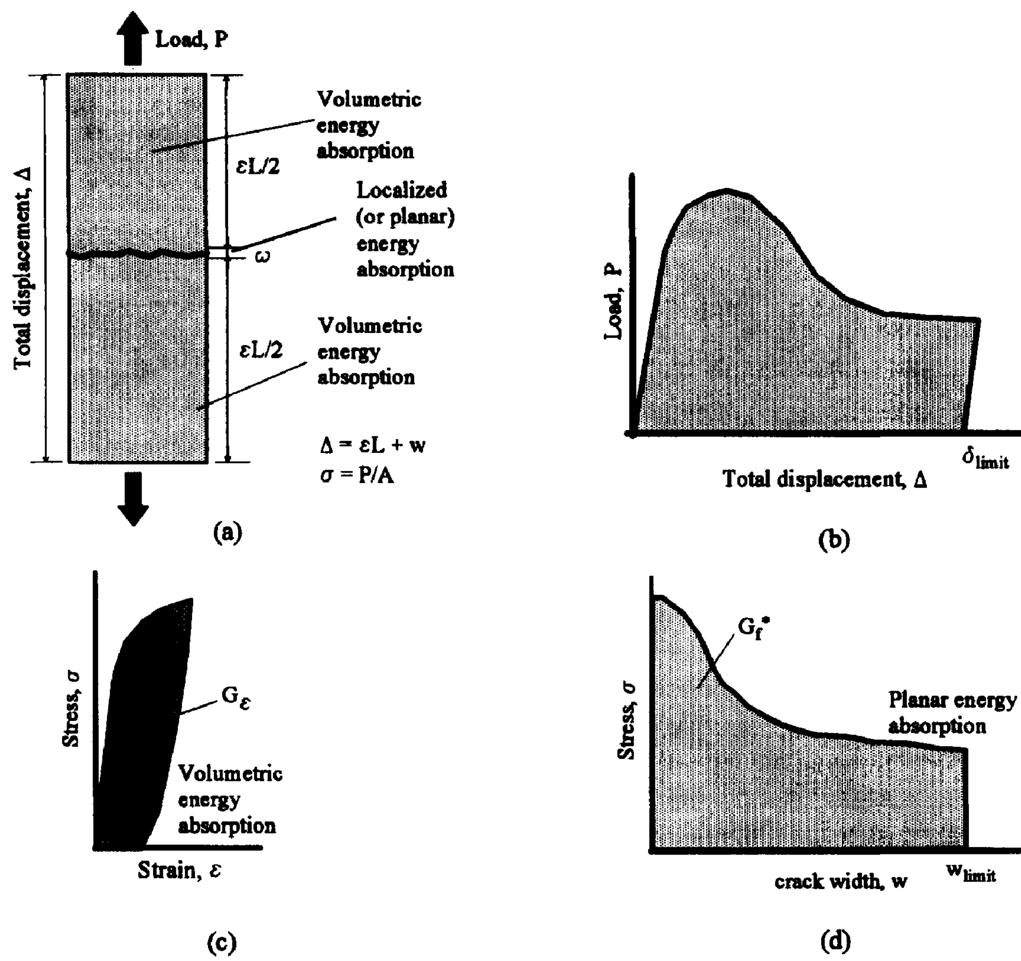

Fig. 6. Model of energy dissipating mechanisms in tensile fracture of FRC: (a) test specimen showing localization of fracture; (b) schematic load–deformation response; (c) volumetric energy absorption; and (d) planar energy absorption.

After deformation localizes, much of the energy is dissipated in the failure zone. This energy dissipation is essentially planar. The total deformation, Δ, of the specimen in the post-peak region consists of the deformation, ω, at the critical section, which is idealized to be of zero thickness, plus the deformation of the zones outside the fracture zone (εL). During the softening (post-peak) response, Δ is primarily related to the increase in ω since the region outside the fracture zone begins to unload after the peal load. The volumetric energy dissipation is denoted as G_ε (units of Nm/m^3) in Fig. 6(c) and the planar energy dissipation, illustrated up to a limiting crack-width of ω_{limit}, is denoted as G_f^* (units of Nm/m^2, Fig. 6(d)). The limiting crack width may be chosen to be application-specific to reflect realistic serviceability levels. For very large values of ω_{limit}, G_f^* approaches G_f, which is defined as the work of fracture and can be considered a material property.[28] The energy absorption parameters are both dependent on specimen size and geometry. Any toughness proposal should adequately take into account the influence of this size effect when extrapolating results from tests on small laboratory specimens to real-size structures.

