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Design of Non-Flexural Members for Shear

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Abstract

Over the past 35 years, or so, extensive experimental programmes have been undertaken to establish the behaviour of reinforced concrete non-flexural members, such as deep beams, nibs, corbels, beam–column joints, etc. The experimental evidence is that non-flexural members display an increased shear capacity relative to flexural members due to the effects of arching. From these early studies a number of empirical models were developed for design. The main limitations from the empirical approach are a limited range of shear span to depth ratios for which the design equations apply and an inability to explain the mechanics of non-flexural behaviour. In the last decade, or so, finite element studies incorporating non-linear material models have been used in combination with well-targeted experimental programmes to determine the mechanics of non-flexural behaviour. These studies have, in turn, led to the development of rational design models, based on the strut and tie approach, for the design of non-flexural members cast with concrete strengths up to 100 MPa. In this paper a strut and tie model is used to describe the observed mechanics of non-flexural behaviour including a local model to assess splitting forces which develop in the concrete struts. Finally, a rationale behind empirically determined minimum reinforcement demands is developed and minimum web reinforcement requirements presented.

Keywords: corbels, deep beams, design, reinforced concrete, shear, strut-and-tie models

INTRODUCTION

It has long been recognised that the shear strength of deep beams and other non-flexural members is considerably higher than for shallow members having the same span. Early studies, concentrated on development of empirical design methodologies, many of which are summarised by Kong[1] and in the ASCE–ACI Joint Task Committee 426.[2] The greater shear strength of the non-flexural members is a result of internal arching. That is the process by which load is transferred directly to a support through the development of compression struts. Figure 1 shows the compression stress results of non-linear finite element (FE) analyses on deep and shallow members.[4] For the deep beam the development of the compression strut directly from the applied load to the support is clearly evident, whereas no direct compression strut exists for the shallow beam.

The limitations of an empirical approach are a restricted range of shear span to depth ratios for which the design equations apply, limited concrete strengths and an inability to explain the mechanics of non-flexural behaviour. Experimental studies have identified three modes in which non-flexural members fail (not including failure due to poor detailing): tension or flexural failure, compression failure (including bearing failures) and diagonal splitting failure. This paper presents the developments in strut and tie models that cover the full range of observed non-flexural behaviour leading to a rational design methodology. Some examples of non-flexural members showing possible strut and tie models are shown in Fig. 2.

A number of strut and tie models have been used to describe the development of the compression struts. One of the earliest is the plastic truss model, first appearing in 1978[5] and used extensively in the design of non-flexural members by Rogowsky & MacGregor.[6] Whilst other truss models can be adopted, the appeal of the plastic truss model lies in its simplicity. Two points distinguish the plastic truss model from other truss models: (1) all members enter the nodal zones at 90°; (2) at the nodes, the members have the largest cross-sectional area available of any truss model. Away from the nodes the struts may bottle outwards, giving the critical sections for compression at the faces of the nodes. The significance of point (1) is that the nodes are in a state of hydrostatic compression and thus do no require checking for shear; whilst point (2) implies that when calibrating the efficiency factor (an empirical factor required to take into account the brittleness of the concrete compressive struts) the plastic truss model gives the lowest factor of the infinite selection of truss models available. A third, less significant, advantage is that if the boundary conditions are satisfied, and if the cross-section is constant in area, then the internal compression members are automatically satisfactory. This is not necessarily so with other truss models and compression members are

