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Toughening in Cement Based Composites. Part I: Cement, Mortar, and Concrete

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Abstract

This paper reviews the mechanisms of toughening in unreinforced cementitious materials, including cement paste, mortar, and concrete. The paper emphasizes the microstructural aspects of the different fracture processes that can potentially take place in these materials, and point out any possible interactions between these processes. Reference is made to three types of fracture process — frontal, crack tip and wake processes, and estimates of contributions to composite toughness of the individual mechanisms are included. It is shown that the fracture mode of a cementitious material closely relates to the types of fracture process that occur in that material. Based on the understanding of the conditions under which certain toughening mechanisms can take place in a given material, it may be possible to control the material fracture mode by tailoring the material microstructure. © 1996 Elsevier Science Limited.

Key words: Toughening mechanisms, microcrack shielding, crack deflection, crack trapping, crack face pinning, aggregate/ligament bridging, frontal processes, wake processes, R-curve, fracture process zone.

INTRODUCTION

Cement is a highly brittle material. Neat cement by itself can self-destruct by stresses created due to plastic shrinkage, without any external loads. The addition of fillers, in the form of sand or aggregates, reduces shrinkage stresses, and at the same time enhances the fracture toughness of the resulting mortar or concrete. Investigation of the fracture toughness of concrete has been on the rise in the last decade, due to the recognition that structural behavior is controlled not only by compressive strength of concrete, but also by the independent material parameter — fracture toughness.

Fracture mechanics based numerical tools, mainly in the form of finite element codes, are now widely available to simulate structural response by explicitly accounting for crack development in the structure. The successful utilization of these tools requires input information on fracture toughness of the material. One of the most important revelations of recent research on fracture mechanics of concrete structures is the placing of experimental evidence of structural size effect on solid theoretical grounds (see, e.g. Ref. 1). These works have advanced so far that serious considerations are being given to code implementations.[2]

A parallel development, and with equal significance, is the investigation of toughening mechanisms of cement based composites. This research is motivated by the need to engineer the toughness property of the material, especially in light of the advances in high strength concrete in the laboratory and in field applications, and the 'brittleness' associated with these newer materials.[3] Scientific curiosity also drives deeper exploration into the source of brittleness or toughening mechanisms. This is greatly aided by recent advances in various experimental tools, such as scanning electron microscopes, laser interferometry and long distance microscopes, to probe the material micro- and meso-structures in fracture specimens.[4]

Perhaps one of the most studied structural element in detail is the steel headed anchor embedded in concrete.[5] It has been demonstrated, both numerically and experimentally, that the performance in the form of structural capacity, of the headed anchor is governed mainly by the fracture toughness of the concrete material, rather than its compressive strength.[6] This distinction is important because the use of fiber reinforcement leads to significant changes in the fracture toughness but typically minor changes in the compressive strength. Surprisingly, no research has yet been carried out with anchors embedded in fiber reinforced cement/concrete (FRC). Structural behavior affected by fracture toughness includes not only structural load carrying capacity, but also structural durability. For example, the phenomenon of concrete spalling around corroding steel reinforcement in concrete structures has been traced to crack extension in the concrete tension loaded by pressure generated by the corrosion debris.[7] It is therefore expected that the durability of R/C structures susceptible to spalling with subsequent strength reduction can be advantageously modified by cement based composites with higher fracture toughness.

From the above discussions, it can be seen that significant advances in concrete structural performance can be gained by our increasing ability to predict their behavior accurately via modern concrete fracture mechanics, and the emerging science and technology of systematically tailoring the concrete microstructure for enhanced fracture and cracking resistance.

In this two-part paper, we shall limit our focus to the mechanisms of toughening in cement based composites, including cement paste, mortar, and concrete in Part I, and FRCs in Part II. We begin Part I by providing a synopsis of the theoretical background on crack growth in brittle matrix composites. This synopsis will be helpful in laying the groundwork for a rational discussion of the toughening mechanisms reported in the literature.

THEORETICAL CONCEPTS

The Cohesive Crack Model proposed by Barenblatt[8] and Dugdale[9] suggests that the crack tip singularity in Griffith[10] and Irwin[11] type ideal brittle material concept will be removed by the presence of a cohesive zone in which non-vanishing traction acts across the crack flanks. This cohesive crack concept has been widely adopted in the concrete fracture literature, subsequent to the enlightening research of Hillerborg *et al.*[12] The cohesive crack model is identified with the situation when the physical mechanisms of crack advance (or the damage process controlling the advance of the cohesive zone) is the same as that governing the cohesive traction and crack opening of the crack wake. In this cohesive model, the stress profile is assumed to rise to the tensile strength of the material at the physical crack tip (Fig. 1(a)), and thereafter decay as the crack opens in the wake. The traction profile maintains continuity across the transition of the intact material and cohesion zone material. This implies a vanished stress intensity factor K_I at the tip of the cohesive zone.

In concrete, crack advance may include processes such as microcracking, crack front trapping, tip deflection, crack branching, crack face pinning and aggregate rupture. These processes will be explained in more detail in the following section. For now, we recognize that some of these processes are also responsible for the presence of the cohesive traction in the crack wake. These cohesive tractions are expected to decrease with effective 'opening' of the crack. Hence the Cohesive Crack Model is indeed suitable for describing crack growth in concrete. The microcracking process ahead of the physical crack tip is usually ignored, or lumped into a generalized 'fracture process zone' (FPZ).