Generally, in all specimens and structures, both volumetric and planar energy dissipating mechanisms contribute to the mechanical behavior.[19,22,28] The relative contribution of each mechanism depends on the structural geometry, loading configuration and size. It should be noted that the volumetric component normally occurs prior to localization and does not directly influence failure. The determination of these contributions and the parameters that quantify the mechanisms will offer a way to link engineering applications of toughness and its characterization.

3.1.3 Effect of strength and deformation capacity on specimen size

For plain concrete, the brittleness of structural failure can be quantified through the use of a parameter such as d/l_{ch}, where d is a characteristic size of the structure (e.g., beam depth), and l_{ch} is defined as the characteristic size of the material microstructure, which is related to the fracture energy, G_f, tensile strength, f_t and elastic modulus, E_c through the relation $l_{ch} = E_c G_f / f_t^2$.[28] The strength (i.e., maximum stress at peak load) and post-peak load-carrying capacity of concrete structures decrease with an increase in size d and structural brittleness d/l_{ch}. This implies that the first-crack strengths used for toughness characterization will also decrease with specimen dimensions. The size dependence of the composite strength, however, appears to decrease with increased fiber efficiency.[31]

Other size effects on the load–deformation response of FRC have also been observed. For example, the post-cracking stiffness of some fiber concretes increases with specimen size; for smaller sizes a softening behavior is obtained while hardening is observed for larger sizes.[32,33] Also, the relative contribution of the volumetric energy dissipation is expected to decrease with increase in structure size.[22] Therefore, it must be noted that the toughness characterization based on the load–deformation response is largely dependent on the size and geometry of specimen used.

As most of the size effects in failure have been attributed to fracture mechanisms, fracture parameters that account for such effects can be used to characterize toughness. For example, the parameter l_{ch} represents the pseudo-ductility of the material. This and other material parameters based on fracture mechanics are practically independent of specimen size and geometry.[40] Accordingly, a parameter analogous to l_{ch}[28] has been proposed for FRC, where the use of energy absorption up to a prescribed deflection or crack-width[19] is more appropriate than the energy required for complete fracture.

3.2 Experimental issues related to toughness characterization

3.2.1 Deflection measurement accuracy

Deflection measurement and its influence on toughness measurement has received considerable attention in many recent studies.[19,20,23,27] Figure 7 shows plots of the load–deflection

