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A Two-Phase Composite Materials Approach to the Workability of Concrete

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Abstract

The present article is an attempt to establish basic composition–property relations for fresh concrete, using the two-phase composite materials approach and the law of plastic viscosity. Results of common practical tests of unit weight, slump test and Vebe test on systematic fresh concrete series, performed by the authors, are presented and are expressed through the following analytical models: the simple mixture rule for the unit weight:

$$\delta_c=\delta_m+(\delta_a-\delta_m)V_a$$

and the proposed plastic viscosity model for the workability tests:

The slump test $$s_c=s_m\frac{1-M\cdot V_a}{1+k\cdot V_a+k'\cdot V_a^2}$$

The Vebe test $$T_c=T_m\frac{1+k\cdot V_a+k'\cdot V_a^2}{1-M\cdot V_a}$$

with good agreement. The indices m, a and c refer to the mortar, the aggregate, and the concrete, respectively, and V_a *is the volume fraction of the coarse aggregate. The material constants* M, k, *and* k' *depend on the characteristics of the mortar and the coarse aggregate phases. It is concluded that with the development of this analytical model a reasonable solution for the workability of fresh concrete has been obtained.*

Keywords: Concretes, composite materials, slump, workability, aggregates, matrix phase, aggregate phase, water–cement ratio.

NOTATION

M,k,k'	Material constants
s_c	Slump of concrete
s_m	Slump of mortar
T_c	Vebe time of concrete
T_m	Vebe time of mortar
V_a	Volume fraction of coarse aggregate
λ_c	Plastic viscosity of concrete
λ_m	Plastic viscosity of mortar
δ_a	Unit weight of aggregate
δ_c	Unit weight of concrete
δ_m	Unit weight of mortar
σ_0	Yield stress of composite

1 INTRODUCTION

Many practical test methods have been developed in the past to measure the workability of fresh concrete. Among them the slump test for plastic mixtures and the Vebe test for the no-slump concretes are the most common. During the 50 year period of 1920–1970 formulae have been set up to predict the water requirement of fresh concrete for a given workability. The most important ones being the Abrams' formula on the basis of the fineness modulus,[1] the Bolomey formula,[2] Plums' formula,[3] Popovics' formula,[4] and Powers' formulae related to his exponential law.[5] Among the attempts to develop relations between workability and the composition of fresh concrete one can mention Lyses' rule of the constancy of water requirement for different cement contents,[3] Powers' assumption of proportionality between force and deformations in fresh concrete,[6] Popovics' parabolic consistency equation on the basis of the ratio of the volume of water to the volume of the mixture,[7] and Powers' exponential consistency equation on the basis of the ratio of the volume of solids to the volume of water.[5] Some attempts to apply plastic-viscosity to the workability of fresh concrete have also been reported.[5,8,9] More recent publications deal with problems like development of a new probe test,[10] the slump loss,[11] the effect of superplasticizers,[12] and extension of the no-slump range for roller compaced concrete.[13] Tattersall's work confirms the applicability of the Bingham model and shows that two parameters — yield value and plastic viscosity — are needed to characterize fresh concrete.[14–17] One new article reports on the use of a linear additive statistical model between slump and the composition factors of fresh concrete.[18] But a universally accepted basic composition–property relation for fresh concrete seems to be lacking. Legrand *et al.* have considered concrete as a two phase material.[19]

The first name author and co-workers have published results of systematic investigations on composition–property relations for hardened concrete, where they treated it as a two-phased composite material, made up ot a dispersed coarse aggregate phase in a cement mortar matrix.[20,21] The present article is a continuation of those investigations, and is an attempt to establish basic composition–property relations for fresh concrete, by using the two-phase composite materials approach and the law of plastic viscosity.