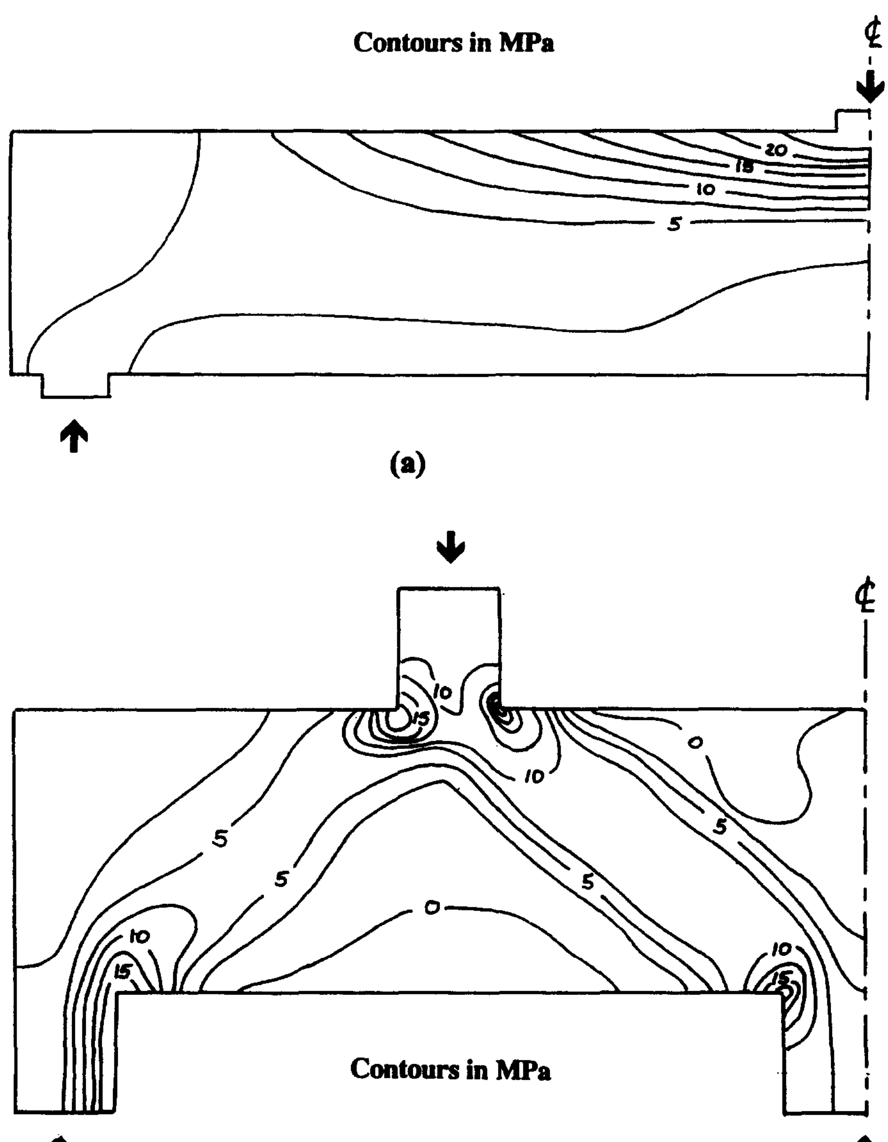

Fig. 1. Principal compression stress contours obtained from a non-linear FE model for (a) Bresler and Scordelis beam A1;[3] (b) Foster beam 1.[4]

required to be individually checked for normal and shear stresses.

Under increasing load conditions the compression struts change from an exaggerated bow shape before cracking, to a relatively narrow band at ultimate, provided that splitting forces are kept under control. Splitting forces are tracked by developing a local strut and tie model within the global plastic truss model. From this model the magnitudes of splitting forces that exist in the compression struts can be obtained for both the serviceability and strength limit states, enabling sufficient reinforcing steel to be detailed to control bursting. Finally, a rationale behind empirically determined minimum reinforcement requirement is developed and minimum web reinforcement requirements presented.

DESIGN FOR TENSION AND COMPRESSION

A number of equilibrium models could be used for the design of deep beams. One such model is outlined in the CEB–FIP Model Code[7] for the modelling of disturbed regions. A similar model is described here where deep beams are categorised into one of three types with the model selected taken as a function of the shear span to the internal lever-arm a/z.

Type I: in this model the shear is carried from the load points to the supports directly by major compression struts and, thus, no account is taken of forces that may exist in any vertical web reinforcement.

Type II: the shear is taken to the supports by a combination of major and minor compression