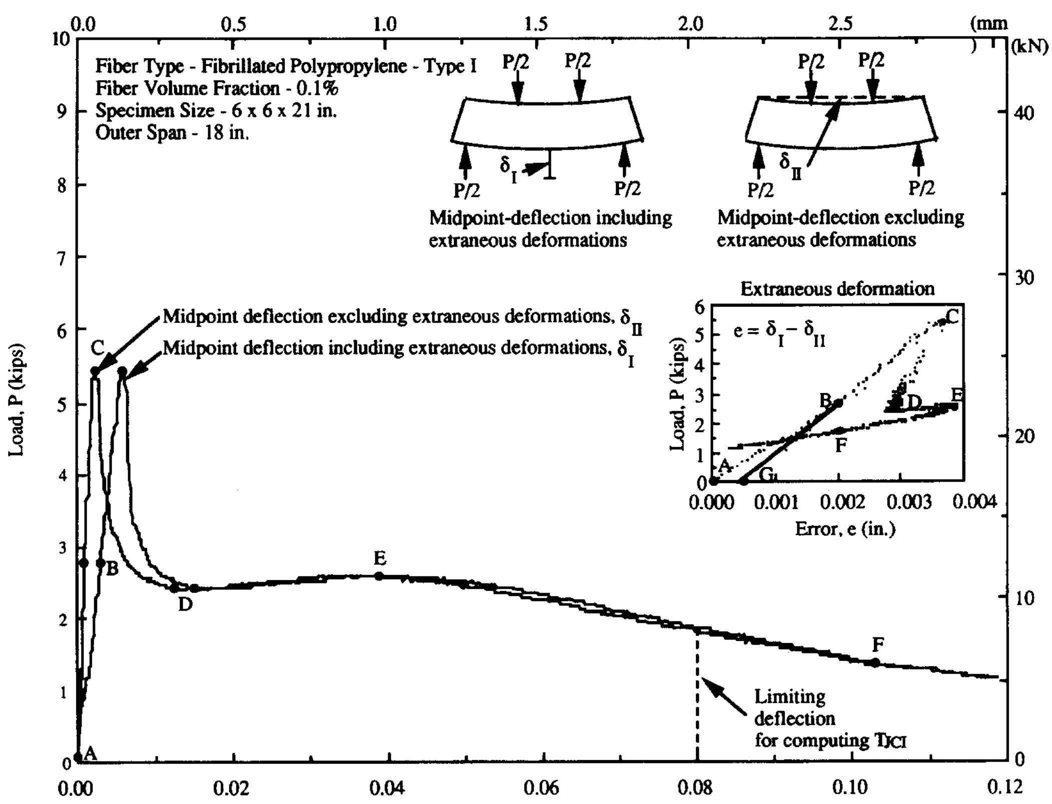

Fig. 7. Load–deflection response of a polypropylene fiber reinforced concrete beam showing net and gross deflection behavior as well as the geometry of the cracked beam. Inset shows that the error is elastic up to first-crack. Thereafter, it is a function of the post-cracking load capacity.[26]

response for a 152 × 152 × 533 mm polypropylene fiber reinforced concrete beam (fiber volume fraction, $V_f = 0{\cdot}1\%$, fiber length 51 mm, fibrillated fiber) using gross (δ_I) and net (δ_{II}) beam midspan deflections. An outer span of 457 mm was used. The inset to the figure shows that the error in gross deflection measurement ($e = \delta_I - \delta_{II}$) is largely linear up to first-crack. Analytical expressions for this effect have been obtained by computing the elastic deformations of the beam at its supports.[27] In the post-cracking regime, the error is related to the load-carrying capacity of the beam (i.e., reduces or increases with a reduction or an increase in the load carrying capacity, respectively).

Errors in the measurement of first-crack deflection can result in significant errors in the toughness measures that rely on the first-crack deflection. For example, the toughness at first-crack, which is the denominator in the ASTM toughness index, is directly influenced by inaccuracies in the measurement of δ_f. Also, since the limiting deflections used for the computation of the ASTM toughness indices are defined as multiples of δ_f, the numerator in these indices is also affected in a significant manner. These two influences have been shown[20,27] to offset each other slightly but inconsistently, depending upon the magnitude of the extraneous deformation as well as on the shape of the load–deflection response.

While the older ACI 544 toughness index is significantly influenced by errors in the measurement of δ_f, the toughness index I_t, is not. Many standards,[9,10,12–14] in contrast, avoid problems originating from inaccuracies in the measurement of δ_f by defining toughness measures that are independent of first-crack toughness and by using deflection limits that are prescribed fractions of the span (l/300 and l/150) or fixed limits for prescribed beam sizes. Energy absorption capacities measured up to deflection limits that are approximately one order of magnitude larger than the first-crack deflection (includes Belgian, Dutch, German, Japanese and Spanish specifications) are not affected significantly by the type of deflection measurement error discussed above.

While it is possible and convenient to measure the net deflection at the midspan accurately using one transducer mounted at the beam mid-width,[27] correlation of the area under the load–deflection curve thus obtained to the actual energy absorbed by the specimen is not possible, especially in the post-cracking regime. Significant errors result in attempting to convert midspan deflection to load-point deflections through a constant multiplication factor obtained from elastic analysis, particularly where the crack is unsymmetric.

3.2.2 The third-point loading configuration

Third-point loading, where the middle-third of the beam is subjected essentially to pure bending free of transverse shear loads, is a suitable configuration for the determination of the flexural strength of plain concrete, where the peak load occurs approximately at crack initiation. Unsymmetric cracking is of negligible influence in the analysis of results from a valid test. Moreover, results from tests where the crack is outside of the middle third by more than 5% of the span length are considered invalid or the modulus of rupture is computed using the moment based on the actual location of the crack.

However, in FRC beams the loads at first-cracking and at the peak may be substantially different. The critical crack is often unsymmetric and varies in length. Given the rotational freedom of the loading fixtures (Fig. 1(a)) it is possible to ensure that nearly equal loads are applied at the loading points. However, unsymmetric cracking results in significantly different deflections at the two loading points (differences of 15% have been typically measured[37]). In concretes with lower fiber volume fractions, where fiber distribution is likely to be nonuniform, the differences may be even more significant. This makes correlation of the measured toughness to the actual energy absorption capacity arbitrary at best. Also, since toughness tests for FRC are carried out up to deflection limits that are at least one order of magnitude larger than the deflection at first-cracking, this problem is further aggravated. None of the standards address this issue[1–14] at all. This problem highlights yet another inherent disadvantage of the four-point bending configuration.