2 RESULTS OF COMMON PRACTICAL TESTS ON FRESH CONCRETE

In a recent investigation by the second named author,[22] systematic series of concrete have been prepared in which the composition of the cement mortar has been kept constant, but the volume fraction of the coarse aggregate phase V_a has been increased in successive steps, from zero up to a practically possible maximum. Besides properties in the hardened state, also unit weights, slumps and Vege values in the fresh condition have been measured. Their variation with the composition factor V_a has been studied and shown in Fig. 1. In this figure experimental points shown with different shapes indicate different series, either with different mortars (i.e. different water–cement ratios) or

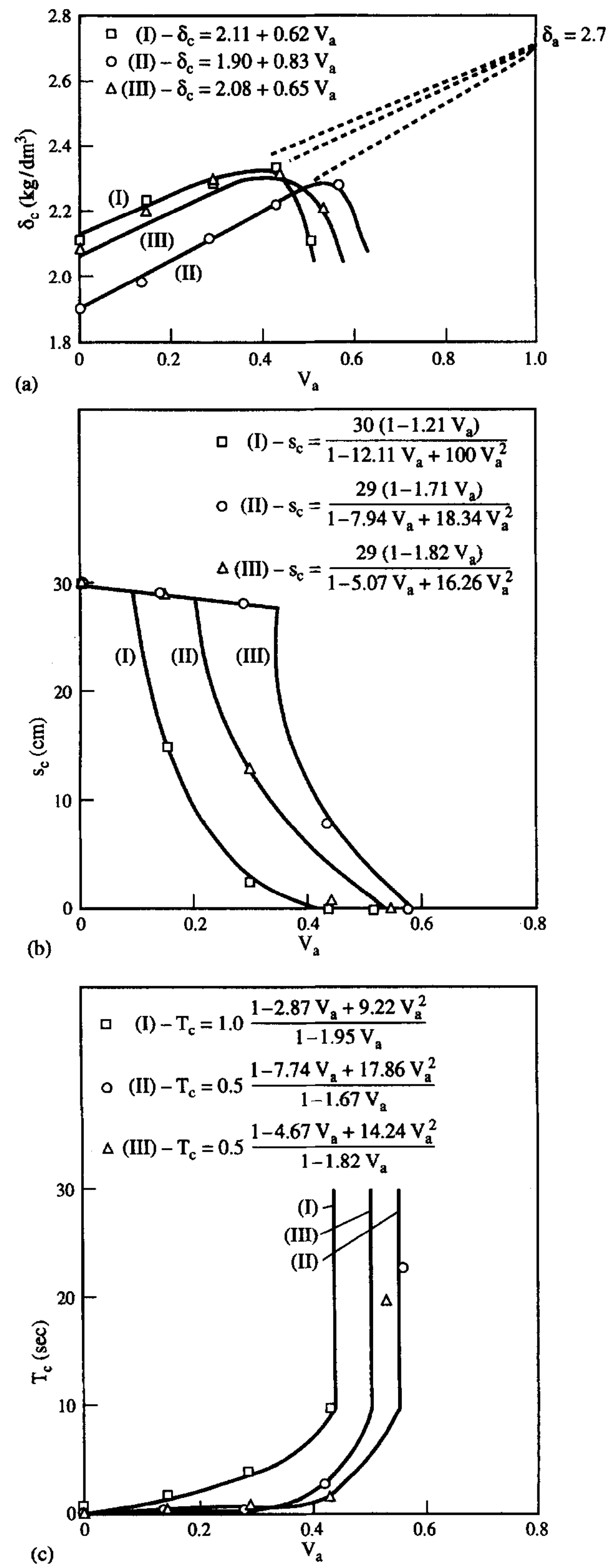


Fig. 1.

Table 1. Experimental results

Series	Composition parameters			Fresh concrete properties		
	Type and size of aggregate	*Water–cement ratio of mortar (w)*	*Volume fraction of aggregate (V_a)*	*Unit weight δ_c (kg/dm^3)*	*Slump s_c (cm)*	*Vebe T_c (s)*
I	0–16 mm crushed limestone	0·40	0·000	2·113	30	1
			0·147	2·228	15	2
			0·292	2·278	2·5	4
			0·436	2·334	0	10
			0·513	2·112	0	>>
II	0–16 mm crushed limestone	0·60	0·000	1·900	30	0·1
			0·137	1·987	29	0·3
			0·286	2·116	28	0·5
			0·429	2·217	8	3
			0·569	2·287	0	23
III	4–16 mm or 8–16 mm crushed limestone	0·40	0·000	2·080	30	0·1
			0·146	2·198	29	0·5
			0·296	2·296	13	1
			0·437	2·332	1	2 or 7
			0·538	2·218	0	20

