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Mixed-mode Fracture Properties of Concrete Reinforced with Low Volume Fractions of Steel and Polypropylene Fibers

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Abstract

Mixed-mode fracture characteristics of plain concrete and concrete with 0.3% volume fraction of polypropylene and steel fibers were investigated experimentally. Different test configurations and notch locations were used. The test results were interpreted using linear and nonlinear fracture mechanics concepts. Nonlinear fracture parameters appropriately described the positive effects of low volume fractions of fibers on the mixed-mode fracture characteristics of concrete. The results help assess the effectiveness of fibers in controlling the opening and slippage of cracks. © 1998 Published by Elsevier Science Ltd. All rights reserved.

Keywords: Fiber reinforced concrete; fracture energy; fracture toughness; polypropylene fibers; steel fibres.

INTRODUCTION

Shear strength is a property of major importance for a wide range of civil engineering materials and structures. Current design procedures for shear resistance are still based on empirical results despite decades of research which have been carried out on the evaluation of the shear performance of concrete members. One of the major problems with shear strength studies is the difficulty of determining the shear strength directly.[1] Under shear forces cracks tend to propagate in both mode I (opening) and mode II (sliding) configurations. Attempts to apply fracture mechanics concepts to study mixed mode failure of concrete have been made.[2] For mixed-mode failure of concrete, the determination of the final failure path and the criteria for crack instability are more complicated than in pure mode I failure. Currently, attempts are being made to extend the methods of fracture mechanics to predict crack propagation in concrete when mode II or mixed modes I and II are prevalent at crack tips.[3] In an early attempt,[4] it was pointed out that the effect of aggregate interlock which leads to volume dilation should preclude crack propagation in any mode other than opening. Nevertheless, there has been an extensive amount of experimental work done over the years which points at the importance of mode II and mixed-mode fracture of concrete.[3] Fiber reinforcement is an effective means of enhancing the fracture characteristics of concrete. The use of low fiber volume fractions in concrete has increased significantly in recent years for the control of opening and slippage of concrete cracks. To understand fully such effects of fibers in concrete, it is important to assess their effects on the mixed-mode fracture of concrete.

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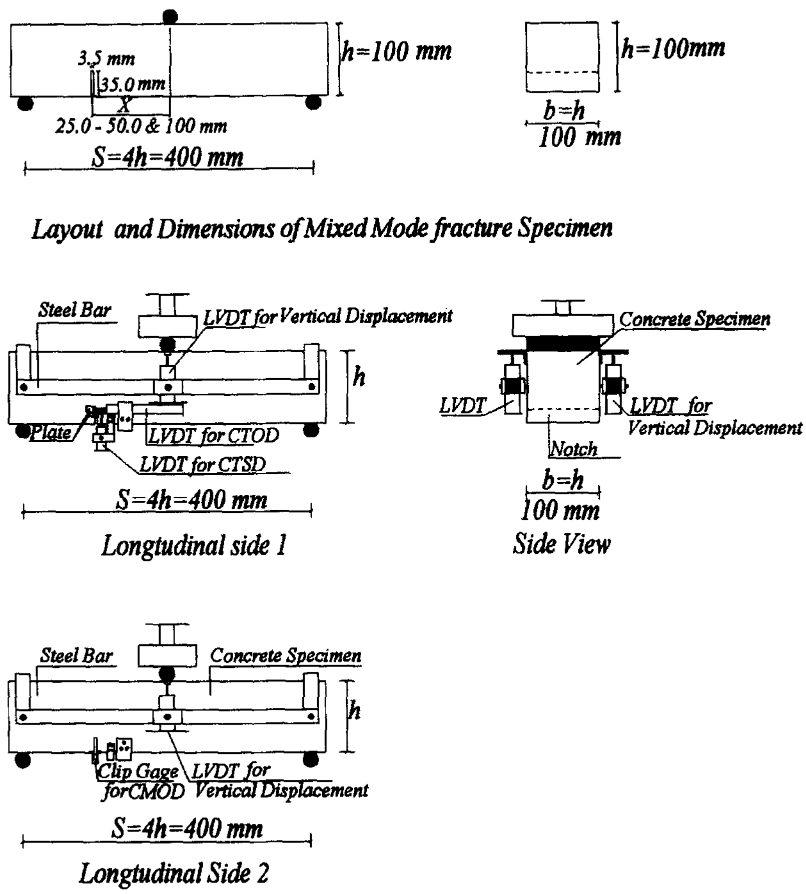

Fig. 1. Mixed mode fracture with the 3PBB test set-up.

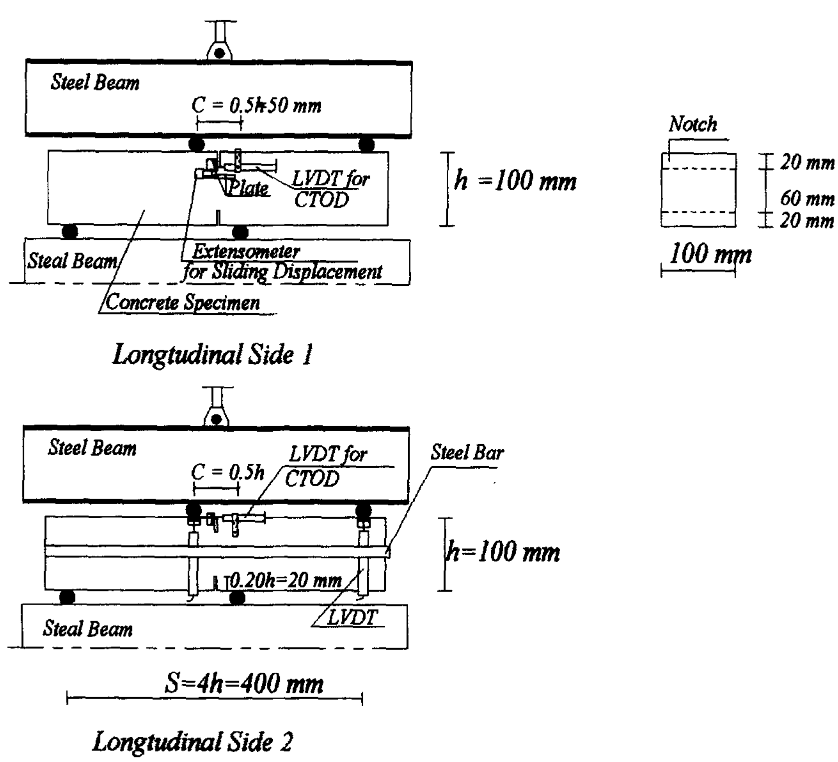

Fig. 2. Mixed mode fracture with the 4PS test set-up.

