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Flexural strength of reinforced concrete T-beams with steel fibers

Ramzi B. Abdul-Ahad, Omer Qarani Aziz *

Building and Construction Department, University of Technology, Tel-Mohammed, P.O. Box 35289, Baghdad, Iraq

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Abstract

The ultimate strength of reinforced concrete T-beams reinforced with conventional steel bars and short discontinuous steel fibers are studied. It is found that the presence of steel fibers reduced effectively the deflection, width of cracks and also improved the ductility and flexural rigidity of the concrete beams. Hence, an appreciable increase to the ultimate compressive strain is observed as well as the increase in the ultimate compressive strength. These are reflected by an increase in the value of the compressive block parameters. In addition, an increase in tensile strength is achieved and a rectangular tensile stress distribution is proposed. It was found that a negligible difference in moment capacity between overreinforced and underreinforced concrete beams. Therefore, it may be economical to use more amount of tension reinforcement than that allowed by the codes. Theoretical equations are developed to calculate the ultimate strength of reinforced concrete T-beams taking into account the effect of amount of compression reinforcement and amount of steel fibers. Theoretical equations show good agreement when compared with experimental results.

Keywords: Fibre concrete beams; Ultimate strengths; Reinforcement ratio; Steel fibres; Structures

Notations

- A_s area of bar reinforcement at tension zone
- A'_s area of bar reinforcement at compression zone
- A_{sf} area of steel bar used to balance compression force in flange
- a depth of equivalent rectangular compressive stress block
- b_f width of flange of the beam
- b_w width of web of the beam
- C distance from extreme compression fiber to neutral axis
- C_f compressive force in the flange
- C_w compressive force in the web
- d effective depth of the beam
- f'_c cylinder compressive strength of concrete
- f'_s stress in compression steel
- f_y yield stress of reinforcing steel bars
- h overall depth of the beam
- h_f depth of the flange of the beam
- V_f volume fraction of steel fiber
- β factor defining the depth of the equivalent rectangular stress block for ordinary concrete
- β_f factor defining the depth of the equivalent rectangular stress block for fiber reinforced concrete
- γ factor defining the intensity of compressive stress of the equivalent rectangular stress block for ordinary concrete
- γ_f factor defining the intensity of compressive stress of the equivalent rectangular stress block for fiber reinforced concrete
- ε_u ultimate concrete strain
- ε strain in concrete
- ε_s strain in tension steel
- ε_y yield strain of steel

1. Introduction

Fiber reinforced concrete is an ordinary concrete with randomly dispersed discrete fibers. Different types of fibers have been used to strengthen the concrete and the cement mortars in the field. It has been shown by many researches [1–4] that the addition of steel fibers improves concrete properties such as tension, compression, shear, flexural strength, ductility, impact resistance and first cracking strength.

Few studies dealt with the flexural strength of beams reinforced with a combination of ordinary steel bars and steel fiber reinforcement [5–7]. In all these researches the

* Corresponding author.

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contribution of steel fibers in tension and compression zone of beams was studied. These studies used the conventionally reinforced concrete beams with rectangular cross-section and different methods for predicting the ultimate flexural strength were developed.

The main objective of this investigation is to obtain experimental data on the deformation characteristics and strength of reinforced concrete T-beams with different percentages of steel bars at compression zone and different percentages of steel fibers. Equations for predicting the ultimate strength of such beams are proposed.

A total of eight conventionally reinforced concrete T-beams were studied. They were divided into two groups (group one and group two), group one was divided into four overreinforced concrete T-beams (G10, G11, G12 and G13) with volume fraction of steel fibers of 0%, 0.5%, 1% and 1.5%, respectively. Group two was also divided into four underreinforced concrete T-beams (G20, G21, G22 and G23) with volume fraction of steel fibers of 0%, 0.5%, 1% and 1.5%, respectively.