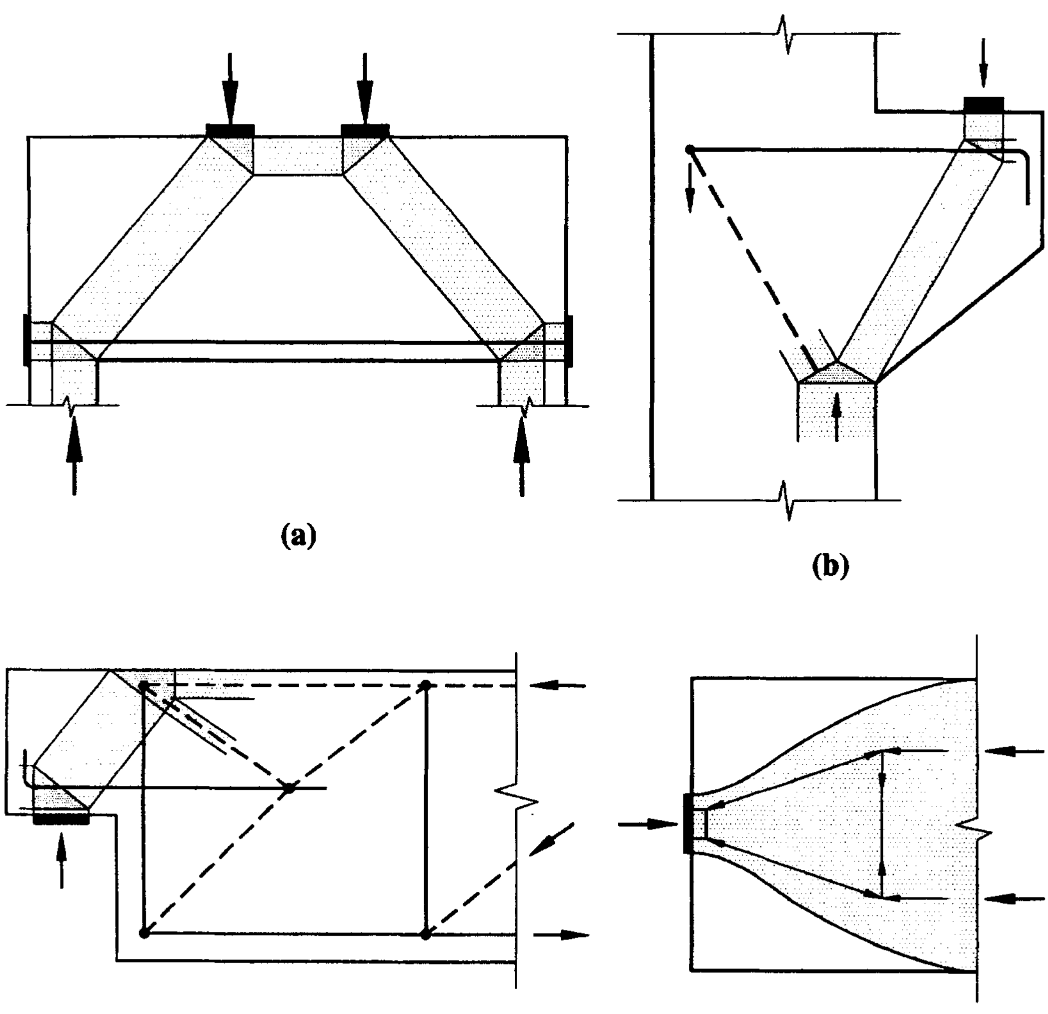

Fig. 2. Examples of non-flexural members showing typical strut and tie models for: (a) deep beam; (b)corbel; (c) dapped connection; (d) prestresses anchorage zone.

struts. In the type II model, hanger reinforcement is required to return the vertical components of forces developed in the minor compression struts to the top of the member.

Type III: the shear is carried to the supports via a series of minor compression struts with hanger reinforcement used to return the vertical components of the compression strut forces to the top of the member.

The three types of plastic truss model, together with their design equivalents, are shown in Fig. 3. The appropriate design model depends on the ratio of the shear span a to the internal lever-arm z and, as a general guide, may be selected from eqn (1).

$$\begin{aligned}&\text{Type I} && a/z \le 1\\ &\text{Type II} && 1 < a/z \le \sqrt{3}\\ &\text{Type III} && a/z \ge \sqrt{3}\end{aligned} \qquad (1)$$

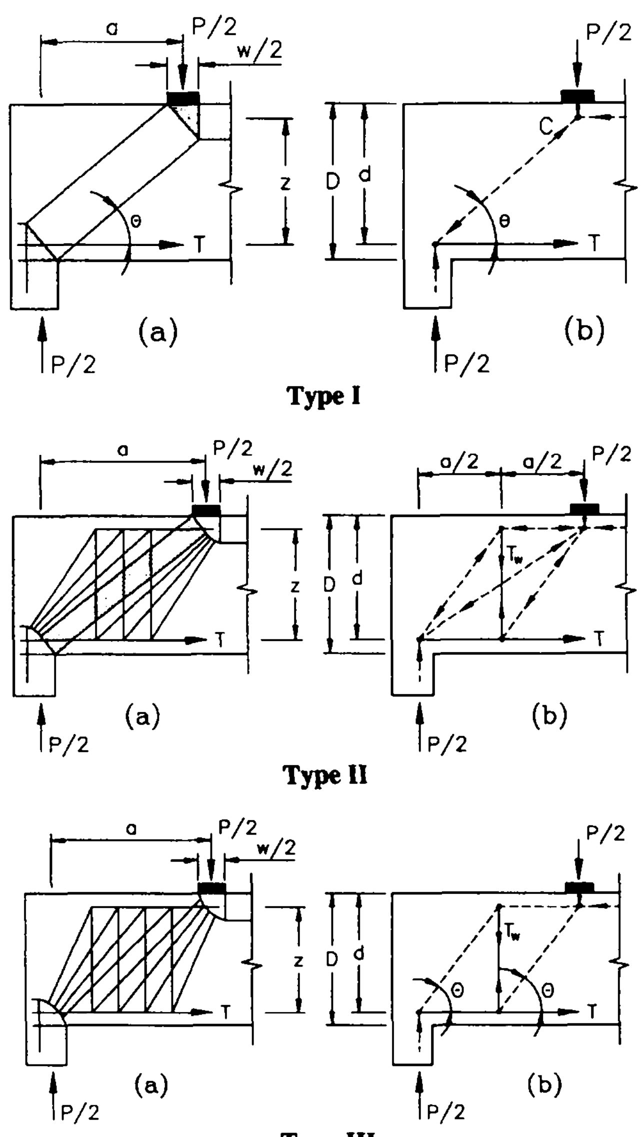

Fig. 3. Types I, II and III stut and tie models: (a) plastic truss model; (b) simplified design model.