3.2.3 Instability after matrix cracking

For concretes with low fiber volume fractions it is not uncommon that matrix cracking is following by momentary loss of stability in the test, under displacement control, even while using relatively stiff conventional testing machines. Toughness values evaluated at small deflection levels (< 1 mm) such as those at 3 δ_f and 5·5 δ_f

typically fall within this regime. Clearly, the load–deflection response recorded in this regime cannot be considered as the actual response but is an artifact of the compliance of the testing machine as well as of the frequency response of the recording device. Little is mentioned in the referenced standards[1–14] whether results from such tests are acceptable and if so how to interpret results from such tests. Energy absorption capacity evaluated at larger limiting deflections are affected to a lesser degree by this instability. Stability at all times during a test for all types of concretes cannot be guaranteed even in a servo-controlled test of an unnotched beam. However, in tests where a notched beam is used and where the test is controlled using the crack mouth opening displacement (Fig. 1(b)) it is possible to guarantee stability even for high-strength low fiber-content concretes.[30–34] There are numerous other advantages to this type of a test configuration as discussed in Section 2.4.

3.2.4 *Identification of first-crack and associated problems*

In addition to problems associated with the accurate measurement of net deflection at first-crack, often the identification of the first-crack is not objective. The first-crack point is defined[4] as 'the point of the load–deflection curve at which the form of the curve first becomes non-linear'. The location of this point in practice is ambiguous and depends upon, among other things, the resolution of the recording device and the judgement of the operator.[23] This is particularly true of beams that exhibit post-cracking strengthening. For such responses many standards[3,6–8,10,11] adopt a more objective definition of the first-crack analogous to the specification for yield strength of steel that does not exhibit a well-defined yield plateau. Figure 5 shows the definition of the first-crack[6–8] for three common types of load–deflection responses observed. While a fixed offset deflection for the definition of first-crack for a prescribed beam size is appropriate, an offset deflection expressed as a function of beam size will provide a more general definition.

In addition, the first-crack strength and, in general, the load–deflection response are known to be sensitive to concrete age. Toughness should hence be specified at prescribed ages and not treated as an age-independent property. The first-crack strength may also be influenced to some extent by fiber type and content. Hence, it would be best to avoid toughness definitions that use the first-crack strength or the first-crack deflection.[12,32,33,36]

3.3 Practical implementation for design

3.3.1 *Equivalent flexural strength concept in design*

It may be possible in the future to develop procedures for some applications that explicitly use energy absorption capacity (or alternate energy-based parameters) as a design parameter. Meanwhile, it appears that a pseudo-toughness approach based on equivalent flexural strength offers a practical approach to evaluate the serviceability and post-cracking response of FRC structures. There are several design guides and recommendations that already incorporate this concept for the design of slab-on-grade[6,11,24,38] and tunnel linings.[8,39] A simple analytical procedure that demonstrates the influence of the toughness of FRC and its contribution to the flexural response of beams made using conventional reinforcement has recently been proposed by El-Shakra *et al.*[29] Design aids to select appropriate fiber types, volume fractions and concrete properties can be readily developed using this kind of analysis. The availability of these procedures and others to come, along with a better understanding of the mechanics of failure, should result in improvements in the characterization of toughening due to fiber incorporation.

3.3.2 *Application-specific deflection or crack-width limits*

It is apparent from the foregoing discussions that toughness measures should be general enough so that they can be effectively applied to different situations depending upon the desired performance and serviceability considerations. In this context, limits on deflection, crack-width or compliance/stiffness serve essentially the same purpose and, in principle, could be related to each other.

4 CONCLUSIONS

(1) The following points are suggested to improve toughness characterization while using the four-point bend test on unnotched beams: (a) avoid the use of

first-crack deflection to define toughness. If first-crack strength is needed, it should be objectively defined (Fig. 5); (b) measure net-deflections at load-points; (c) use shallow beams (*l*/*d* ratios > 5) to minimize structural effects, and large shear spans to minimize the effect of shear stresses; (d) ensure stability of the test at all times through the use of a stiff machine and/or a servo-controlled machine; (e) use absolute energy and associated equivalent flexural strength at prescribed deflection limits for a standard beam as sensitive measures of toughness; and (f) use deflection limits that are relative to specimen size.

(2) The notched beam test using CMOD control offers a promising alternative to characterize toughness of FRC. Though a servo-controlled testing machine is required, the test avoids many of the problems associated with the four-point test on unnotched beams. The results can be related to fundamental material parameters and to practical design parameters. Using an appropriate toughness measure along with suitable serviceability limits (CMOD, deflections, or compliance), results from the test could be applied to structural design. Further work is needed to develop specific proposals for this purpose.

(3) The equivalent post-cracking strength approach offers an elegant way to incorporate energy absorption capacity in design. Limits for deflection or crack-widths used in conjunction with this approach should be serviceability-related and application-specific.

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