2. Experimental program

All the beams were geometrically similar having cross-sections of $b_f = 250$, $b_w = 100$, $h_f = 60$ and $h =$ 210 mm, total length of 2000 mm and a span length of 1800 mm between supports. The cross-section and the reinforcement details for tested beams are shown in Fig. 1.

Six cylinders (150 × 300) mm were cast for determination of compressive and indirect tensile strengths and also three beams (100 × 100 × 500) mm were cast for modulus of rupture.

The mix proportion of 1:2:2 (cement, sand, crushed aggregate) with a water cement ratio of 0.57 all by weight was used. The well graded sand and crushed aggregate with the maximum size of 9.5 mm were used. The steel fibers used in this study were low carbon hooked (0.5 × 50) mm in dimension with a tensile strength of 1150 MPa.

All the mixes were batched in a rotary mixer of 0.1 m^3 capacity. First, the aggregate, sand and cement were mixed dry for about one minute, the water was added and then fiber was dispersed to the mixer with hands in order to achieve homogenous mixes. After 28 days the beams were tested using a universal testing machine with a maximum capacity of 250 ton, the effective span of tested beams was 1800 mm and the load was applied at mid-span by two-point load (500 mm space between them). Load was applied gradually and incrementally in steps of 0.5 ton.

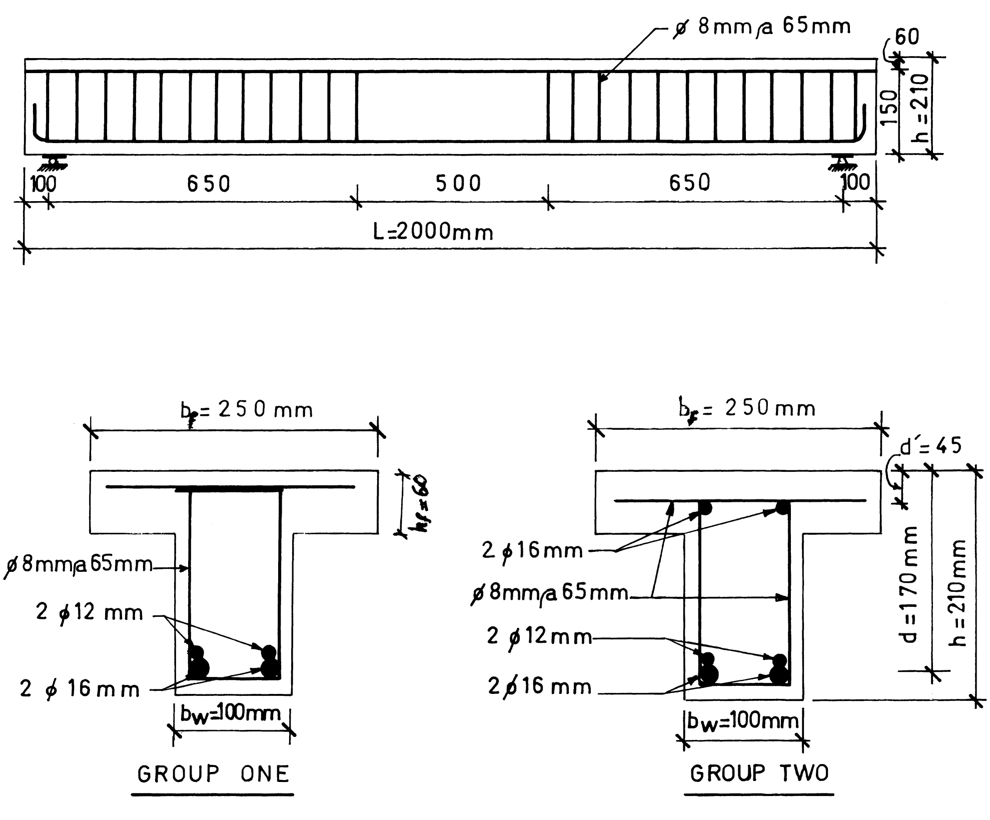

Fig. 1. Dimensions and reinforcement details.