For the Type II model, it is for the designer to select the ratio of shear forces carried by the major and minor compression struts within the limits $0 \le T_w \le F$, where T_w is the vertical component of the force carried by the minor compression struts and F is the total vertical component of the external load carried through the shear span. A simple linear relationship may be adopted for estimating T_w:

$$T_w = \left[\frac{(a/z) - 1}{\sqrt{3} - 1} \right] F \qquad (2)$$

The remainder of the shear force is carried directly to the supports by arching action through the major compression strut.

Figure 3 shows an idealised plastic truss model with the development of the major tensile and compressive components. From the model the design for tension and compression failure follow; that is tension reinforcement is detailed such that

$$A_{st} \ge \gamma_s T / f_{sy} \qquad (3)$$

where A_{st} is the area of reinforcement required to be smeared uniformly through the tension zone, γ_s is a partial safety factor for steel reinforcement, and f_{sy} is the yield stress of the reinforcing bars used. For compression

$$C \le \nu f'_c d_c b / \gamma_c \qquad (4)$$

where C is the force in the compression strut, γ_c is a partial safety factor for concrete, ν is an efficiency factor, where $\nu \le 1$, f'_c is the characteristic cylinder strength, d_c is the width of the compression strut and b is the width of the beam.

Using the balanced design approach all members (in theory at least) fail simultaneously. If ductility is taken to be a desirable feature, then a factor is needed to give the built structure a bias towards the more ductile tension failure wherever possible. It is important, therefore, to distinguish between compression field and tension field failures. All else being equal, the γ factors adopted in European and Canadian codes lead to a bias in the design towards the more favourable tensile failure mode. Strength reduction, or ϕ, factors used in the ACI[8] and Australian[9] codes penalise the member rather than the material. Thus, a large number of factors are needed to account for the various member behaviours. In non-flexural members, as in other members, it is desirable that the reinforcing steel yields before the concrete crushes. Whilst the ultimate behaviour may not be described as ductile, distress in the member is evidenced by significant cracking in the concrete. Thus, it is important that different partial safely factors γ or strength reduction factors ϕ be adopted for the tension and compression components of the strut and tie model.

THE CONCRETE EFFICIENCY FACTOR

To corroborate the design model with experimental results a single, empirically derived, efficiency coefficient is introduced. First applied by a Danish research group under the direction of Nielsen,[5] the efficiency factor ν takes into account the non-perfect assumption that concrete is a perfectly plastic material, together with other documented phenomena such as the effect of transverse tensile strains on the concrete compressive strength.[10]

Various investigators have proposed different values for the efficiency factor. Marti[11] suggests that a value of $\nu = 0.6$ be adopted, whereas Rogowsky & MacGregor[6] suggest $\nu = 0.85$. More recently Yun & Ramirez[12] and MacGregor[13] have proposed various values for the efficiency factor based on the stress state of the nodes. A number of attempts have been undertaken to establish the efficiency factor for normal and high strength concrete. Chen[14] undertook a study on 412 beams with shear span to depth ratios less than 2.5 and proposed the expression for normal strength concrete ($f'_c \leq 60$ MPa)

$$\nu = \frac{0.60(1-0.25h)(100\rho+2)[2-(0.4a/h)]}{\sqrt{f'_c}} \quad \text{for} \begin{bmatrix} a/h \ngtr 2.5 \\ \rho \ngtr 0.02 \\ h \ngtr 1.0 \end{bmatrix} \tag{5}$$

where h is the height of the beam in metres, ρ is the reinforcement ratio for the main longitudinal steel and f'_c is in megapascals. Of the 412 beams studied, however, many failed in flexure or by diagonal splitting, and although many of the specimens were reported to have failed in shear, in many of these specimens the shear mode probably occurred after yielding of the primary tension steel. Foster and Gilbert[32] compared eqn (5) against a number of high strength concrete deep beams and corbels failing in compression and showed that the correlation was not particularly good, but in all cases conservative.

Based on the extensive panel tests of Vecchio & Collins,[10] Collins & Mitchell[16] proposed the relationship

$$\left.\begin{aligned} \nu &= \frac{1}{0.8+170\varepsilon_1} \\ \varepsilon_1 &= \varepsilon_x + (\varepsilon_x - \varepsilon_2)/\tan^2\theta \end{aligned}\right\} \tag{6}$$

where ε_1 is the major principal strain (tension normal to the compression strut), ε_2 is the minor principal strain (parallel to the compression strut), ε_x is the strain in the horizontal direction and θ is the angle of the strut to the horizontal. Eqn (6) is adopted in the Canadian Code CSA94[17] with ε_x replaced with the tensile strain in the tensile tie ε_s and the limitation that $\nu \leq 0.85 f'_c$. Based on eqn (6), Foster & Gilbert[18] developed the simplified relationship

$$\nu = \frac{1}{1.14+0.75(a/d)^2} \tag{7}$$

where a/d is the shear span to effective depth ratio (refer to Fig. 3).