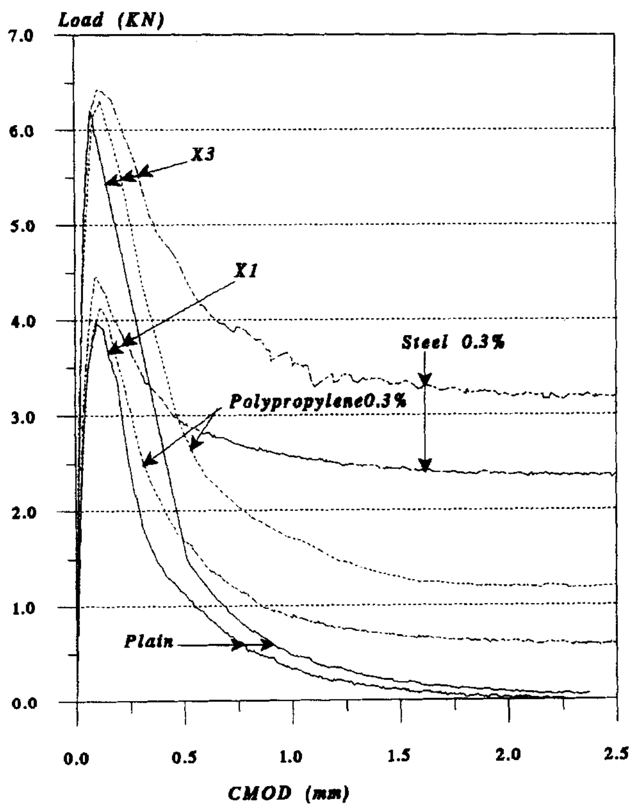

Fig. 3. Load–*CMOD* curves for the 3PBB test configuration.

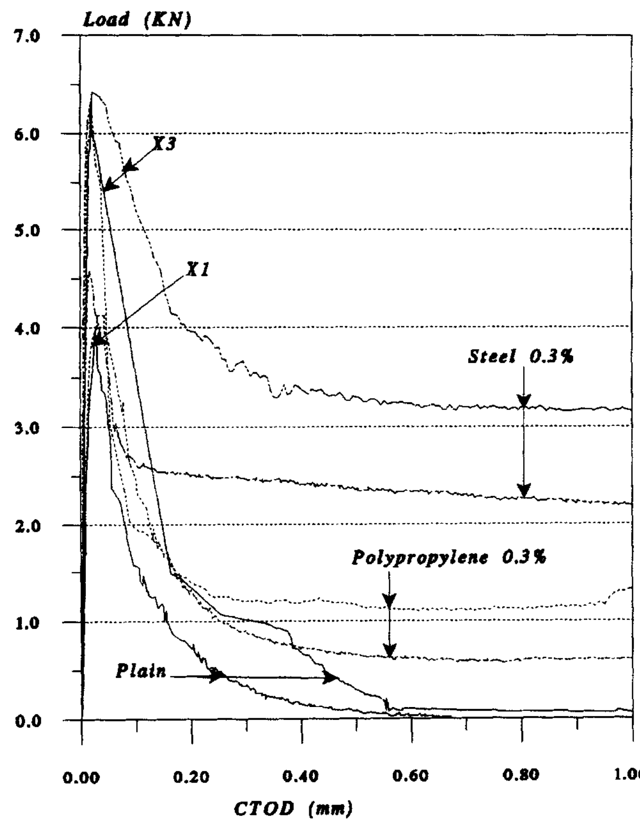

Fig. 5. Load–*CTOD* curves for the 3PBB test configuration.

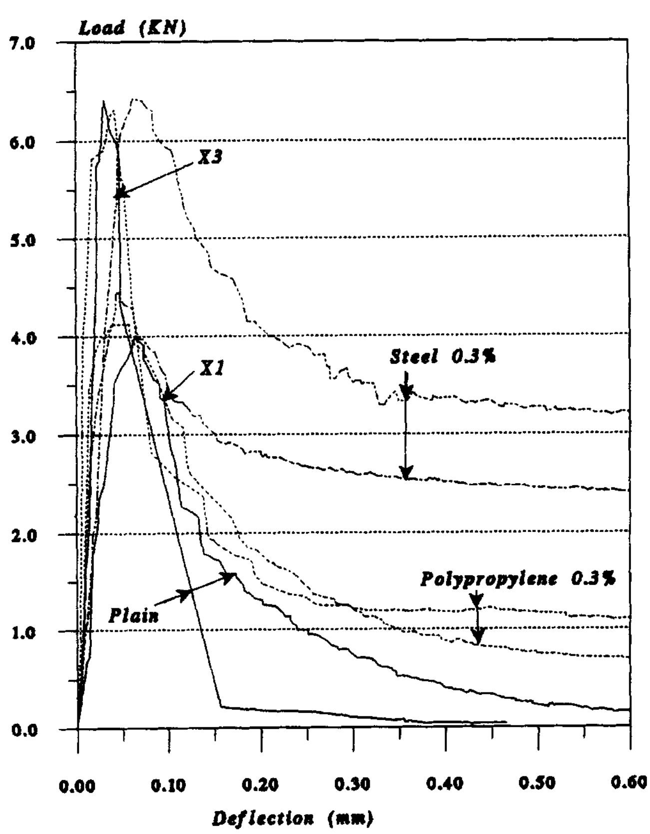

Fig. 4. Load–deflection curves for the 3PBB test configuration.

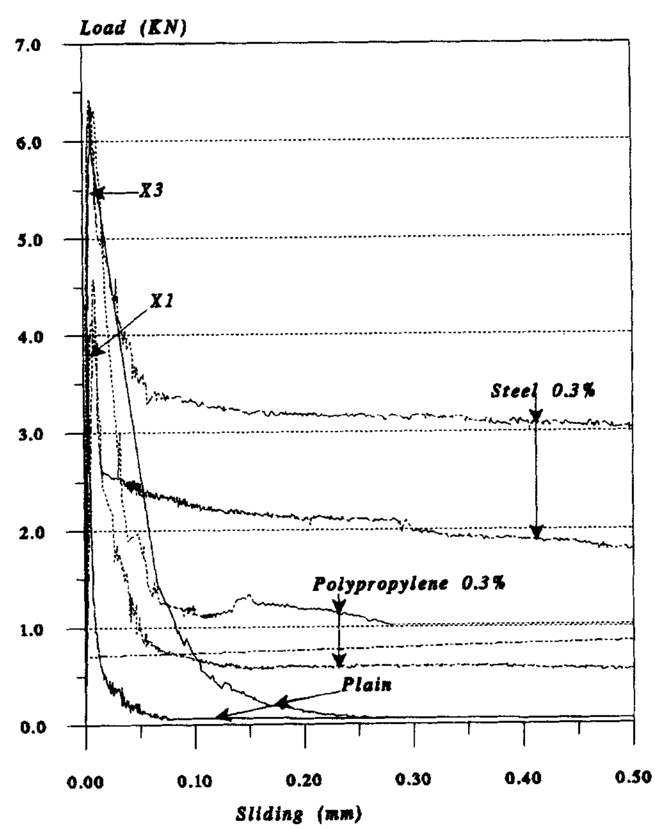

Fig. 6. Load–sliding curves for the 3PBB test configuration.