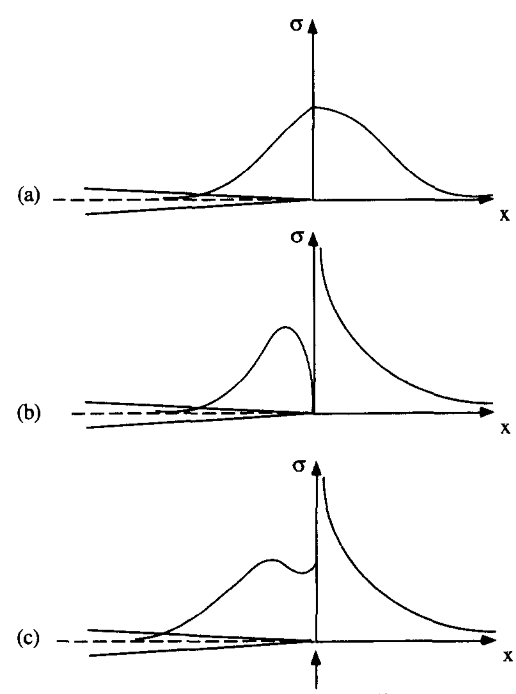


Fig. 1. Stress profile of (a) cohesive crack model; (b) bridged crack model; (c) FRC.

Recently, Cox and Marshall[13] clarified the Bridged Crack Model as being distinct from the Cohesive Crack Model. Thus in the Bridged Crack Model, the bridging mechanisms acting on the crack flanks are assumed to derive from a different source from that responsible for crack advance. This is certainly the case in fiber reinforced cement. In this material, crack advance is associated with the damage localization in the cement matrix. This process is different from the mechanisms of fiber bridging, which includes fiber/matrix interface debonding, friction sliding, and other fiber/matrix interaction mechanisms which will be discussed in more detail in Part II. For this reason, development of the bridging zone does not necessitate the cancellation of the crack tip singularity. Further, the stress profile on the crack line (Fig. 1(b)) does not need to maintain continuity across the physical crack tip. Indeed, the bridging stress acting on the crack flanks can be a rising function of the crack opening, corresponding to processes dominated by fiber/matrix debonding. Only after a certain amount of crack growth, if steady state[14] is not reached will the bridging stress descend, corresponding to processes dominated by frictional sliding of fibers.

It may be surmised that the cement paste has its own innate toughness against crack growth. Even though this may incur only a small fraction of the total energy absorption in concrete or especially in FRC, it may be useful, for the sake of completeness, to account for this toughness explicitly. If the inelastic process (e.g. microcracking) associated with this toughness takes place in a small (in the 'small scale yielding' sense of linear elastic fracture mechanics) zone, it will be suitable to allow a crack tip singularity with singularity strength reflecting the toughness of the cement. Later on, we will recognize that this singularity strength can be modified by interactive processes between the aggregates and cement at the crack tip.

Thus, the stress profile on the crack line in a FRC, due to the presence of aggregates and fibers, may combine the effects of crack tip singularity, aggregate tension softening and fiber bridging, as shown in Fig. 1(c). Some experimental evidence of this combined behavior is available.[15] The Cohesive Crack Model and the Bridged Crack Model provide a clear theoretical definition of the crack tip location, which is usually not easily identifiable in experimental investigations. This definition is useful for discerning two families of energy absorption processes associated with crack growth. The family of physical processes acting generally ahead of this crack tip are referred to as frontal processes, and those generally behind it are referred to as wake processes. Frontal processes involve inelastic deformation occurring over a volume of material, whereas wake processes involve inelastic deformation which are effectively occurring on the crack plane.

The localized nature of wake processes usually imply that such processes govern the softening branch of a well controlled uniaxial tension test. Hence much information concerning energy consuming inelastic processes have been obtained from such a uniaxial tension test in cement based composite materials. Indeed, if these processes can be characterized by a tensile stress vs crack opening (σ–δ) relationship, then the fracture energy consumed G is given by

$$G=\int_0^l \sigma(x)\frac{\partial\delta(x)}{\partial x}\,dx=\int_0^{\delta_c}\sigma(\delta)\,d\delta \quad (1)$$

The first integral is a result of wrapping a contour around the crack wake based on a J-integral analysis,[16] with the x-axis origin located at the crack tip, and l is the length of the crack wake along which traction is transferred across the crack flanks. The second integral is just the area under the σ–δ curve from a uniaxial tension test. Equation (1) connects the energy consumed in the wake processes of a crack to the energy consumed in the softening process of a uniaxially failed specimen. In concrete and FRC materials, increasing amounts of experimental data on σ–δ curves have been obtained in the last decade (e.g. Refs 15, 17–22)

MECHANISMS IN CONCRETE

In this section, we focus exclusively on the toughening mechanisms in concrete. The contributions of fibers will be covered in Part II of the paper. This discussion will concentrate on identifying the physical micromechanisms, experimental evidence, and estimates of toughness contribution. Figure 2 shows the various operating mechanisms and their approximate location relative to the crack tip. For concrete, frontal processes may include microcrack

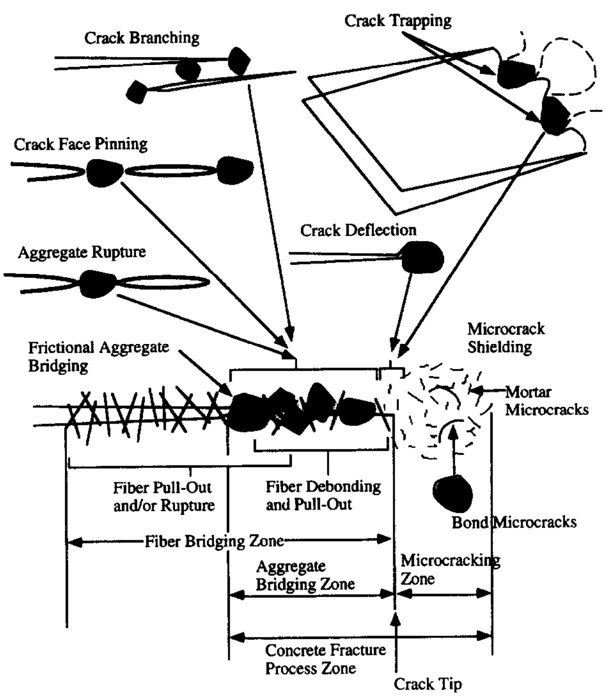


Fig. 2. Possible operating toughening mechanisms in a FRC.

shielding. Wake processes in concrete include crack face pinning, and aggregate/ligament bridging. Crack front trapping and crack deflection may be considered crack tip processes. These various processes are generally referred to in the concrete fracture literature as 'fracture processes' in a 'fracture process zone'. A review of a number of different experimental techniques for fracture process zone detection can be found in Ref. 4.