Undertaking a series of non-linear finite element studies, Warwick & Foster[19] proposed the relationship

$$\left.\begin{aligned} &\nu = 1.25 - \frac{f'_c}{500} - 0.72\left(\frac{a}{d}\right) + 0.18\left(\frac{a}{d}\right)^2 \leq 0.85 \quad \text{for } a/d < 2 \\ &\nu = 0.53 - \frac{f'_c}{500} \quad \text{for } a/d \geq 2 \end{aligned}\right\} \tag{8}$$

where $f'_c \leq 100$ MPa. Eqn (8) and eqn (7) give similar results for $f'_c = 100$ MPa, but for lower strength concretes eqn (8) gives higher values than eqn (7) for the efficiency factor. Both eqns (7) and (8) give a fair correlation with experimental data for non-flexural members where the failure mode is governed by the strength of the concrete struts. Eqn (8) has been shown to model accurately corbels failing in a compression mode and fabricated with normal and high strength concrete.[15]

DESIGN FOR WEB SPLITTING

Whilst the simple models shown in Fig. 3 demonstrate the tension and compression failure modes, more detail is required to describe splitting failure accurately. For a linear material, continuity requires that rather than being parallel sided the compression struts are bowed. Non-linear FE studies, combined with experimental observations, indicate that the maximum width of the struts occurs prior to cracking. Once cracking occurs the struts become significantly narrower, approaching that of the theoretical model.[20,4] Whilst immediately after cracking the force across the splitting plane may be reduced, splitting cracks are wide and become a serviceability issue.

An equilibrium model to describe splitting failure, consistent with the plastic truss approach, is shown in Fig. 4. Previous studies[21] have indicated that before cracking the angle of deviation of the compression strut α to the average strut angle θ is $\alpha \approx \arctan(1/2)$. This compares favourably with experimental and numerical observations of Crawford & Foster.[20] However, after cracking the numerical and experimental evidence is that $\alpha \approx \arctan(1/5)$. Two design requirements need to be evaluated, that is serviceability and strength. Foster[4] showed that for deep beams cast integrally with columns the distribution of stresses at the nodes is not uniform as the plastic truss model suggests, and that significantly higher stresses occur at the column edges. This leads to a steeper strut angle θ than shown in Fig. 4; however, the model is sufficiently accurate for design purposes.

For the model given in Fig. 4, the total bursting force is

$$T_b = C \tan \alpha \tag{9}$$

Ignoring any capacity of the concrete to carry bursting forces, and adopting an orthogonal layout of reinforcing steel, the areas of bursting reinforcement required are

$$\left.\begin{aligned} A_{wh} &= \gamma_s T_b \sin \theta / f_s \\ A_{wv} &= \gamma_s T_b \cos \theta / f_s \end{aligned}\right\} \tag{10}$$

where A_{wh} and A_{wv} are the areas of horizontal and vertical web reinforcements respectively; γ_s is the partial safety factor for steel (taken as $\gamma_s = 1$ for serviceability design) and f_s is the stress in the reinforcing steel. For service load design it may be desirable to limit the strain in the reinforcement (typically to 1000 $\mu\varepsilon$) and hence the stress. For the strength limit $f_s = f_{sy}$, where f_{sy} is the yield strength of the steel and γ_s is specified in building codes. The deviation angle can be taken as

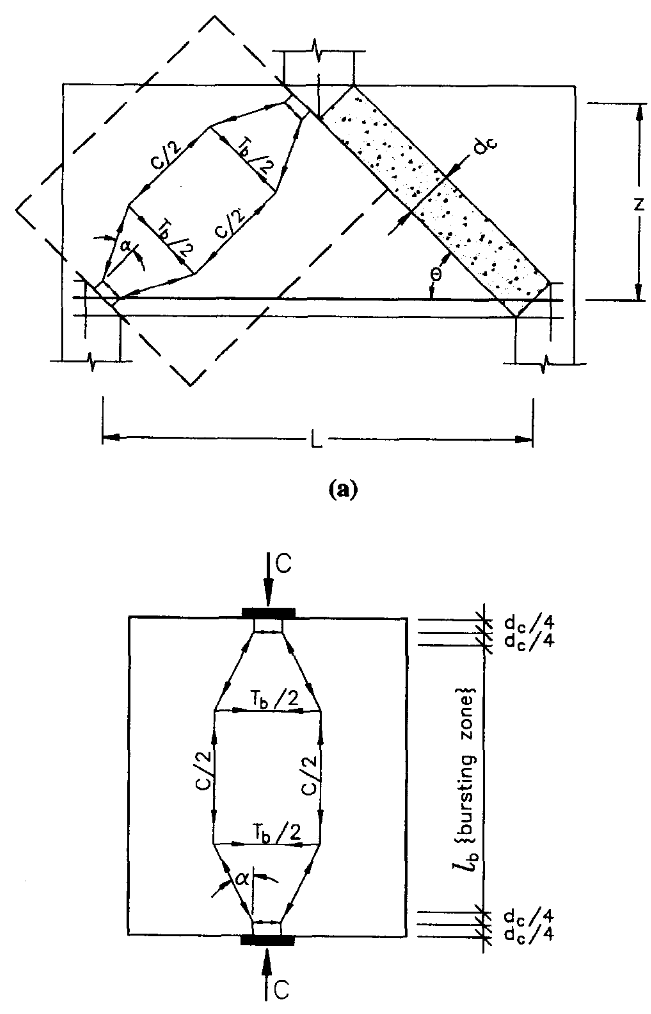

Fig. 4. Strut and tie model used to describe web splitting.