Table 1. The maximum load U, the energy release rate and α test results for the 3PBB test configurations.

Type of concrete	$X^{(1)}$	*Max. Load KN*	*U N.m*	*Energy release rate N/m*			$\alpha°$
				G_{C1}	G_{C2}	G_{C3}	
	$X1$	3.9550	0.22985	33.3315	28.9840	36.2720	22.00
		3.5949	0.17470	27.845	24.2130	29.118	20.50
Plain	$X2$	3.97957	0.12199	16.5545	14.3951	18.76478	28.00
		4.21624	0.16314	23.2633	20.229	25.3145	20.00
	$X3$	5.74042	0.2202	31.8342	27.6819	34.953	20.00
		6.0195	0.20354	25.6509	22.3052	31.314	29.75
Steel	$X1$	4.06957	0.245988	36.06669	31.36234	40.0355	22.50
		4.040645	0.223408	31.14568	27.08321	34.37045	25.25
fiber	$X2$	4.27202	0.2489285	33.219315	28.88636	38.997	31.50
		4.80746	0.2556585	35.846675	31.17102	40.58068	28.25
0.3%	$X3$	6.39637	0.288898	39.77614	34.58795	45.45647	26.50
		6.156106	0.261199	36.86154	32.05351	42.7297	30.50
Polypropylene	$X1$	3.87504	0.189457	27.03354	23.507425	29.7525	25.50
		3.455915	0.261358	34.985025	30.421755	31.7055	29.75
fiber	$X2$	3.941669	0.172561	24.29998	21.13042	27.4035	27.50
		4.141183	0.158916	23.330225	20.28715	26.486	27.50
0.3%	$X3$	6.509910	0.1678430	23.324750	20.282395	25.8245	24.25
		6.150900	0.183464	27.560255	23.965440	29.3650	19.50

(1) = Distance between center line of beam and notch place.
$X1$ = 25 mm = 1/16 times loaded span of beam.
$X2$ = 50 mm = 1/8 times loaded span of beam.
$X3$ = 100 mm = 1/4 times loaded span of beam.

RESEARCH SIGNIFICANCE

Low volume fractions of polypropylene and steel fibers are increasingly used in concrete flatwork construction for the control of crack widths and vertical slippage. The test results reported herein provide information for understanding the role of fibers in controlling crack opening and slippage.

EXPERIMENTAL PROGRAM

The purpose of this experimental program was to investigate the effects of a low volume fraction of polypropylene and steel fibers on the mixed-mode fracture mechanics properties of concrete. Two different test procedures were used for mixed mode fracture mechanics. The first one used a three-point-bending beam (3PBB) with an eccentrically placed notch, and the second one used a four-point shear specimen (4PS) with two notches. The variables for the three-point-bending beam (3PBB) were: the type of reinforcement (steel and polypropylene fibers), and the distance between the center line of the beam and the place of notch. For the second test procedure, the only variable was the type of reinforcement. Three specimens were used for each batch; two batches (replications) were considered for each mix.

Table 2. The maximum load U, the energy release rate, the stress intensity factor and α test results for the 4PS test configurations.

Type of concrete	*Max. load KN*	*U N.m*	*Energy release rate N/m*			*Stress intensity factor MPa.m$^{1/2}$*			$\alpha°$
			G_{C1}	G_{C2}	G_{C3}	K_{C1}	K_{C2}	K_{C3}	
Plain	20.135	0.1357	22.256	19.353	26.643	0.7826	0.6805	0.8502	32.00
	21.35	0.1291	20.937	18.206	25.863	0.7593	0.6603	0.8369	34.33
Steel fiber	21.479	0.2375	32.476	28.24	39.579	1.001	0.8704	1.103	34.50
0.3%	23.257	0.1937	26.433	22.985	31.894	0.9133	0.7942	1.003	34.00
Polypropylene fiber	19.167	0.1600	22.367	19.450	27.033	0.8170	0.7104	0.8997	34.00
0.3%	21.467	0.1667	21.700	18.8870	27.967	0.8040	0.6991	0.9133	39.33

MATERIALS AND MIX PROPORTIONS

The basic mix ingredients were Type I Portland cement, natural siliceous fine aggregate, crushed limestone coarse aggregate, steel or polypropylene fibers, water and air-entraining agent. The specific gravity of fine and coarse normal-weight aggregates were 2.60 and 2.65, respectively. The fineness modulus of fine aggregate and the maximum size of coarse aggregate were 2.84 and 10 mm, respectively.

Steel fiber reinforced normal-weight concrete with 0.3% volume fraction of steel fibers (hooked-end steel fiber with 30 mm length and 0.3 mm diameter), and polypropylene fiber reinforced normal-weight concrete with 0.3% volume fraction of polypropylene fibers (collated fibrillated polypropylene fibers with 19 mm length) were used in the mixed-mode fracture tests.

The weight ratios of water, sand and coarse aggregate to cement were 0.46, 2 and 2.5, respectively.