Microcrack shielding

Microcracking usually occurs at aggregate/mortar interfaces ahead of the physical crack tip. Such interfaces represent weak zones due to bleeding[23] on the bottom surfaces of aggregates. In addition, material processing defects, and microcracks due to differential shrinkage are likely to be activated by the high tensile stress field near the crack tip region. Microcracking has been detected with a variety of techniques, including X-ray with contrast medium,[24] 3-D acoustic emission,[25] quantitative acoustic-emission (AE),[26,27] laser holographic interferometry and image analysis,[28] optical interferometry with laser light,[29] and Moiré interferometry.[30] Using optical interferometry with laser light Cedolin *et al.*[29] inferred the existence of a volumetric microcracking zone ahead of the crack tip. Figure 3(a) shows the boundary of the microcracking zone (solid line) where the measured strains exceed 0·00015. A similar microcracking zone, defined by AE sources, is shown in Fig. 3(b).[31]

The microcracks that develop ahead of a crack tip lower the effective elastic modulus of the undamaged material. This generally results in a reduction in the crack tip stress intensity factor K_I.[32] Therefore, the material appears to be tougher than the original undamaged material. This toughening mechanism is known as microcrack shielding. Another interpretation of microcrack shielding as a toughening mechanism is that microcrack formation and propagation requires additional external energy.[33] Following the J-integral analysis of Evans and Faber,[34] and the self consistency analysis of Budiansky and O'Connell,[35] Huang and Li[36] estimated the effective fracture toughness $K_{IC}{}^{m}$ of concrete with distributed interfacial microcracks as a function of volume fraction of aggregates V_a and the fracture toughness of the material without interfacial microcracks K_{IC}. For V_a=0·3, 0·5, and 0·7, $K_{IC}{}^{m}/K_{IC}$=1·10, 1·19, and 1·30. However, it is possible for K_I to increase because of macrocrack–microcrack interactions and the coalescence of coplanar microcracks into the advancing crack tip.[37] This leads to crack tip damage which effectively reduces the significance of microcrack shielding as a toughening mechanism.

Crack deflection

The tortuousity (non-planar geometry) of the crack path (e.g. see Fig. 4) observed in concrete has been related to the mechanism of crack deflection.[36,38,39] When a matrix crack intercepts an aggregate, an alternative is presented for the direction of continued crack propagation. In this case the crack is likely to deflect along the matrix/aggregate interface if the latter constitutes the path of least resistance. This is generally the case in normal strength concrete (NSC) where the matrix/aggregate interface is the weakest phase in the material. However, it is possible for the crack to propagate through the aggregate creating a less tortuous fracture surface. This is the case in lightweight concrete, where the mortar is stronger than the aggregate,[38–40] and high strength concrete (HSC), where the matrix/aggregate interface is much stronger in comparison to that of NSC.[41]

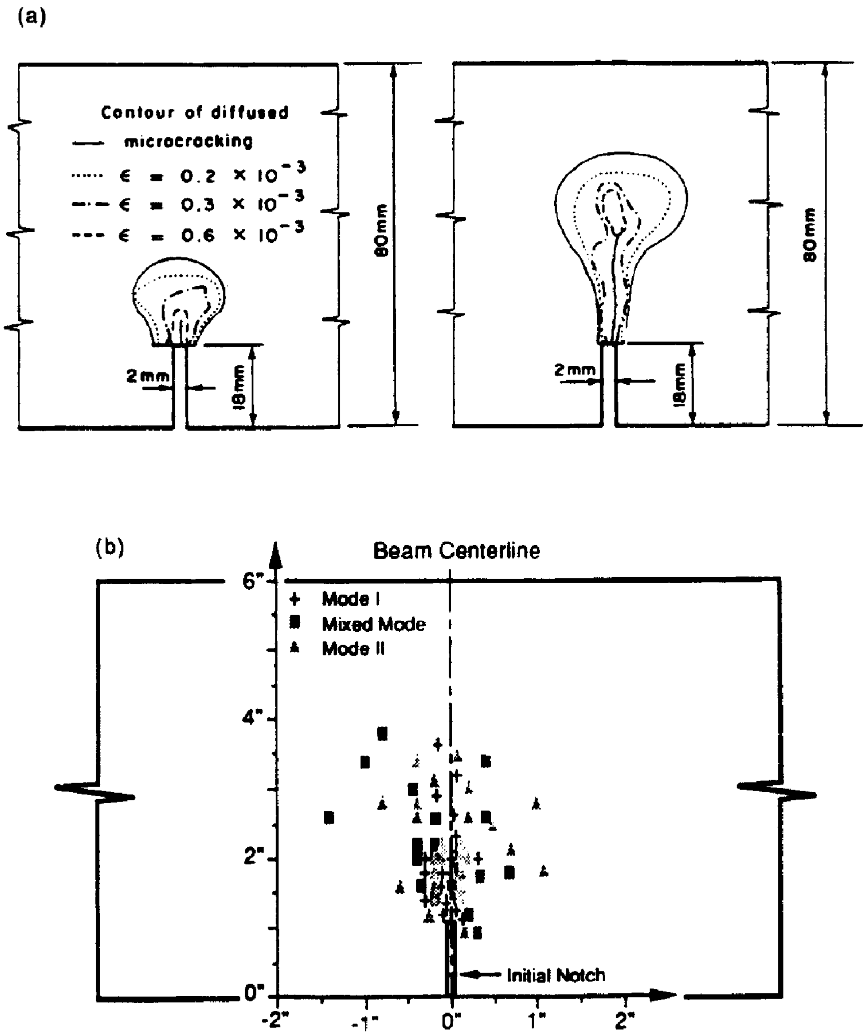


Fig. 3. Microcracking zone as detected by (a) laser interferometry,[29] and by (b) AE source locations.[31]

Fig. 4. Crack deflection revealed as tortuous crack path.[38]

An example of a crack deflecting around an aggregate is shown in Fig. 5.[42] In this figure, the direction of crack propagation is from bottom to top. We can see that a crack starting-off from a sand grain propagated in a direction along the pre-notch, then deflected to pass around the aggregate (I).