$$\tan\alpha = 1/2 \text{ for serviceability}$$
$$\tan\alpha = 1/5 \text{ for strength} \qquad (11)$$

Bursting reinforcement is to be evenly distributed through the bursting zone l_b (see Fig. 4), where

$$l_b = \sqrt{z^2 + a^2} - d_c \qquad (12)$$

and z and a are the lever arm and shear span respectively (refer also to Fig. 3).

Ramakrishnan & Ananthanarayana[22] identified splitting failure as similar to the failure of a split cylinder. In numerical studies by Crawford & Foster[20] it was observed that prior to cracking the transverse distribution of stress in the bursting region was approximately uniform. This fits well with the observations of Ramakrishnan & Ananthanarayana and the strut and tie model described in Fig. 4. Taking a uniform distribution of transverse stress through the bursting region and assuming that $f_t = fn(\sqrt{f'_c})$, where f_t is the tensile strength of the concrete, then the bursting force at cracking is

$$T_{b,cr} = l_b b K \sqrt{f'_c} \qquad (13)$$

where b is the thickness of the beam and K is a splitting coefficient. The results of an evaluation of the splitting coefficient are given by Foster & Gilbert[23] for experimental data where diagonal splitting was identified as the mode of failure. Whilst some variation exists between different test series, the results are consistent within each series with the variations between series within the normal tensile strength variation for concrete.

If it is taken as desirable that the beams not fail suddenly upon diagonal cracking of the concrete, then the minimum reinforcement required is that needed to carry the bursting forces at the time of cracking. Taking the splitting coefficient as $K = 0.6$ and substituting eqn (11) into eqn (13) leads to a minimum reinforcement requirement of

$$p_{min} = \frac{\sqrt{f'_c}}{4 f_{sy}} \qquad (14)$$

where p_{min} is the minimum reinforcement ratio. Eqn (14) is plotted in Fig. 5 for a 400 MPa grade web reinforcing steel, together with a reduced value if it is assumed that the concrete carries a residual tensile capacity $0.3f_t$. Figure 5 and eqn (14) demonstrate the need to increase

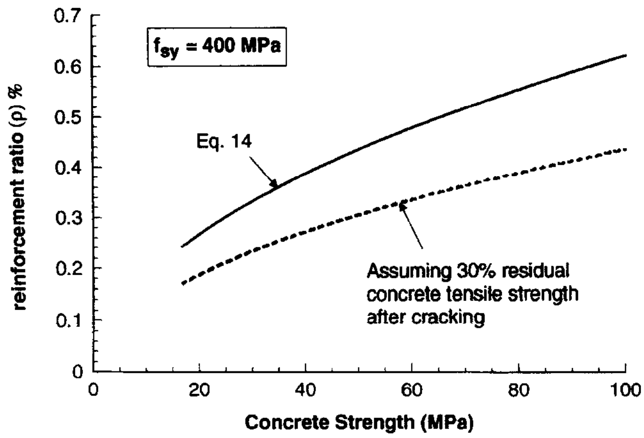

Fig. 5. Minimum reinforcement ratio versus concrete strength for f_{sy} = 400 MPa.

the volume of web reinforcement with increasing concrete strength. Higher strength members have the capacity to carry higher axial loads and, thus, have larger bursting forces in the struts at the time of concrete cracking.

DESIGN OF CONTINUOUS DEEP BEAMS

Rogowsky[24] reported that the first tests on continuous deep beams were undertaken by Nylander & Holst;[25] however, the first major study on the behaviour of continuous deep beams is that by Leonhardt & Walther.[26] Since then some major studies include those by Rogowsky & MacGregor,[6,27] Ricketts & MacGregor,[28] Rogowsky *et al.*[29] and Foster.[4] Continuous deep beams are stiff members and on the surface look unforgiving. This has led some[24] to suggest that differential support settlements are of fundamental importance in predicting the behaviour of continuous deep beams. The author is unaware, however, of any experimental research undertaken to support this conclusion.

The main difficulty in designing continuous deep beams is in determining the support reactions. If the support reactions can be obtained then a strut and tie model can be developed and the steel reinforcement detailed to carry the prescribed forces. Rogowsky & MacGregor[6] noted

> "A plasticity truss with top and bottom chord forces assigned arbitrarily will not lead to safe predictions of beam capacity. The most appropriate truss will be the one

which produces the expected distribution of support reactions."