Series	Calculated material constants: Slump, eqn (13)			Calculated material constants: Vebe, eqn (15)		
	M	*k*	*k'*	*M*	*k*	*k'*
I	2·105	−12·105	100·000	19·490	−2·871	9·22
II	1·709	−7·943	18·336	1·667	−7·738	17·855
III	1·818	−5·070	16·260	1·818	−4·665	14·235

different coarse aggregate sizes (series as shown in Table 1).

In Fig. 1(a), showing the relation between unit weight and composition, it is seen that the unit weight is first equal to the unit weight of the mortar δ_m for $V_a=0$, and that it increases according to the simple mixture rule linearly with increasing volume fraction of the coarse aggregate in the direction of the weight δ_a of the aggregate. But before reaching this value at point $V_a=1{\cdot}0$ it passes from a maximum at a critical value of the aggregate volume fraction, and then starts to decrease again. This critical point indicated that $(V_a)_3$ is the place where the total solid volume of aggregates in the mixture becomes a maximum. In order to show the effect of this fact upon workability Fig 1(a)–(c) have been plotted with identical horizontal scales and their origins placed on the same vertical, one below the other. In Fig. 1(a) it is further seen that an increase in the water–cement ratio of the mortar phase results in a rotation of the unit weight straight line around the point δ_a, towards the direction of lower values.

In Fig. 1(b), where the relation between slump and composition is shown, it is seen that the slump is first nearly equal to the value of its mortar phase, but with increasing volume fraction of the coarse aggregate, it separates itself from this value at another critical volume fraction indicated with $(V_a)_1$, and then decreasing hyperbolically reaching zero at the first critical volume fraction $(V_a)_3$ of the coarse aggregate. In Fig. 1(b) it is also seen that an increase in the water–cement ratio of the mortar phase, results in an upwards translation of the hyperbolic curve, that is in an increase of the slump values. Also an increase in the average size of the coarse aggregate has the same result.

In Fig. 1(c), where the relation between Vebe readings and the composition is shown, it is seen that while the Vebe of the mortar phase is zero, or nearly zero, it increases with increasing volume fraction of the coarse aggregate in an inversely proportional manner to the slump and extends towards infinity at the critical volume fraction $(V_a)_3$ of the coarse aggregate. It is also seen that an increase in the water–cement ratio of the mortar phase, or in the average size of the coarse aggregate phase, results in a downwards translation of the Vebe curve, decreasing the reading values.

According to these observations, it can be concluded that the relation between slump and composition has an inversely proportional character to the relation between Vebe and composition. Thus if an analytical model for the relation between slump and composition could be developed, by inverse transformation the relation between Vebe and composition would also be obtained.

3 ATTEMPT FOR DEVELOPING AN ANALYTICAL MODEL FOR SLUMP

The stress–strain–time relation of a two-phase composite, made up of dispersed coarse aggregate particles in a viscous cement mortar matrix may be assumed to obey the law of plastic viscosity, shown with eqn (1) below; this formula is an alternative formulation of the Bingham model given by Tattersall:[14]

$$\frac{d\varepsilon}{dt}=\frac{1}{\lambda_c}\cdot(\sigma-\sigma_0) \quad (1)$$

Here λ_c represents the plastic viscosity and σ_0 the yield stress of the composite. At the instant, when the slump cone is taken off, the stresses in the fresh concrete are due to gravity, and their value at a depth x from the top is equal to the product of the unit weight δ with this depth x:

$$\sigma_x=\delta\cdot x\cdot g \quad (2)$$

During the deformation of the fresh concrete under the effect of these stresses, the value of the stress σ_x decreases because of the diminution of the depth x, and because of other factors producing damping. Observations show that the value of the stress σ_x decreases in a very short time interval T from its initial value σ_1, to the level of the yield stress σ_0, where the deformation stops. This variation of the stress σ_x in the short time interval T may be assumed to obey the parabolic relation shown below in eqn (3):