3. Test results

The results of the tested beams are shown in Table 1 and the Load-deflection relationship for each group have been plotted as shown in Figs. 2 and 3. It has been noted that for a given deflection the beams with steel fibers resist higher load than the beams without fibers. The deflection at first crack are also less for the fiber reinforced beams even though the first cracking load is much higher (see Table 1(a)). By using steel fiber content of 0.5%, 1% and 1.5%, the ultimate load increased by 7.2%, 12.4% and 10.6% for group one and 5.64%, 7.74% and 10.35% for group two, respectively. The compression reinforcement decreases the deflection before ultimate load, but at ultimate there is a negligible difference in deflection between beams with and without compression reinforcement. Beam G13 has lower ultimate load when compared with beam G12, because when the discontinous short fibers are used there is a limit beyond which fiber addition does not improve the strength of the composite. This limit depends on fiber characteristics as well as the method of fabrication used in the preparation of the composite beyond a certain amount of steel fiber is ascribable to an increase in its porosity. Fiber bundles are porous themselves and the introduction of large quantities of such reinforcements also requires grater amount of mixing water, the two effects combine to reduce the strength of the material.

4. Analytical study of reinforced concrete T-beams

Analytical method for the analysis of concrete beams reinforced with a combination of steel bars and randomly distributed steel fibers, based on the ultimate strength approach of the American Concrete Institute for ordinary reinforced concrete [8], takes into consideration the contribution of steel fibers to the flexural compressive strength of concrete. The actual and assumed stress and strain distribution of failure for fiber reinforced beam is shown in Fig. 4.

The value of γ_f was increased from 0.85 for plain concrete to 0.88 for fiber concrete. β_f increased from 0.85 for plain concrete to 0.9 for fiber concrete and the ultimate compressive strain (ε_u) increased from 0.003 for plain concrete to 0.004 for fiber concrete. The value of γ_f, β_f and ε_u can be calculated as follows [9].

$$\gamma_f = 0.85 + 0.03\frac{WL/D}{450} \leqslant 0.88. \quad (1)$$

$$\beta_f = 0.85 + 0.05\frac{WL/D}{450} \leqslant 0.90 \text{ for } f'_c \leqslant 27.56 \text{ MPa}. \quad (2)$$

$$\beta_f = 0.85 + 0.05\frac{WL/D}{450} - \left[1 + 0.25\frac{WL/D}{450}\right]\left[0.05 - \frac{f'_c - 27.58}{6.895}\right] \quad (3)$$

for $27.56 \leqslant f'_c \leqslant 56.16$ MPa.

Table 1
Results of the tested beams

(a) Deflection characteristic at first crack and ultimate load

	Beam No.	Load (kN)		Mid-span defl. (mm)	
		First crack	Ultimate load	First crack	Ultimate load
Group one	G10	19.30	130.00	1.30	20.20
	G11	19.70	140.00	1.12	20.08
	G12	20.30	146.80	1.05	21.20
	G13	20.70	144.50	0.92	21.70
Group two	G20	18.4	133.40	1.41	21.30
	G21	18.50	141.00	1.26	21.7
	G22	18.80	143.80	1.12	21.40
	G23	19.00	147.00	1.10	21.70

(b) Ultimate moment

	Beam No	V_f (%)	A_s (mm^2)	A'_s (mm^2)	f'_c (MPa)	M_{exp} (kN m)
Group one	G10	0.00	628	–	21.30	42.45
	G11	0.50	628	–	21.40	45.50
	G12	1.00	628	–	21.79	47.71
	G13	1.50	628	–	21.98	46.96
Group two	G20	0.00	628	402	17.32	43.38
	G21	0.50	628	402	17.72	45.83
	G22	1.00	628	402	18.20	46.74
	G23	1.50	628	402	18.81	47.87