TEST PROCEDURES

Normal-weight concrete mixtures were mixed in a rotary drum mixer following ASTM C192 procedures. The specimens were moist cured at 100% relative humidity and 22°C (72°F) for 35 days. The notches for these specimens were produced by means of a circular saw with 3.5 mm thickness at 30 days of age, and the specimens were returned back to the curing room until one day before the time of testing

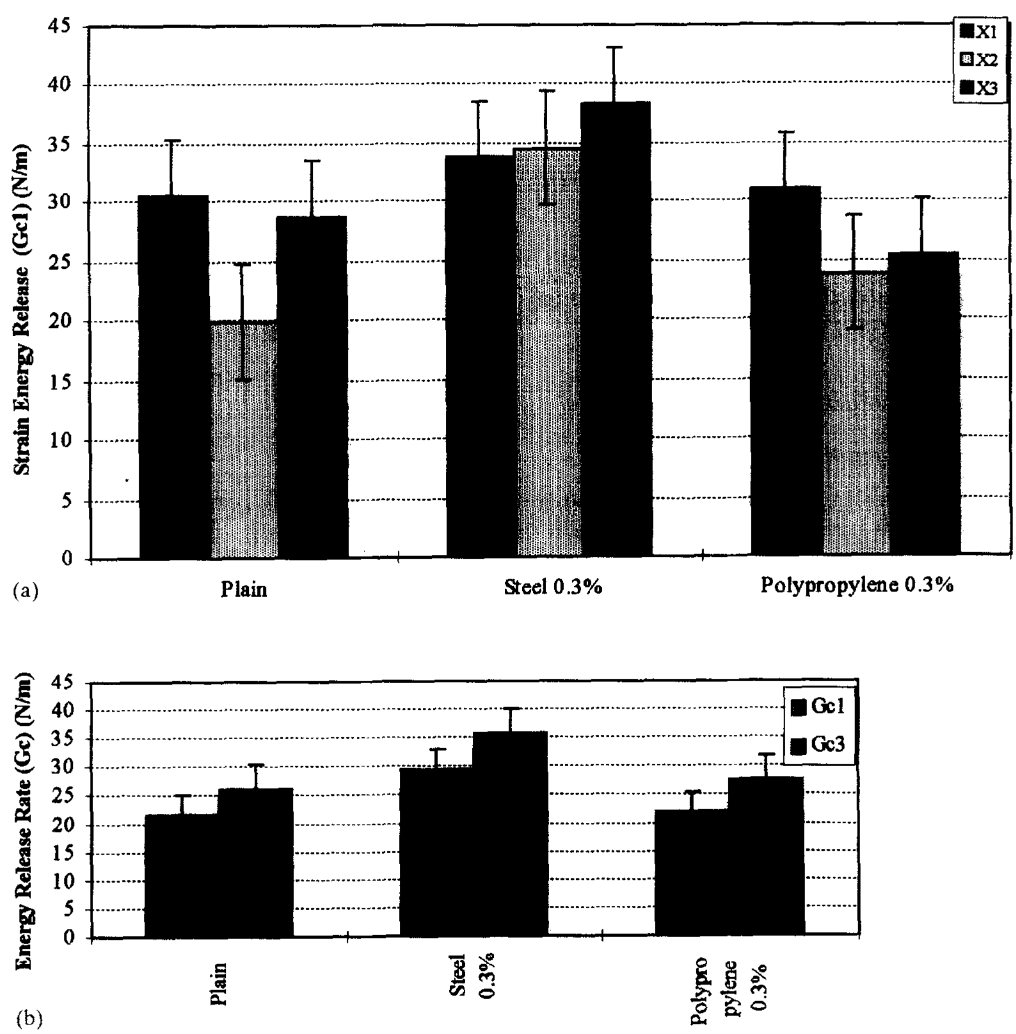

Fig. 7. Mean values and pooled 95% confidence intervals of energy release rate (G_{C1}) test results: (a) the 3PBB test configuration; (b) the 4PS test configuration.

Table 3. The stress intensity factor, and the non-linear fracture parameter test results for the 3PBB test configuration.

Type of concrete	$X^{(1)}$	*Modulus of elasticity KN/mm²*	*Stress intensity factor MPa m^1/2* K_{C1}	K_{C2}	K_{C3}	K_{IIC}	*Total fracture energy N/m* G_{F1}	G_{F2}	G_{F3}	A_{CMD} *N m*
	*X*1		1.02904	0.95955	1.10986	0.15935	111.19	96.6865	119.92	1.7936
			0.95841	0.89372	1.02321	0.13190	89.9355	78.2040	96.016	1.3296
Plain	*X*2	31.975	0.72755	0.67844	0.82400	0.16818	75.4924	65.6456	85.5004	1.44498
		32.9875	0.87580	0.81683	0.93201	0.17737	97.7359	84.9878	104.008	2.20346
	*X*3		1.00891	0.94081	1.07366	0.24260	100.24	92.456	106.673	3.43899
			0.91987	0.85778	1.05952	0.27040	81.231	70.6356	93.5627	3.14242
Steel	*X*1		1.07306	1.0008	1.16147	0.15319	2697.58	2345.72	2919.84	46.1551
			0.99825	0.93088	1.10370	0.17076	2290.51	1991.75	2532.47	44.1187
fiber	*X*2	31.149	1.01387	0.94544	1.18910	0.17427	2674.80	2325.91	3137.08	57.0103
		32.00	1.06481	0.99294	1.20879	0.18894	2798.66	2433.62	3177.08	59.5808
0.3%	*X*3		1.21024	1.12855	1.32736	0.27032	2981.22	2592.36	3269.73	70.8772
			1.21509	1.13308	1.41022	0.23006	2929.88	2547.73	3400.40	79.8328
Polypropylene	*X*1		0.90011	0.83935	0.99726	0.15376	1414.37	1229.89	1567.02	22.6450
			1.01908	0.95029	1.17379	0.14605	1051.89	914.684	1211.58	17.1415
fiber	*X*2	30.008	0.85393	0.79629	0.96271	0.15485	1307.67	1137.10	1474.24	23.5399
		30.004	0.83487	0.77852	0.94122	0.14742	1030.39	895.992	1161.64	18.8829
0.3%	*X*3		0.83626	0.77981	0.91719	0.27512	1235.2	1118.3	1354.74	30.0213
			0.90932	0.84794	0.96465	0.23757	1157.09	1006.17	1227.50	28.5546

when the anchorage plates were glued at the points of measurement. The notch depth in the 3PBB test was 0.35*h*, where *h* is the height of the beam, and in the 4PS mixed mode it was 0.2*h* (two notches).