$$\sigma_{(x,t)}=\frac{\sigma_1-\sigma_0}{T^2}\cdot(T-t)^2+\sigma_0 \quad (3)$$

If this representation of stress as a function of time is inserted in place of σ in eqn (1) and integrated with respect to time from 0 to T, the following result is obtained for the strain at the end of the time interval T:

$$\varepsilon_x=\frac{(\sigma_1-\sigma_0)\cdot T}{3\cdot\lambda_c} \quad (4)$$

Inserting the initial value of the stress $\delta_c\cdot x\cdot g$ for σ_1 in eqn (4), the strain at the depth x, at the end of the time interval T may be obtained as:

$$\varepsilon_x=\frac{(\delta_c\cdot x-\sigma_0/g)\cdot T\cdot g}{3\cdot\lambda_c} \quad (5)$$

Considering how an infinitesimally small layer of thickness dx, at depth x within the slump test sample of the fresh concrete, as shown in Fig. 2, the decrease $\Delta(dx)$ in the thickness of this infinitesimal layer may be calculated from eqn (5) as:

$$\Delta(dx)=\varepsilon\cdot x=\frac{(\delta_c\cdot x-\sigma_0/g)\cdot T\cdot g}{3\cdot\lambda_c}\cdot dx \quad (6)$$

If such decrease may be integrated now with respect to x for the total height of the slump sample from zero to H, then the total decrease ΔH in the height of the slump sample, at the end of the time interval T may be obtained as:

$$\Delta H=\frac{H^2\cdot T\cdot g}{6\cdot\lambda_c}\cdot\left(\delta_c-\frac{2\cdot\sigma_0}{H\cdot g}\right) \quad (7)$$

The last term in the parentheses in eqn (7) may be represented for simplicity by δ_{0c}, resulting in the more symmetric final expression (8):

$$\Delta H=\frac{H^2\cdot T\cdot g}{6\cdot\lambda_c}\cdot(\delta_c-\delta_{0c}) \quad (8)$$

The three composite properties λ_c, δ_c and δ_{0c} shown in this equation may be expressed by

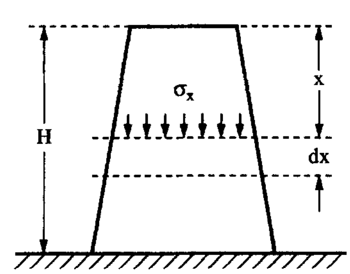


Fig. 2.

their known or proposed equals in terms of composition. For instance, the plastic viscosity through an expression of the type of viscosity of suspensions:

$$\lambda_c=\lambda_m(1+k\,V_a+k'V_a^2) \tag{9}$$

the unit weight with an expression of the simple mixture rule type:

$$\delta_c=\delta_m+(\delta_a-\delta_m)V_a \tag{10}$$

and for δ_{0c} also a similar expression to the unit weight may be proposed

$$\delta_{0c}=\delta_{0m}+(\delta_{0a}-\delta_{0m})V_a \tag{11}$$

After introducing (9)–(11) into (8) and with some final arrangement the following slump–time–composition relation may be obtained:

$$\Delta H=\frac{H^2\cdot T\cdot g}{6}\cdot\frac{(\delta_m-\delta_{0m})}{\lambda_m}\cdot\frac{1-\left[1-\left(\dfrac{\delta_a-\delta_{0a}}{\delta_m-\delta_{0m}}\right)\right]V_a}{1+k\cdot V_a+k'\cdot V_a^2} \tag{12}$$