Crack deflection is considered as a toughening mechanism which can take place whenever interaction between the matrix crack front and the aggregate produces a non-planar crack, subject to a stress intensity lower than that encountered by an otherwise planar crack.[43] Huang and Li[36] adopted the crack deflection analysis of Faber *et al.*[44] for spherical inclusions to estimate the toughening of concrete due to crack deflection. As for microcrack shielding, the effective fracture toughness due to crack deflection K^d_{IC} depends on the volume fraction of aggregates V_a and the fracture toughness of the cement matrix K_{IC}. For V_a=0·3, 0·5, and 0·7, K^d_{IC}/K_{IC}=1·12, 1·20, and 1·27.

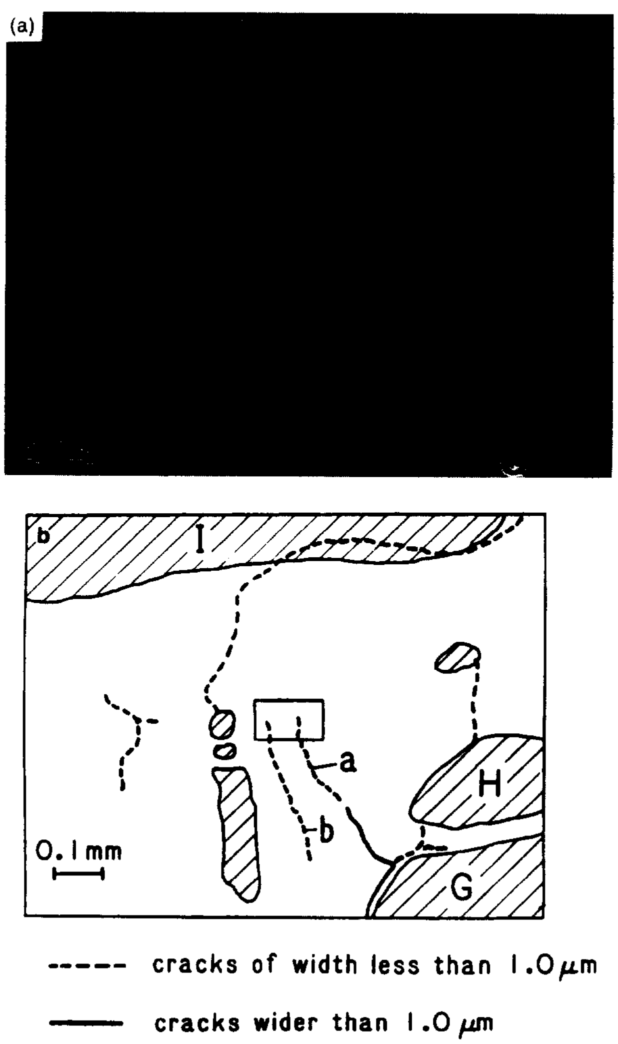


Fig. 5. Crack tip deflected around coarse aggregate:[42] (a) actual image and (b) schematic showing details.

Crack trapping

When a matrix crack approaches an aggregate but can neither deflect nor go through the aggregate, a trap site is created against crack propagation. This mechanism is called crack trapping. When it occurs, the crack faces will be constrained to zero opening displacement at discrete locations. This forces the crack tips to grow past the aggregates (trap sites) by creating locally bowed crack fronts.[45–48] This kind of crack propagation can only be achieved by raising the applied load such that $K_I=K_{IC}$ along the bow out portion of the crack front by overcoming the negative K_I^{tip} caused by the imposed trap sites of zero opening displacements. Therefore, crack trapping results in an increase in the effective toughness of the material.

The mechanism of crack trapping can only occur if three conditions are simultaneously satisfied. First, the aggregate toughness must be higher than the matrix toughness, so that crack propagation through the aggregate is prevented. Second, the aggregate must be well bonded to the matrix such that aggregate debonding or interfacial fracture (crack deflection) does not occur. Finally, the elastic modulus of the aggregate must be equal or greater than that of the matrix, such that the opening displacement can be suppressed at regions along the crack front.[48]

Crack trapping has been suggested as a possible toughening mechanism in HSC.[39,46] However, no direct experimental observation of crack trapping in HSC has been confirmed. A simulation of this mechanism with tough polycarbonate particles embedded in a brittle epoxy is shown in Fig. 6.[48] Following the crack trapping analysis of Rice,[45] Li and Huang[46] estimated the fracture toughness of HSC due to crack trapping K^t_{IC}. For V_a=0·3, 0·5, and 0·7, K^t_{IC}/K_{IC}=1·63, 1·80, and 1·93.

Crack face pinning

When the bow out segments of the crack front pass the first row of aggregates, they coalesce with one another leaving behind a row of intact aggregates in the wake of the crack. The crack front then continues to advance until the next row of aggregates is reached. At this stage the mechanism of crack trapping can be repeated. Several rows of intact aggregates may form in the wake of the crack in this manner, thereby pinning the crack faces and reducing the crack tip stress intensity factor.[46,47] This toughening mechanism which follows crack trapping is called crack face pinning.