The testing configuration and dimensions of the 3PBB test specimen are presented in Fig. 1. Displacement transducers (LVDTs) were used to measure the vertical displacement at mid-span, the crack tip opening displacement (*CTOD*), and the crack tip sliding displacement (*CTSD*). An extensometer was used to measure the crack mouth opening displacement (*CMOD*) which was used as the feedback signal

Table 4. The ratios of test results for the 3PBB test to those for the 4PS test

Type of concrete	X_i	*Ratio of U for 3PBB to that for 4PS*	*Ratio of energy release rate for 4PBB to that for 4PS* Ratio of G_{C1}	Ratio of G_{C3}	*Ratio of stress intensity factor for 3PBB to that for 4PS* Ratio of K_{C1}	Ratio of K_{C3}	*Ratio of mixed mode stress intensity factor to that for mode I* K_{C1}/K_{IC}	K_{C3}/K_{IC}	*Ratio of α° for 3PBB to that for 4PS*
	*X*1	1.6938	1.4976	1.3614	1.3149	1.3054	1.3878	1.4968	0.6875
		1.3532	1.3299	1.1258	1.2622	1.2226	1.5072	1.6091	0.5971
Plain	*X*2	0.8990	0.7438	0.7043	0.9298	0.9692	0.9812	1.1113	0.875
		1.2637	1.1111	0.9788	1.1534	1.1136	1.3773	1.4657	0.5826
	*X*3	1.6227	1.4304	1.3119	1.2892	1.2628	1.3606	1.4480	0.625
		1.5766	1.2252	1.2108	1.2115	1.2660	1.4466	1.6663	0.8666
Steel	*X*1	1.0357	1.1106	1.0115	1.0720	1.0530	1.3370	1.4472	0.6522
		1.1534	1.1783	1.0776	1.0930	1.1004	1.3093	1.4476	0.7427
fiber	*X*2	1.0481	1.0229	0.9853	1.0129	1.0781	1.2632	1.4816	0.913
		1.3199	1.3561	1.2724	1.1659	1.2052	1.3966	1.5854	0.8309
0.3%	*X*3	1.2164	1.2248	1.1485	1.2090	1.3260	1.5079	1.6538	0.7681
y		1.3485	1.3945	1.3397	1.3304	1.5441	1.5937	1.8496	0.8971
Polypropylene	*X*1	1.1841	1.2086	1.1006	1.1017	1.1084	1.3482	1.4936	0.750
		1.5678	1.6122	1.1337	1.2675	1.2852	1.3649	1.5721	0.7564
fiber	*X*2	1.0785	1.0864	1.0137	1.0452	1.0700	1.2790	1.4419	0.8089
		0.9533	1.0751	0.9470	1.0384	1.0306	1.1182	1.2606	0.6992
0.3%	*X*3	1.0490	1.0428	0.9553	1.0236	1.0194	1.2525	1.3737	0.7132
		1.1006	1.2700	1.050	1.1310	1.0562	1.2179	1.2920	0.4958

in closed-loop control. All beam tests were conducted using a servo-hydraulic closed-loop system with a stiff frame and a maximum load capacity of 250 KN (55 Kips). The loading rate was 0.002 mm/sec in the *CMOD*-controlled tests.

For the 4PS test specimens, Fig. 2 shows the testing configuration. An I-beam was interposed between the testing machine and the specimen in order to distribute the load. Crack tip sliding displacement (*CTSD*)-controlled testing with an imposed rate of 0.001 mm/sec was used. The vertical displacement at load point, *CMOD*, *CTOD* and *CTSD* were measured.

FAILURE BEHAVIOR AND RESPONSE TO LOADING

The load versus *CMOD*, load-point displacement, *CTOD* and *CTSD* for the 3PBB tests are shown in Figs 3– 6. X_1 and X_3 in these Figs refer to the 25 and 100 mm distances between the center line of specimens and the location of notch; we also performed tests with X_2 = 50 mm, the results of which were rather comparable to those obtained with X_1 = 25 mm. The ductility and energy absorption capacity increased drastically when steel and polypropylene fibers were used. As the distance between the center line of specimens and the location of notch (X_i) increased, the maximum load increased and both the vertical displacement and *CMOD* decreased, more clearly before the peak. At higher X_i values, the crack for plain concrete became unstable in spite of the fact that we adopted displacement-controlled testing with *CMOD* used as the feedback signal.

ENERGY RELEASE RATE

The energy release rate was calculated using the direct energy method. This method considers

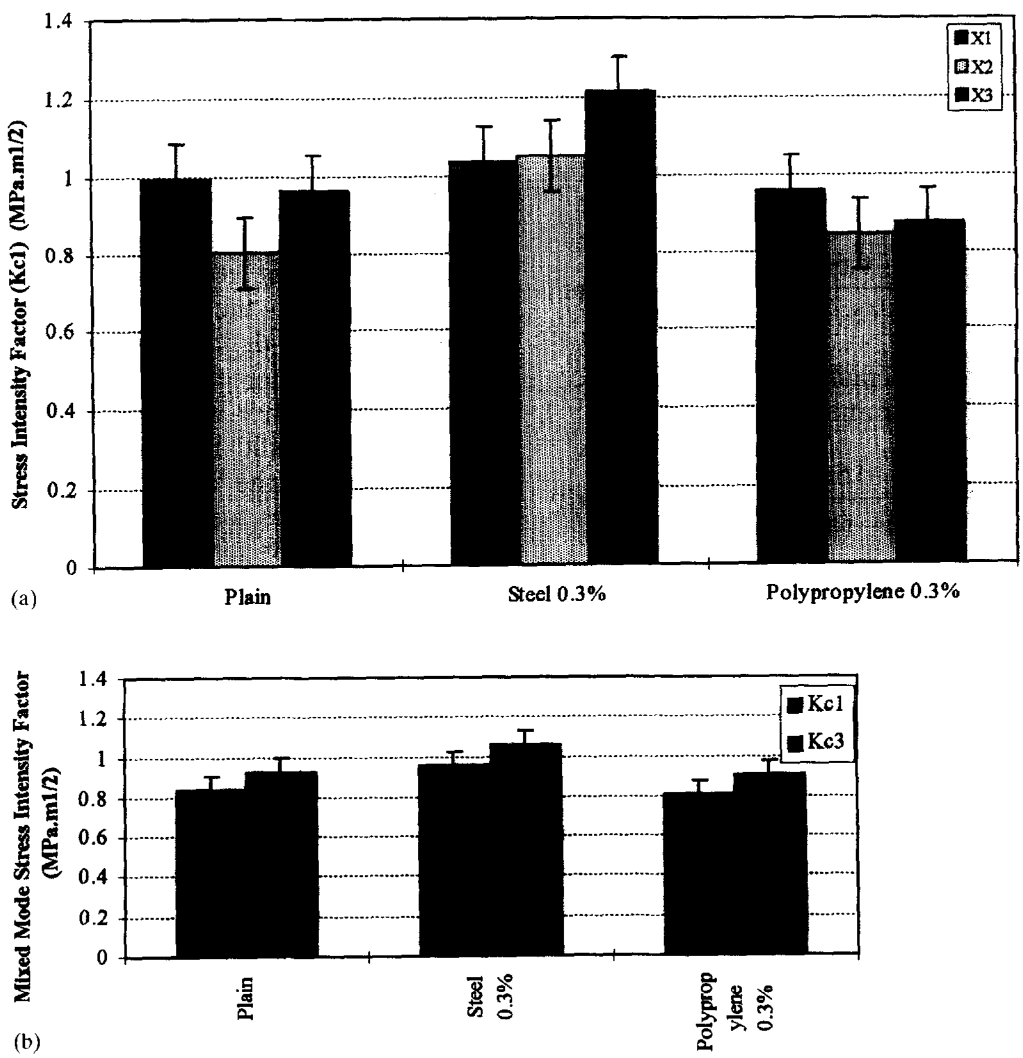

Fig. 8. Mean values and pooled 95% confidence intervals of mixed-mode stress intensity factor (K_{C1}): (a) the 3PBB test configuration; (b) the 4PS test configuration.