Here the indices m, a and c represent the matrix, the aggregate, and the composite, respectively, and V_a is the volume fraction of the coarse aggregate. If in eqn (12) the term ΔH for the slump is denoted with s_c, and the result of eqn (12) for $V_a=0$, the term $(H^2\cdot T\cdot g/6)\,((\delta_m-\delta_{0m})/\lambda_m)$ representing the slump of the mortar phase is denoted with s_m, and the bracket term in the nominator of the last ratio $[1-(\delta_a-\delta_{0a}/\delta_m-\delta_{0m})]$ is represented with M, and if it is assumed that the short time interval is equal for the mortar and for all concrete mixtures, then the following final analytical model for the slump–composition relation may be obtained:

$$s_c=s_m\frac{1-M\cdot V_a}{1+k\cdot V_a+k'\cdot V_a^2} \tag{13}$$

4 ATTEMPT AT DEVELOPING AN ANALYTICAL MODEL FOR VEBE

The derivations made in Section 3 for the slump may easily be applied also to the results of the Vebe tests. But in doing this, it is necessary to consider that the vibrations transmitted to the fresh concrete by the Vebe apparatus increase the stresses to a certain multiple of the gravity stresses, and decrease the coefficients of friction and the plastic viscosity. Therefore the values of the constants λ_m, k, k', δ_m, δ_a, δ_{0m} and δ_{0a} involved in the material properties λ_c, δ_c and δ_{0c} will be different from those obtained for the slump. But the shape of the expressions will be the same and eqn (12) will still be valid. In contrast to the slump test, the deformation ΔH is known and constant in the Vebe test, and the unknown time interval T is measured. Therefore it is necessary to solve this time interval T from eqn (12) as:

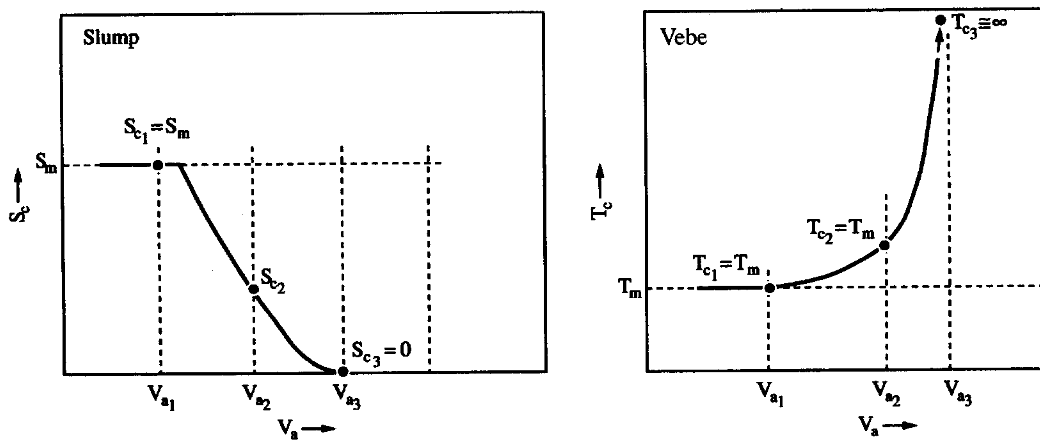
$$T=6\frac{\Delta H}{H^2\cdot g}\,\frac{\lambda_m}{\delta_m-\delta_{0m}}\,\frac{1+k\cdot V_a+k'\cdot V_a^2}{1-\left[1-\left(\dfrac{\delta_a-\delta_{0a}}{\delta_m-\delta_{0m}}\right)\right]V_a} \tag{14}$$

Here it is again possible to denote the term $6(\Delta H/H^2\cdot g)\ (\lambda_m/\delta_m-\delta_{0m})$ representing the Vebe of the mortar phase with T_m, and the bracket term in the denominator of the last ratio with M, and to obtain the following analytical model for the Vebe–composition relation:

$$T_c=T_m\frac{1+k\cdot V_a+k'\cdot V_a^2}{1-M\cdot V_a} \tag{15}$$

5 COMPARISONS, CONCLUSIONS AND RECOMMENDATIONS

Table 1 shows the results of four systematic series of concrete mixtures, obtained in an investigation by the second named author.[22] They belong to concretes made with mortar of natural sand of 0–4 mm size and with crushed limestone coarse aggregates of 0–16, 4–16, and 8–16 mm sizes. Table 1 records values of coarse aggregate size, water–cement ratio, and volume fraction of coarse aggregate as composition parameters, and values of unit weight, slump and Vebe as the fresh concrete properties. These values have been plotted in Figs 1(a)–(c) with different sign figures (□, ○, △) representing the different series. For each one of these series, three suitable pairs of coordinates (s_c, V_a) or (T_c, V_a), within the range of the critical volume fractions $(V_a)_1-(V_a)_3$ where the analytical model is valid, have been taken from Table 1