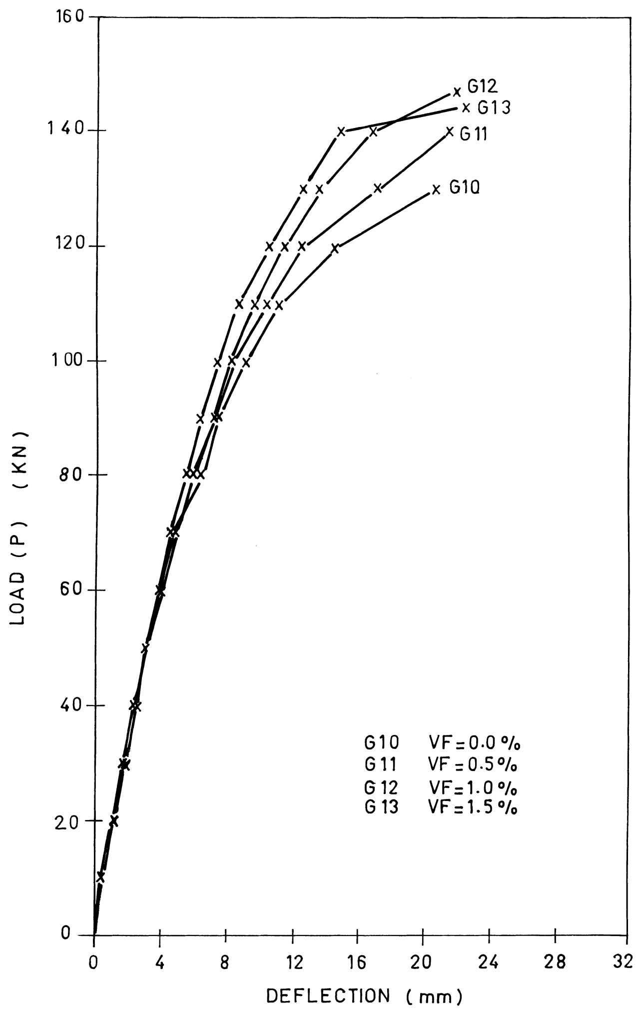

Fig. 2. Load-deflection relationship for group one.

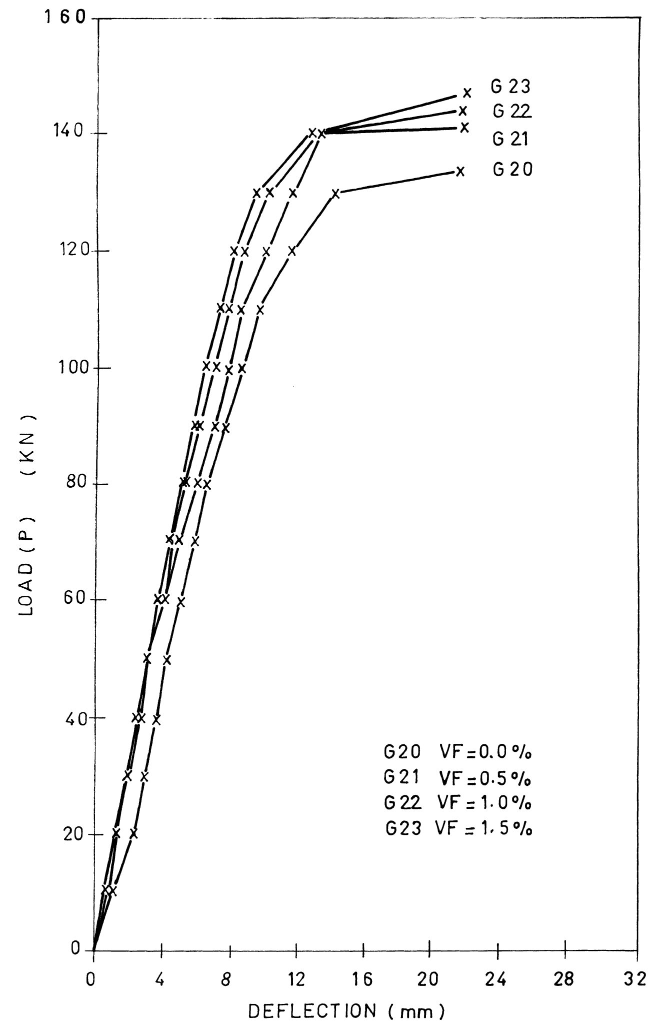

Fig. 3. Load-deflection relationship for group two.