Schliach *et al.*[21] proposed that the strut and tie model be based on linear-elastic stress analyses. For eight two-span deep beams analysed by Foster,[4] the difference in support reactions at ultimate between linear-elastic and non-linear FE analyses was less than 15% for deep beams failing in a compression mode. Further investigations showed that when proportioning the beams such that all elements (tension and compression) failed simultaneously the reactions are within 15% of those of the linear analysis provided that not less than 0.2% horizontal and vertical web reinforcement is used. Whilst for large-scale projects FE analyses may not be a large expense relative to the overall design cost, this may not be the case for smaller projects. Further, numerical data must be analysed with extreme care and should always be viewed with scepticism when detailing non-linear structures failing in non-ductile modes. Whilst it is always desirable that deep beams fail in a ductile manner (here ductility is taken as large strains in one or more tension members before the concrete crushes) this may not always be possible. Take, for example, the case of a wall (which is a deep beam with a shear span to depth ratio $a/D = 0$). Clearly, for a wall the horizontal tension reinforcement will not carry load and cannot have yielded before crushing of the concrete in compression. Often, however, a high level of accuracy (or complexity) is unwarranted. Rogowsky[24] suggests that the ability of any design model to obtain support reaction accuracies better than 25%, in practice, are unwarranted owing to the uncertainty of the effects of support movements.

For a two-span continuous deep beam under equal point loadings applied to the centre of each span (refer to Fig. 6(a)) the central support reaction R_2 must lie within the limits of that for a wall $L/D = 0$ and that for flexural theory $L/D = 5$; that is $P \leq R_2 \leq 1.375P$. For a two-span deep beam with $L/D = 2$ loaded by a single point load applied at the centre of each span an engineering judgement may suggest that the reaction $R_2 = 1.2P$. The maximum error in the support reactions would then be 20%, less than the 25% suggested as acceptable by Rogowsky.[24] Similarly, for a uniformly loaded two-span deep beam (refer to Fig. 6(b)) $P \leq R_2 \leq 1.25P$, where P is the total load on one span. In this case judgement may suggest that for a two-span beam with $L/D = 2$ a central support reaction of $R_2 = 1.15P$ giving an error of less than 15%.

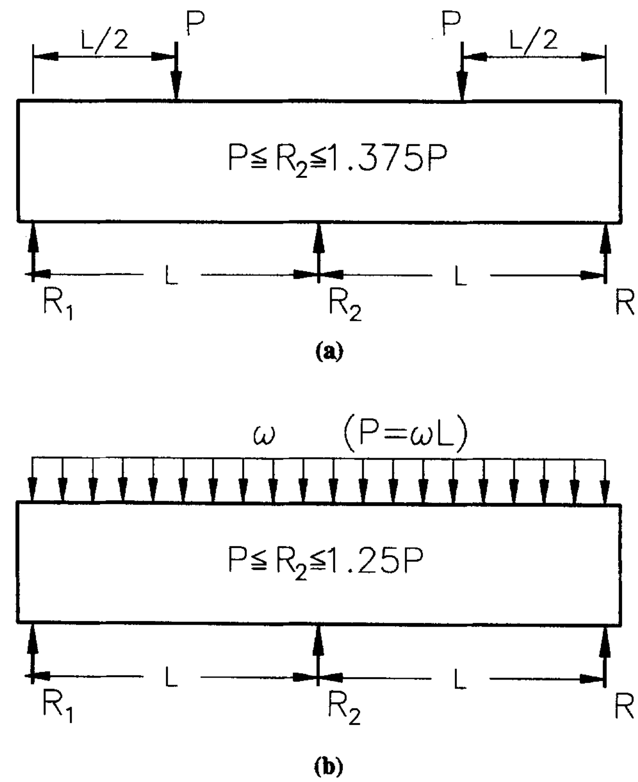

Fig. 6. Two-span deep beam under: (a) midspan concentrated loads; (b) uniform loading.

DESIGN BY STRESS ANALYSIS

There is argument that the strut and tie model should follow the stress path obtained from a linear-elastic FE analysis. Whilst this may be reasonable when looking at the control of crack widths required for serviceability analyses, and while true that an elastic solution is but one of an infinite number of lower-bound plasticity solutions, an elastic analysis may significantly over-estimate bursting forces (as discussed above). Matched experimental/FE analyses by Foster[4] and FE studies by Crawford & Foster[20] show that, after cracking, critical compression struts are narrower than elastic analyses suggest. Compare, for example, the strut width for the internal and external shear spans for the results of a linear-elastic analysis of Foster's[4] beam 1, shown in Fig. 7, with the results of the non-linear analysis shown in Fig. 1(b). Reinforcement detailed to an elastic FE analysis