$$M V_a + \frac{s_c}{s_m} V_a k + \frac{s_c}{s_m} V_a^2 k' = 1 - \frac{s_c}{s_m}$$

$$M = \frac{1}{(V_a)_3}$$

$$k' = \frac{\left(\frac{s_m}{s_{c_2}} - 1\right)\left[1 - \frac{(V_a)_2}{(V_a)_3}\right]}{(V_a)_2\,[(V_a)_2-(V_a)_1]}$$

$$k = -[M + (V_a)_1 k']$$

$$M V_a + \frac{T_m}{T_c} V_a k + \frac{T_m}{T_c} V_a^2 k' = 1 - \frac{T_m}{T_c}$$

$$M = \frac{1}{(V_a)_3}$$

$$k' = \frac{\left(\frac{T_{c_2}}{T_m} - 1\right)\left[1 - \frac{(V_a)_2}{(V_a)_3}\right]}{(V_a)_2\,[(V_a)_2-(V_a)_1]}$$

$$k = [M + (V_a)_1 k']$$

Fig. 3.

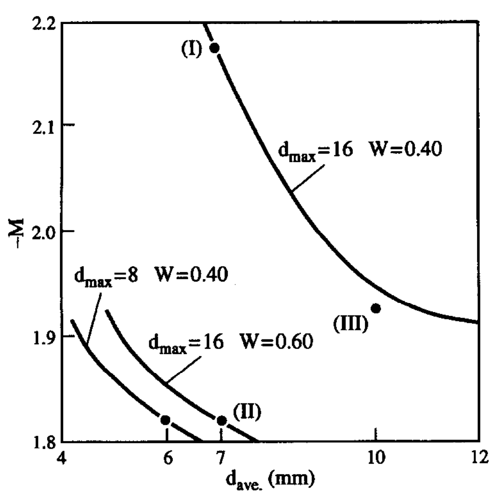


Fig. 4.

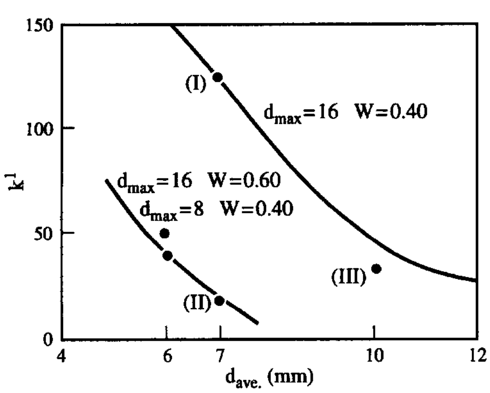


Fig. 5.

and introduced into eqn (13) or (15). From each of the obtained three simultaneous equations the unknown values for the material constants have been calculated. An easy method for this calculation is shown in Fig. 3. The calculated material constants which are shown in the last lines of Table 1 show that with increasing water–cement ratio or average aggregate size, the constants M related to unit weights and yield stresses decrease, and also the constants k' involved in the plastic viscosity decrease. These relations are also shown in Figs 4–6. The composition–property relations obtained with the calculated material constants are shown in Fig. 1(a)–(c) and they are also plotted as continuous curves, together with the experimental points, indicating good agreement. Comparisons with other similar experimental results also showed the same good agreement.

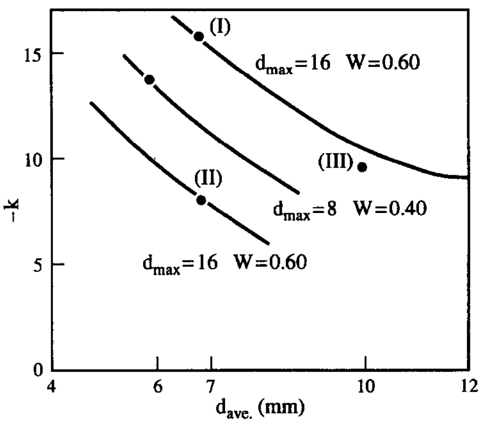


Fig. 6.