The aggregates which pin the crack faces cannot remain intact indefinitely, and must fail by one of two mechanisms. In the first mechanism, the interface between the aggregates and the matrix fails before the aggregates fracture. The aggregates are subsequently pulled out in the wake of the crack. In the second mechanism, the aggregates are so well bonded to the matrix that they fracture in the wake of the crack. This case was analyzed[46] assuming a linear-brittle behavior of the aggregates. Li and Huang[46] found that the effective fracture toughness of HSC due to crack face pinning (crack bridging in their analysis) K^{p}_{IC} depends on the aggregate tensile strength S, average aggregate diameter (assuming spherical shape) a, aggregate volume fraction V_a, and matrix fracture toughness K_{IC}. Assuming $a=10$ mm and $S=10$ MPa, Li and Huang[46] found that for $V_a=0{\cdot}3$, 0·5, and 0·7, $K^{p}_{IC}/K_{IC}=1{\cdot}39$, 1·31, and 1·16. The decrease in toughening with aggregate volume fraction is probably due to a reduction in the length of the pinning zone.[49]

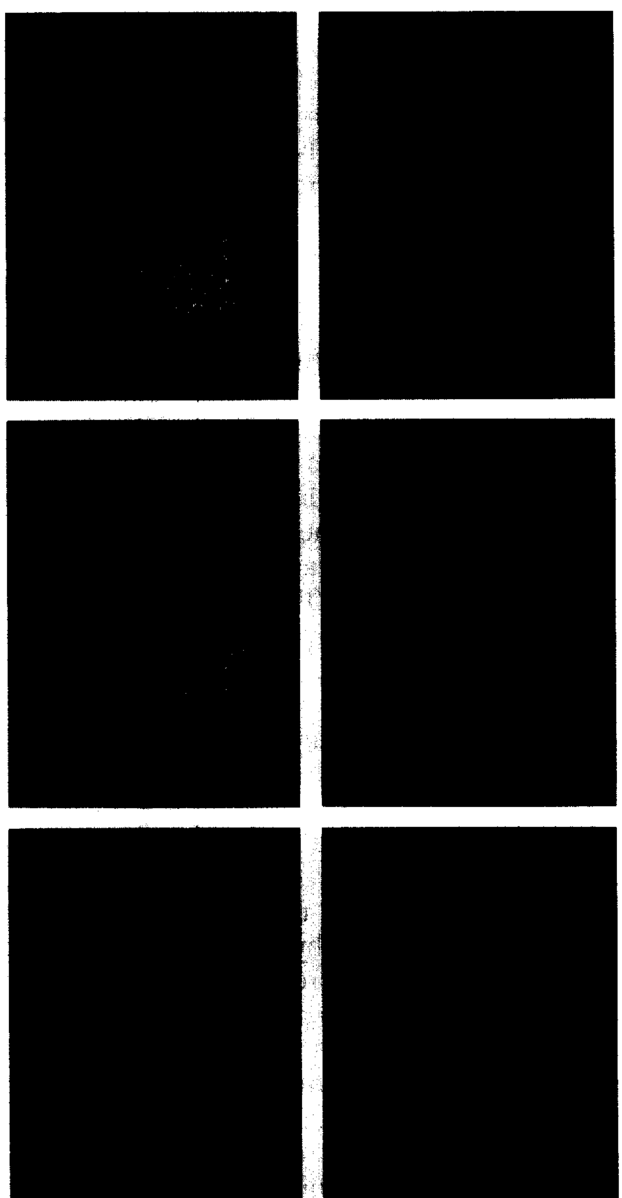
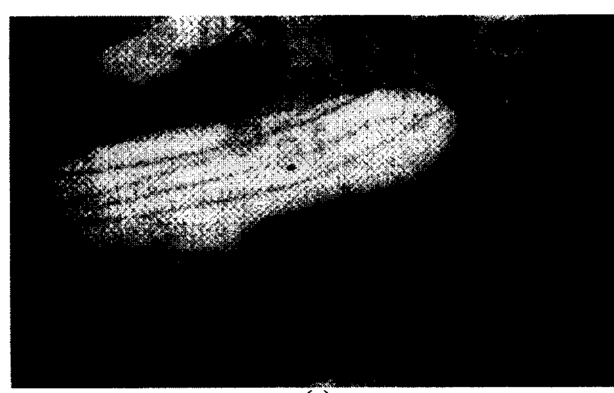


Fig. 6. Simulation of the crack trapping mechanism in a brittle transparent epoxy containing tough polycarbonate particles.[48]

Crack face pinning has not been observed experimentally but may be expected to occur in some high strength or very high strength concrete.

Aggregate/ligament bridging

Aggregate bridging and interlocking is believed to be the reason behind the softening behavior of concrete.[50–52] An example of an aggregate bridging a crack is shown in Fig. 7(a).[53] The



(a)

(b)

Fig. 7. Bridging in the crack wake:[53] (a) aggregate bridging and (b) crack overlap and branching.

bridging action of aggregates causes energy dissipation and delays the propagation of the main crack. According to Ref. 40, the softening behavior of NSC where the crack deflects around the aggregates, is more significant than that of lightweight concrete (LWC) where the crack goes through the aggregates. This indicates that aggregates left intact in NSC accounts for the difference in tension softening behavior observed between NSC and LWC. van Mier[53] attributed the tension softening behavior of concrete to the development of crack overlaps and crack branching (Fig. 7(b)). Intact concrete ligaments bridge the faces of a developing crack. The size of these ligaments were found to depend on the maximum aggregate size in the concrete. Evidence for such mechanisms was obtained by two different techniques. The first consists of using vacuum impregnation of tested specimens with fluorescent epoxy, and observation of internal cracking by the use of Ultraviolet Photography. The second uses a long distance microscope to monitor surface cracking.