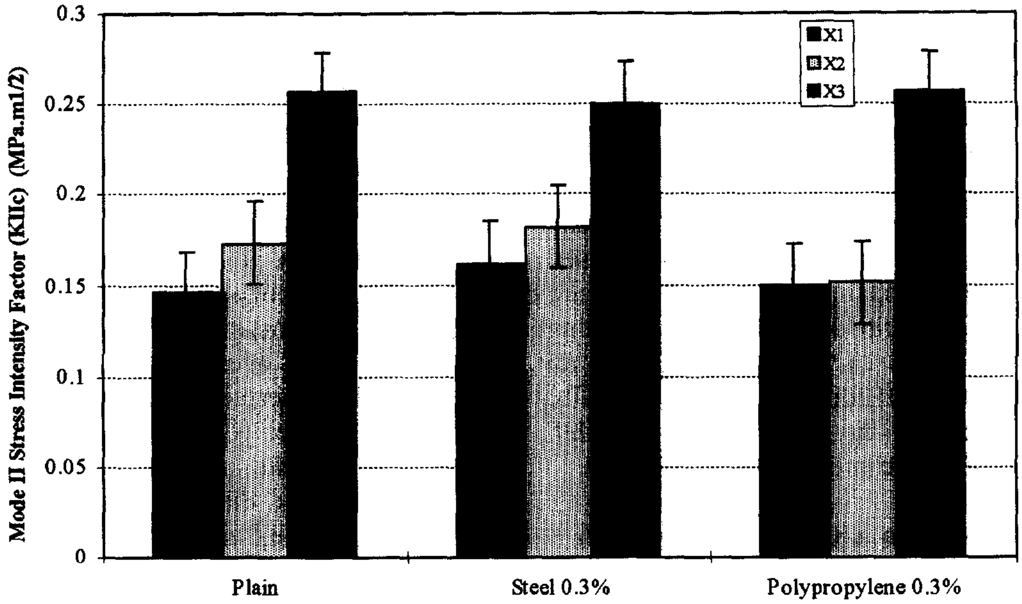

Fig. 9. Mean values and pooled 95% confidence intervals of mode II stress intensity factor (K_{II}) for the 3PBB test configuration.

only the energy absorbed by the beam to the point of instability.[5] The point of instability is defined as the point on the load against the vertical deflection curve where the load begins to drop off. When the crack is inclined, the energy release rate formula is:

$$G_{C_1} = \frac{U - 0.5mg\bar{\delta}_0}{bh[1-(a_0/h)]/\cos\alpha}. \quad (1)$$

Because of the rough crack area in inclined sections, G_C can be determined as:[5]

$$G_{C_2} = \frac{U - 0.5mg\bar{\delta}_0}{1.15bh[1-(a_0/h)]/\cos\alpha}. \quad (2)$$

When the crack is normal (i.e., $\alpha = 0$), eqn (1) simplifies to:

$$G_{C_3} = \frac{U - 0.5mg\bar{\delta}_0}{bh[1-(a_0/h)]} \quad (3)$$

where U is the area under the load versus vertical deflection curve up to the point of instability, δ_0 is the vertical deflection at the instability point, α is the angle between the vertical plane and the crack plane, mg is the unit weight of the specimen, a_0 is the initial notch depth, and b and h are the specimen width and height, respectively.

In this study, the unit weight of the specimen was neglected. Using eqns (1–3), the energy release rates G_{C1}, G_{C2} and G_{C3} were calculated, respectively. The values of G_{C1}, G_{C2} and G_{C3} for the 3PBB and 4PS test specimens are presented in Tables 1 and 2 and Fig 7(a) and (b). Analysis of variance and comparison of means indicated that the addition of low volumes of steel or polypropylene fibers did not have statistically significant effects on the energy release rate. However, the mean value of the area under the load–deflection curve up to instability point (U) was higher with steel fiber than those obtained for other mixes. The notch location in the 3PBB test did not have statistically significant effects on the U values. Table 4 shows the ratio of U values for the 3PBB test to those for the 4PS test. Generally, the ratios of U values were larger than 1.0; so, the U values for the 3PBB test were larger than those for the 4PS test.

STRESS INTENSITY FACTOR

Based upon the principles of linear elastic fracture mechanics, the mixed mode critical stress intensity factor may be expressed as:

$$K_C = \sqrt{G_C E}. \quad (4)$$

where E is the modulus of elasticity.

By using the different values of G_C in Table 1 and Table 2 for the 3PBB and 4PS tests, the values of K_C in mixed mode fracture were calculated. The resulting values of K_C are presented in Tables 2 and 3 and Fig. 8(a) and (b) for all beams tested. The values of K_{C1}, K_{C2} and K_{C3} were calculated using G_{C1}, G_{C2} and G_{C3}, respectively in eqn (4). Statistical analysis of variance and comparison of means indicated that there were no statistically significant effects on the mixed mode stress intensity factor when steel or polypropylene fibers were added to plain concrete. The location of notch in the 3PBB test also did not have statistically significant effects on the mixed-mode stress intensity factor for all mixes used in this study.