It is concluded that with the development of eqns (13) and (15) a reasonable analytical model for the workability–composition relation of fresh concrete has been obtained.

REFERENCES

1. Abrams, D. A., *Design of Concrete Mixtures Bulletin 1*. Structural Materials Research Laboratory, Lewis Institute, Chicago, 2nd Ed., 1925.
2. Bolomey, J., *Bestimmung der Druckfestigkeit von Mörtel und Beton*, Schweizerische Bauzeitung, 1926, Bnd. 88, S. 41.
3. Lyse, I., Test on consistency and strength of concrete having constant water content. In *Proc. ASTM*, Vol. 32, Part II, 1932, pp. 629–36.
4. Popovics, S., Concrete consistency and its prediction. *RILEM Bulletin* (31) (1966) 235–52.
5. Powers, I. C., In *The Properties of Fresh Concrete*, Wiley, 1968, pp. 137–8; 297–8; 480–98.
6. Powers, T. C., Workability of concrete, ASTM Committee C-9: Report on Significance of Tests of Concrete Aggregates, 1943, pp. 43–53.
7. Popovics, S., Relation between the change of water content and the consistence of fresh concrete. *Magazine of Concrete Research*, **14** (1962) 99–108.
8. L'Hermite, R., Idêes actuelles sur la tecnologie du bêton. In *La Documentation Technique du Bätiment et des Travaux Publics*, 1955, p. 9.
9. Shalom, M. I. & Greendberg, S. A., The rheology of Portland cement pastes. In *Proc. of the 4th Int. Symp. on the Chemistry of Cement*, Washington, 1960, pp. 731–96.
10. Nasser, K. N. & Rezk, N. M., New probe for testing workability and compaction of fresh concrete. *ACI J. Proc.*, **69** (1972) 270–5; **73** (1976) 561–5.
11. Tuthill, T. H., Slump loss. *Conrete Int.*, **1** (1979) 30–5.
12. Malhotra, V. M., Superplasticizers, their effect on fresh and hardened concrete. *Concrete Int.*, (1981) 66.
13. Cannon, R. W., Proportioning no slump concrete for expanded applications. *Conrete Int.*, (1982) 43–5.
14. Tattersall, G. H., The rationale of a two-point workability test. *Magazine of Concrete Research*, **25** (1973) 169–72.
15. Tattersall, G. H. & Bloomer, S. J., Further development of the two-point test for workability and extension of its range. *Magazine of Concrete Research*, **31** (1979) 202–10.
16. Tattersall, G. H. & Baker, P. H., The effect of vibration on the rheological properties of fresh concrete. *Magazine of Concrete Research*, **40** (1988) 79–88.
17. Tattersall, G. H. & Baker, P. H., An investigation on the effect of vibration on the workability of fresh concrete using a vertical pipe apparatus. *Magazine of Concrete Research*, **41** (1989) 3–9.
18. Abbasi, A. F., Munir, M. & Mohammed, W., Optimization of concrete mix proportioning using reduced factorial experimental technique. *ACI Materials J.*, (1987) 55–63.
19. Legrand, In *Rheology of Fresh Cement and Concrete*, ed. P. F. G. Banfill, 1991.
20. Özturan, T. & Kocataskin, F., Abrasion resistance of concrete as a two-phase composite material. *Int. J. Cement Composites and Lightweight Concrete*, **9** (1987) 169–76.
21. Kocataskin, F., Özturan, T. & Ersoy, H. Y., A composite materials approach for the prediction of concrete properties. *Bulletin of the Technical University of Istanbul*, **41** (1988) 333–47.
22. Topçu, İ. B., Investigation of lightweight concrete properties as composite material. PhD thesis. Technical University of Istanbul, 1987.