Tension softening behavior of concrete

The tension-softening σ–δ curve reflects a number of the simultaneously operating mechanisms described above in the wake zone. Attempts at modeling the tension softening curve have been carried out by a number of researchers.[36,54–57] Most of these meso-mechanical models account for one or more of the mechanisms involved in the wake zone. However, the complexity of the various mechanisms, operating simultaneously, often renders the models oversimplified. Stang[58] suggested an empirical approach which has the appeal that a wide range of experimental data can be fitted extremely well. In this model the aggregate bridging stress σ_a is expressed as a function of the crack opening δ:

$$\sigma_a = \frac{\sigma_m^u}{1+\left(\dfrac{\delta}{\delta_0}\right)^p} \qquad (2)$$

where σ_m^u is the maximum bridging stress due to aggregate action at $\delta=0$. The parameter p describes the shape of the softening process with increasing crack opening, and has been determined to be close to unity for most concrete tested to date. The parameter δ_o corresponds to the crack opening when the

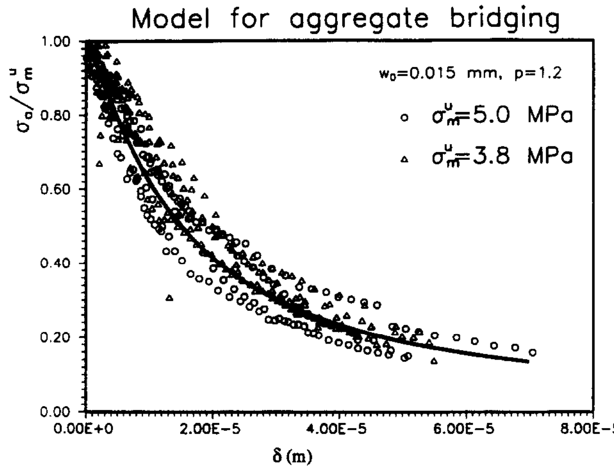


Fig. 8. σ–δ data for two types of concrete,[15] with eqn (2).

stress has dropped to half of σ_m^u eqn (2) is shown in Fig. 8, with $p=1{\cdot}2$, and $\delta_o=0{\cdot}015$ mm. Experimental data for two concrete types, one with normal strength $\sigma_m^u=3{\cdot}8$ MPa, the other with high strength $\sigma_m^u=5{\cdot}0$ MPa are also shown. Equation (2) is shown to describe both concrete data very well, at least for the range of δ indicated ($\delta \leq 0{\cdot}07$ mm).

The fracture energy from wake processes can then be obtained using (1) and (2). With the above parametric values for p and δ_o, integration up to several times δ_o gives approximately 0·1 kJ/m^2. This is roughly the order of magnitude of concrete toughness.

From eqn (2), the fracture energy is seen to scale linearly with σ_m^u. This would argue for an increasing energy absorption in wake processes in higher strength concrete. However, for high strength concrete or lightweight concrete in which aggregates fracture, this description may not hold true.

The fracture energy G was found to depend on the maximum size of aggregate. Mihashi and co-workers[25 59] found that by increasing the aggregate size from 8 to 32 mm the fracture energy G increased by about 40%. Larger aggregates tend to correlate with a wider and more tortuous fracture process zone. Note also that van Mier[53] found the size of bridging ligaments to increase with increasing size of aggregates. The effect of aggregates on G could also be seen from the difference between the fracture energy of mortar, and that of concrete. Gustafsson[60] reports typical G values of 60 N/m, and 140 N/m for mortar and concrete, respectively. This indicates that by adding aggregates to mortar, the fracture energy of the material increases by approximately 130%.

TOUGHENING IN SPECIFIC TYPES OF CONCRETE

Hardened cement paste (HCP)

At the early stages of hydration, the microstructure of cement paste is porous and highly heterogeneous, containing cement gel (C–S–H), calcium hydroxide (CH), and unhydrated cement particles. As cement paste matures, its microstructure becomes less porous and more homogeneous.[61] According to Kendall *et al.*[62] there are two types of pores in hardened cement paste (HCP) — colloidal pores (submicrometer in size) and crack like pores (some millimeters in length). Colloidal pores originate from the cement hydration process. Crack-like pores stem from imperfections in the mixing process where large air bubbles and packing defects are introduced into the mixture. These crack-like pores, however, contribute only about 10% to the total volume fraction of pores.

It is generally known that the fracture path in cement paste is rather planar, in contrast to plain concrete in which the fracture path is tortuous. In addition, hardened cement paste shows an almost perfectly linear elastic, brittle behavior under compression as well as tension.[63,64] This must be related to unstable crack propagation in hardened cement paste. Birchall *et al.*[65] found that the Griffith theory is applicable to ordinary cement paste. A similar result has also been obtained by Mindess *et al.*[66] who found agreement between experimental values of J_{IC} and G_{IC}, indicating the applicability of LEFM to HCP. According to Kendall *et al.*[62] the modulus and toughness of HCP are governed by the volume fraction of colloidal pores in HCP material. The modulus and toughness were found to decrease with increasing porosity. A similar result has also been reported for rock.[64] Crack-like pores on the other hand do not affect toughness or modulus but were found to affect fracture initiation and strength of HCP.

Struble *et al.*[67] observed the existence of a diffused zone of microcracks (about 5 mm in length, and 0·1–0·2 mm in width) ahead of a crack tip extending some 50 mm from the tip. Due to the presence of this microcracking zone, the mechanism of microcrack shielding could be operating in HCP, however, the magnitude of its contribution may not be very significant because of the small size of the microcracking zone (as a reference, the microcracking zone observed by Maji and Shah[68] in concrete extends about 25 mm ahead of the crack tip). The other toughening mechanisms discussed above (such as crack deflection, crack trapping, and crack face bridging) are not expected to be present in HCP. Therefore, microcrack shielding alone is not expected to change the brittle behavior of the base HCP material.

Mortar

When fine aggregates are added to a cement paste matrix, a mortar material is obtained. Mindess and Diamond[69] examined mortar compact tension specimens in an SEM chamber. According to the authors, shrinkage microcracks appear to pre-exist in the material. When the mortar specimens are loaded, cracks were seen to develop at the sand–cement interface. In addition, cracks were seen to follow a tortuous course to avoid the sand grains. Crack branching has also been observed to occur in the mortar specimens. Based on the above observations, microcrack shielding, crack deflection, and aggregate/ligament bridging could be important toughening mechanisms in mortar. According to Jenq and Shah,[70] when 46% volume fraction of fine aggregates are added to a cement paste matrix (everything else remained the same), the fracture toughness of the material increases by 55%. This increase in fracture toughness must be related to the occurrence of the mechanism of microcrack shielding and crack deflection in the mortar. Gustafsson[60] reports typical G_f values of 15 N/m and 60 N/m for cement paste and mortar, respectively. This indicates that by adding fine aggregates to cement paste, the fracture energy of the materials increases by 300%. This increase in fracture energy must be related to the occurrence of aggregate/ligament bridging in mortar.