It was mentioned[2] that the conventional single-parameter fracture approach cannot be used to characterize the general mixed-mode fracture behavior of concrete. Table 4 also shows the ratios of mixed-mode stress intensity factor for 3PBB to that for 4PS and to linear elastic fracture mechanics mode I stress intensity factor for specimens with 0.35h notch depth. From this table, the mixed-mode stress intensity factors for the 3PBB test in the case of plain concrete and steel fiber concrete were, respectively, 20% and 20–40% higher in the 3PBB test when compared with the 4PS test. In the presence of polypropylene fibers, however, the mixed mode stress intensity factor for the 3PBB test was almost equal to that for the 4PS test. The mixed-mode stress intensity factor thus generally depends on the test configuration; this provides support for the conclusions derived.[2] The mixed-mode stress intensity factor (K_{C1}) was higher than that of conventional mode I stress intensity factor, K_{IC} (LEFM) by 34, 40 and 26% for plain, steel fiber concrete and polypropylene fiber concrete, respectively;

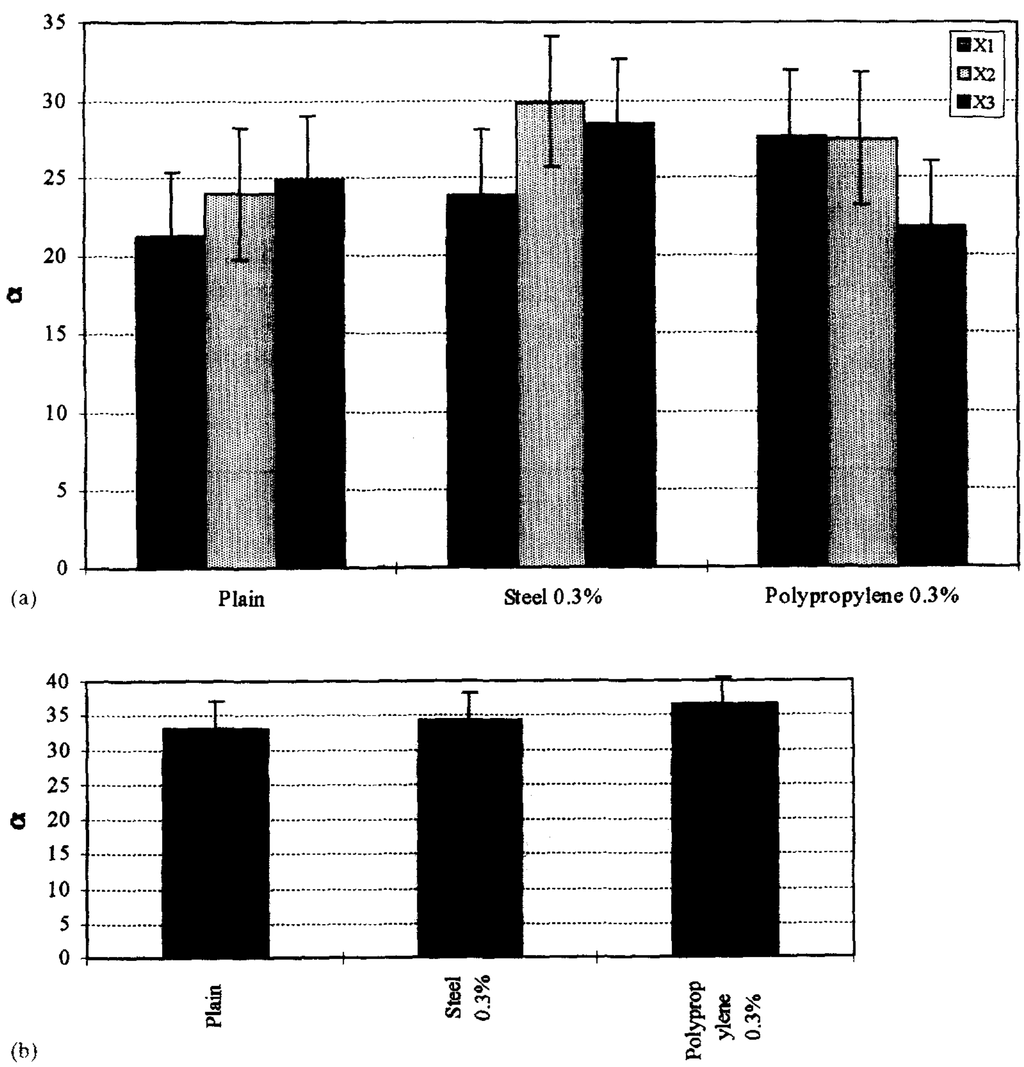

Fig. 10. Mean values and pooled 95% confidence intervals of initial crack angle (α): (a) the 3PBB test configuration; (b) the 4PS test configuration.

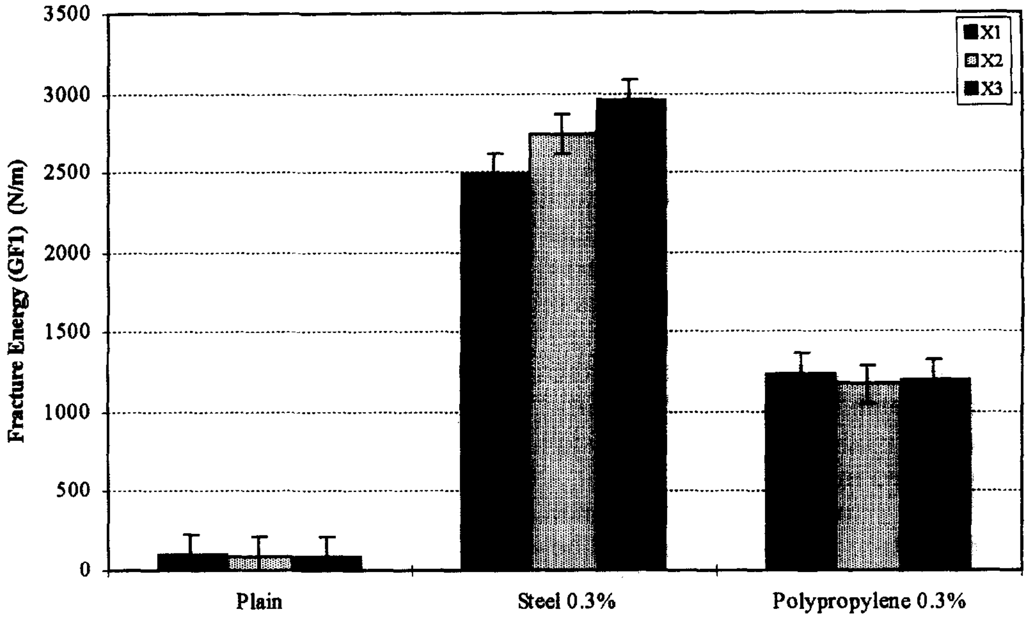

Fig. 11. Mean values and pooled 95% confidence intervals of fracture energy (G_{F1}) for the 3PBB test configuration.

the values of K_{C3} were also higher than those of conventional mode I stress intensity factor (K_{IC}) for these mixes by 47, 58 and 40%, respectively.