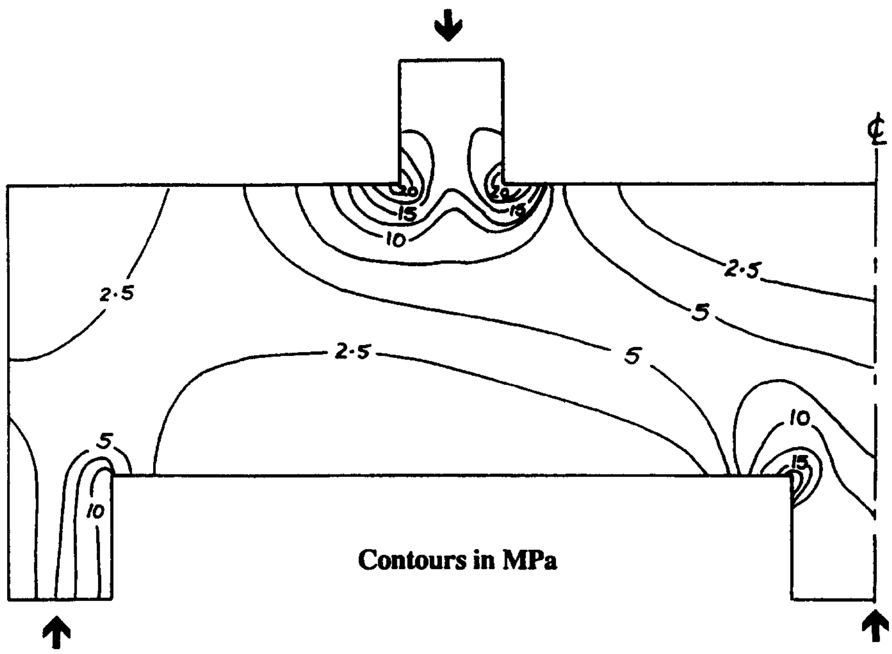

Fig. 7. Stress contour results from a linear-elastic FE analysis of Foster's[4] beam 1.

would lead to the provision of significantly more web steel than has been historically used in practice.

If using an elastic analysis, care should also be exercised in discretising of the FE mesh and in detailing of the longitudinal reinforcement. Rogowsky & MacGregor[27] showed that main longitudinal reinforcement curtailed to an elastic stress analysis may give an unsafe design. This is not to say that elastic analyses cannot be well utilised in the design of non-flexural members, but simply to suggest that caution is needed in the interpretation of the FE results. In the writers opinion, the best use of stress analyses is in obtaining estimates of the support reactions. Once the support reactions have been established the designer is better advised to adopt a strut and tie modelling approach combined with common sense' detailing.

Many non-linear FE modelling packages are available in the market, most containing very powerful and graphically impressive pre- and post-processors. It must be remembered, however, that it is not the graphical features of these packages that are of greatest importance when evaluating the usefulness of the package, rather the ability of the elements to model accurately the behaviour of the structure. When modelling planar concrete structures, two of the non-linear aspects of concrete stand head and tail above the array of non-linear features that may be included in the numerical model. They are:

- the ability of the element to model concrete cracking;
- compression softening of the concrete due to transverse tension fields (that is the modified compression field theory of Vecchio & Collins[10]).

Other non-linearities, such as biaxial compression strengthening, increases in the apparent dilation due to concrete micro-cracking, shear retention, bond-slippage, etc., are less important in most cases. For the reinforcing steel a simple bilinear stress–strain relationship is sufficient. The constitutive model may have the reinforcing steel included with the concrete to give a reinforced concrete element or the steel elements may overlay the concrete elements; either will give sufficiently accurate solutions. Finally, the FE model should have been demonstrated to model scaled experimental data, of the problem type being considered, at least to an accuracy consistent with design practice.

CONCLUDING REMARKS

Over the past 35 years, the mechanics of the behaviour of deep beams and other non-flexural actions has been well established. Early research based on experimentation has lead to an array of empirical design methodologies. Although the seeds of rational models are seen early in the literature,[5] they have not, until recently, made it into the design codes and into routine design practice. In the last 15 years, or so, extensive research has improved the extent of strut and tie models to a stage where they have become a useful tool in the designers armoury, particularly when combined with finite element analyses. Some would argue that all but the simplest codification of strut and tie modelling is undesirable[30] as the power and versatility of the method is diminished. Yet, for many years, full or partial strut and tie models have been routinely codified for the design of members in shear and torsion and codification in the non-flexural realm is underway.[31] Whilst research continues in the strut and tie modelling of non-flexural members, it is generally limited to refinement of the models, or a better understanding of the interaction of various components of the models. In this paper strut and tie models have been presented covering the full range of observed non-flexural behaviour. A rational formulation has been developed to obtain minimum web reinforcement requirements with the results consistent with web reinforcement ratios that have proved satisfactory for normal strength concrete members over many years. Finally, whilst the examples used here have been deep beams, the same basic principles are applicable to any member, or section of a member, where non-flexural behaviour exists.

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