Normal strength concrete (NSC)

It is known that the interface between the mortar and the aggregate is the weakest phase in concrete. The interfacial zone is characterized by its high porosity compared to the mortar matrix.[71] Tests have shown that the tensile bond at the interface between mortar and aggregate is only a fraction of the tensile strength of the plain mortar.[72] Carrasquillo *et al.*[41] observed the pre-existence of interfacial bond microcracks in concrete. In addition, they found that these types of microcracks are more predominant in NSC, than in HSC. Because of the pre-existence of microcracks in concrete, and the weakness of the interfacial zone, microcrack shielding, crack deflection, and crack face bridging are expected to be important toughening mechanisms in this material. The higher toughness observed in concrete (over mortar) could be explained by the occurrence of microcrack shielding and crack deflection at aggregate sites in this material. Jenq and Shah[70] reported that when 33% volume fraction of aggregates are added to a mortar matrix (everything else remained the same), the fracture toughness of the material increased by 37%. The significant softening behavior observed in NSC by many researchers could be explained by the occurrence of the mechanism of aggregate/ligament bridging. As pointed out earlier, the addition of aggregates to a mortar matrix resulted in an increase in the fracture energy G_f of the material.

Lightweight concrete (LWC)

Experimental observations reported in literature[38,53] indicate that the crack path in LWC is rather straight. Cracks in LWC usually propagate through the aggregates. This is the case because the bond between porous lightweight aggregates and mortar is strong whereas the aggregates are weaker than the surrounding mortar.[38,53] Therefore the mechanism of crack deflection is not expected to be an important toughening mechanism in LWC. On the other hand, microcrack shielding is expected to remain an important toughening mechanism as in NSC. Furthermore, the mechanism of aggregate/ligament bridging is expected to be present in LWC, except that its contribution to toughness will not be as large as in NSC. This mechanism will mostly occur in the mortar phase of the material. Fracture energy data reported in literature[73] indicate that G_f for LWC is lower than G_f for NSC and comparable to that for mortar.

High strength concrete (HSC)

HSC overcomes some of the shortcomings of NSC as a structural material by improving the compressive strength and elastic modulus. However, it aggravates the problem of brittleness. In general, it is agreed that the response of HSC is relatively more linear elastic and even more brittle than NSC in the sense that the material has a smaller brittleness number[74] ($K_{IC}/\sigma_u b^{1/2}$) or smaller characteristic length[12] (EG_f/σ_u^2). Yet the fracture toughness of HSC K_{IC} has been measured to be higher than that of NSC.[3,75,76] This may suggest that in HSC the toughening mechanisms are more confined to crack tip process such as trapping. Crack wake processes important to NSC may be insignificant or absent in HSC. In support of this hypothesis, experimental observations indicate that HSC has much fewer microcracks at each stress level when compared to NSC. In particular, by using SEM techniques, it was observed that the interface between the matrix and the aggregate as well as the matrix itself are considerably less porous than those in NSC.[77] The size of the weak matrix/aggregate interface is reduced and the matrix strength is improved due to the reduction of excess bleeding and the filling in of gaps by the commonly used silica fume and low water cement ratio in HSC. The strong interface makes it more difficult for a crack to go around the aggregates, so that as concrete strength increases a higher percentage of cracks tend to propagate through the aggregate rather than around it. As more cracks propagate through the aggregates, the mechanism of aggregate/ligament bridging becomes less and less important leading to brittle failure of HSC.

With the above background, it is expected that the mechanisms of microcrack shielding and crack deflection do not play a significant role in determining the toughness of HSC. Li and Huang[46] suggested that the important toughening mechanisms in HSC could be crack trapping and crack face pinning. Although these mechanisms have not been directly observed in HSC, there is indirect evidence in the literature that suggests that at least crack trapping is present in HSC. Kitsutaka *et al.*[78] showed that by replacing lightweight aggregates in a HSC with stronger sandstone aggregates the tensile strength of the material increased by about 85%, however, the characteristic length has remained essentially the same. In both materials, crack propagation was through the aggregates. The increase in the tensile strength is an indication of an increase in the material fracture toughness which must be related to a frontal zone toughening mechanism. The only toughening mechanism leading to aggregate rupture is crack trapping. Unstable propagation of the bowed crack front at the trap site is expected to be delayed by the presence of the stronger aggregates.

FURTHER DISCUSSIONS AND CONCLUSIONS

In the above review, the different toughening mechanisms were treated as if they occured independently of each other. In reality, the occurrence of one mechanism could influence the existence or effectiveness of other mechanisms.[79] For example, the occurrence of crack deflection would certainly prevent the occurrence of crack trapping. The existence of microcracks in a matrix leads to microcrack shielding. However, the existing microcracks are likely to affect the crack deflection mechanism. Because of such interactions, it is difficult to predict with accuracy, the effective toughness due to the simultaneous occurrence of different toughening mechanisms. For the same reason, it is also difficult to evaluate experimentally the individual contributions of different toughening mechanisms to the total material toughness.

Nevertheless, by understanding the conditions under which a mechanism can take place, it would be possible to predict the importance of different mechanisms for different materials. In addition, it may be possible to modify the material microstructure for the purpose of selectively promoting the occurrence of certain toughening mechanisms.