Mode II stress intensity factor (K_{IIC}) was calculated using Tada's formula [6] for a single-edge-notched beam subjected to shear. This formula is presented below:

$$K_{II} = \frac{2Q}{b\sqrt{\pi a}} \cdot \frac{1.30-0.65(a/h)+0.37(a/h)^2+0.28(a/h)^3}{\sqrt{1-(a/h)}} \quad (5)$$

where Q = applied shear force at notch, $a = a_0$ = notch depth, and b and h = beam width and depth, respectively. Table 3 and Fig. 9 show the mode II stress intensity factors for all mixes used in this study under the 3PBB test configuration. Analysis of variance and comparison

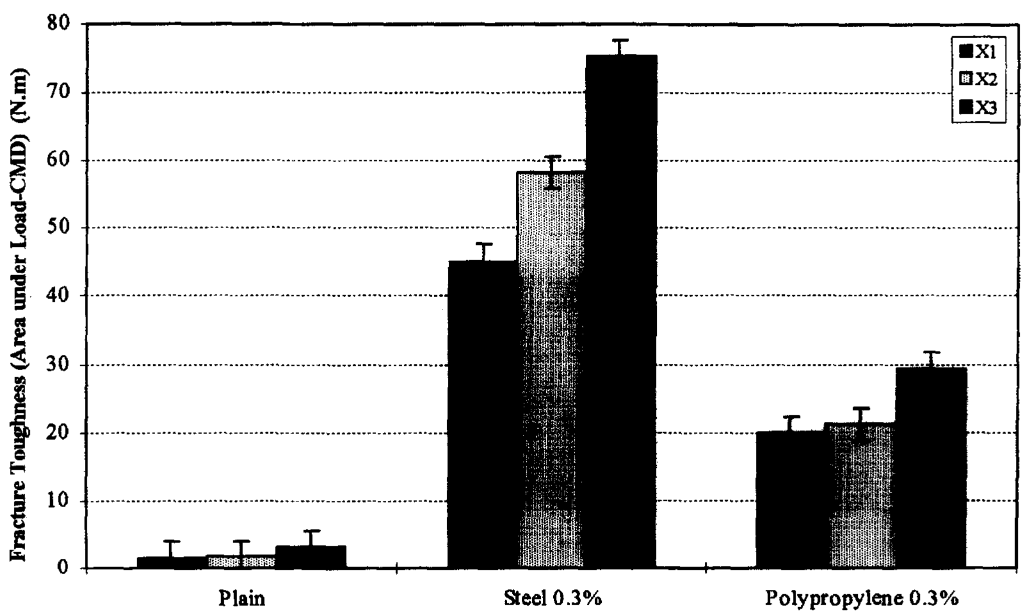

Fig. 12. Mean values and pooled 95% confidence intervals of area under the load–*CMD* curve (A_{CMD}) for the 3PBB test configuration.

of means indicated that the mode II stress intensity factor depends on the location of notch for all mixes; as the distance X increases, the mode II stress intensity factor also increases.

CRACK INITIATION ANGLE (A)

For mixed-mode failure of concrete, the determination of the final failure path and the criteria for crack instability are more complicated than for pure mode I failure. In mode I failure, the crack always propagates along its own plane, which is not the case in mixed-mode crack propagation. So, criteria for determining the crack propagation trajectory and crack instability are necessary for mixed-mode fracture.[2] It can be seen that the crack initiation angles are difficult to measure due to the tortuosity of crack paths. The results of initiation crack angles are presented in Table 1 and Table 2 for the 3PBB and the 4PS tests, respectively. Analysis of variances and comparison of means (see Fig. 10) indicated that the concretes containing polypropylene and steel fibers produced the same crack initiation angles as plain concrete. The location of notch also did not affect the initial crack angle. However, the test configuration influenced the initial crack angle; the 4PS test gave higher crack initiation angles than the 3PBB test. This may be due to the higher shear stress in the 4PS test configuration as compared to that in the 3PBB test.

NON-LINEAR FRACTURE PARAMETERS

Fracture energy

The fracture energy is calculated as the total energy to fracture of the specimen divided by the area of the original uncracked section. The following equation is used to determine the fracture energy:

$$G_F = \frac{W_0}{bh[1-(a_0/h)]} \cdot \cos\alpha \qquad (6)$$

where W_0 is the total area under the load–deflection curve. G_F values are presented in Table 3 and are called G_{F1}. When the cracked rough area of the inclined section is considered, the fracture energy is determined using the following equation and is called G_{F2}:

$$G_{F2} = \frac{W_0}{1.15bh[1-(a_0/h)]} \cdot \cos\alpha. \qquad (7)$$

When the crack is normal, the fracture energy is obtained using the following equation and is called G_{F3}:

$$G_{F3} = \frac{W_0}{bh[1-(a_0/h)]}. \qquad (8)$$

Table 3 and Fig. 11 show the fracture energies G_{F1}, G_{F2} and G_{F3} for the 3PBB test configuration. Analysis of variance and comparison of means indicated that steel and polypropylene fibers have statistically significant effects on fracture energy. The location of notch did not have statistically significant effects, except for steel fiber concrete where the fracture energy increased with increasing distance X.

Fracture toughness (area under load–*CMD* curve)

The crack mouth displacement (*CMD*) is the vector sum of crack mouth opening displacement(*CMOD*) and crack mouth sliding displacement (*CMSD*). The total area under the load–*CMD* curve is called the fracture toughness (A_{CMD}). Table 4 and Fig. 12 show the fracture toughness test results for the 3PBB test configurations. Analysis of variance and comparison of means indicated that steel and polypropylene fibers produced higher values of fracture toughness than plain concrete.

CONCLUSIONS

Mixed-mode fracture test results on plain concrete and concrete reinforced with low (0.3%) volume fraction of steel and polypropylene fibers indicated that:

(1) Fibers significantly influence the post-peak behavior of concrete in mixed-mode fracture tests. Nonlinear fracture mechanics principles are thus appropriate to interpret

the test results. Nonlinear fracture parameters (fracture energy and toughness) showed statistically significant increases with the addition of low fiber volume fractions.

(2) When linear elastic fracture mechanics concepts were used to interpret the test results, only 0.3% volume fraction of steel fibers increased the mean value of the mixed-mode stress intensity factor; this increase was not, however, statistically significant.

(3) Test configuration and location of notch had interactive effects on the mixed-mode fracture properties of concrete.

(4) A general conclusion is that 0.3% volume fraction of steel and polypropylene fibers are effective in the control of both crack opening and slippage; steel fibers are more effective in this regard. These results point at the effectiveness of a proper fiber volume fraction as secondary (non-structural) reinforcement in concrete.

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