Of all the crack tip and frontal processes, the crack trapping mechanism appears to be most potent in magnitude, if the conditions for its existence are present, as likely in HSC. As a result, the crack tip singularity can be amplified in HSC. In very high strength concrete, smaller aggregates are utilized, resulting in a lower bridging toughness in the wake. This may explain why higher K_{IC} are measured in HSC and yet the post-peak behavior is typically brittle.

In terms of R-curve concept, the frontal and crack tip processes may be responsible for increasing the initiation toughness, whereas the wake processes are responsible for the increase in fracture resistance as a stable crack extends. This idea is supported by R-curve data reported in the literature[80,81] for cement paste, mortar, and concrete (Fig. 9). Figure 9 shows that cement paste has an almost constant R-curve. This data supports the earlier discussion that cement paste shows a linear elastic, brittle behavior. For mortar, Fig. 9 shows a higher initiation toughness and a rising R-curve. The higher initiation toughness must be related to the occurrence of microcrack shielding and crack deflection, while the rising R-curve must be related to the occurrence of aggregate/ligament bridging. For concrete, Fig. 9 shows a higher initiation toughness and a more rapid rising R-curve compared to mortar. This is the case because the mechanisms of microcrack shielding, crack deflection, and aggregate/ligament bridging become more significant as coarse aggregates are added to mortar.

Both frontal and wake processes can be responsible for crack path tortuousity. For example, it is expected that crack deflection and aggregate pull-out or branch crack overlaps will all lead to higher fracture surface roughness. These mechanisms, which are associated with the presence of aggregates, are consistent with the findings of Lange *et al.*[82,83] who measured

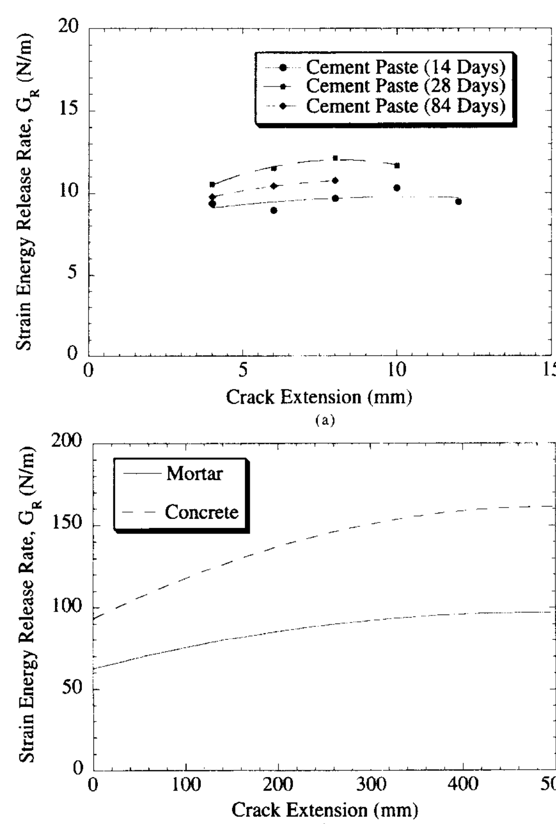


Fig. 9. R-curve for (a) cement paste,[80] (b) mortar and concrete.[81]

the fracture surface roughness of cement paste and mortar using confocal microscopy. They found that the roughness number (RN), defined in their study as the ratio of actual (fracture) surface area to projected surface area, has a high correlation with the fracture toughness of cement and mortar. This indicates that the fracture mechanisms responsible for the tortuousity of the fracture surface (e.g. microcracking, crack deflection, crack branching) are also responsible for the increase of the fracture toughness of those cementitious materials.

The various toughening mechanisms are summarized in Table 1. It is seen that among frontal, crack tip and wake processes, the wake processes appear to provide the maximum amount of toughening. This is in agreement with the theoretical argument of Horii,[52] who concluded that microcrack contribution to total composite toughness is small, less than 20%, based on a coupled analysis of microcrack and crack face bridging. The dominant energy consumption derives from wake processes. Taking all toughening mechanisms together, it may be reasonable to expect that concrete can have toughness value that are several times to one order of magnitude higher than that of cement paste. This range is approximately what has been reported in the literature for different types of concrete. As will be shown later in Part II, aggregate toughening in concrete, while important, is nonetheless much less effective than fiber toughening.[84–90]

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Table 1. Summary of toughening mechanisms in cement based composites

Toughening mechanism	*Location*	*Toughness increase over cement*	*Detection technique*	*Remark*
Microcrack shielding	Frontal	10–20%[47] up to 29%[36]	X-Ray[24,41] Stereomicroscopy[72] 3-D AE[25] Laser holography[28] Quantitative AE[33] SEM[67] Laser interferometry[29]	Concrete MPZ ≈ 25 mm[68] HCP MPZ ≈ 50 μm[67]
Crack deflection	Crack tip	10–20%[47] up to 27%[36]	SEM during loading[38,42] X-ray[24]	
Crack trapping	Crack tip	60–90%[46]	Should be detectable by dyed epoxy impregnation technique	Not yet observed in cement based materials, but expected in HSC
Crack face pinning	Wake	15–40%[46]	Should be detectable by dyed epoxy impregnation technique	Not yet observed in cement based materials, but expected in HSC
Aggregate/ ligament bridging	Wake	~130% (over mortar)[60]	SEM during loading[38,42] Long distance microscope[53] Epoxy impregnation and UV photography[53] X-ray[24]	Concrete FPZ > 50 mm,[68] 20–40 mm,[29] 20–30 mm,[30] 30–40 mm,[84] 12–43 mm,[85] 70 mm,[25] 29 mm long and 20 mm wide[24]
Fiber debonding	Wake-frontal		SEM during loading[42] Moiré interferometry[86]	
Fiber pull-out	Wake	Up to 250 ×[87]	SEM during loading[42]	Bridging zone in cm to m scale depending on fiber length[88]
Multiple micro-cracking	Frontal	Up to 1000 ×[89–90]	Regular photography during loading[89–90]	Volumetric, covers over 1000 cm^2 on specimen surface[89]

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