$$\beta_f = 0.65 \text{ for } f'_c > 56.16 \text{ MPa}. \tag{4}$$

$$\varepsilon_u = 0.003 + 0.001\frac{WL/D}{450} \leqslant 0.004, \tag{5}$$

where WL/D is the percentage of steel fiber by weight.

The tensile strength (σ_t) of fiber concrete can be calculated by the following equation [10].

$$\sigma_t = 0.82\ \tau v_f d_f L/D, \tag{6}$$

where

τ	interfacial bond stress between fiber and matrix (MPa),
V_f	volume fraction of steel fiber (%),
d_f	bond factor of steel fiber ($d_f = 0.75$ for hooked steel fibers) and
L/D	aspect ratio of steel fiber ($L/D = 100$).

The bond stress (τ) can be calculated as follows [11]:

$$\tau = \frac{10PL_p}{21 d_f V_f D_p B L/D}, \tag{7}$$

where

P	ultimate load of the (100 × 100 × 500) beam (kN),
L_p	effective span of the (100 × 100 × 500) beam (mm) and
B, D_p	width and depth of the (100 × 100 × 500) beam, respectively.

The maximum steel ratio is calculated as follows [11]:

$$\rho_b = \frac{\gamma_f \beta_f f'_c + \sigma_t}{f_y}\left[\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}\right] - \frac{h\sigma_t}{df_y}, \tag{8}$$

where

ε_y	ultimate strain of steel bar ($\varepsilon_y = f_y/E_s$) (mm/mm) and
f_y	yield stress of steel bars in MPa; f_y for 12 mm $\phi = 465$ MPa; f_y for 16 mm $\phi = 517$ MPa.

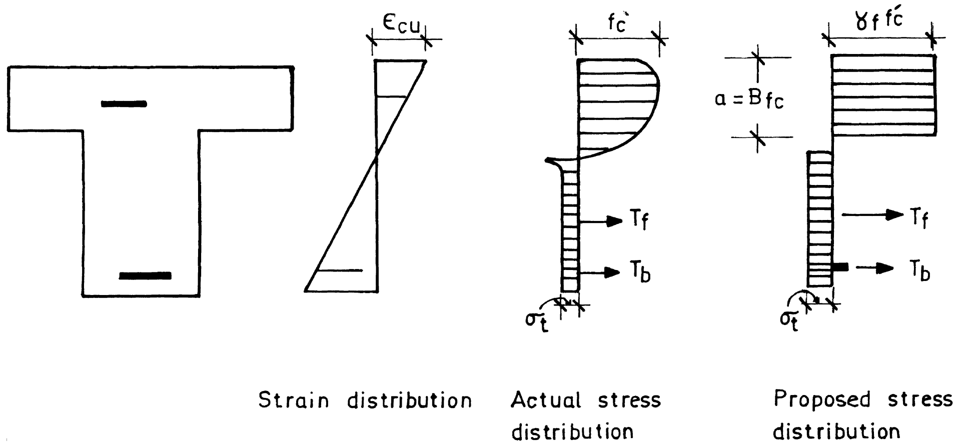

Fig. 4. Stress and strain distribution.

$$\rho_{max} = 0.75(\rho_b + \rho_f) + \rho' f'_s / f_y. \tag{9}$$

$$\rho_f = A_{sf}/b_w d. \tag{10}$$

$$A_{sf} = \frac{\gamma_f f'_c (b_f - b_w) h_f}{f_y}. \tag{11}$$

$$\rho' = A'_s / b_w d. \tag{12}$$

$$f'_s = \varepsilon'_s E_s. \tag{13}$$

$$\varepsilon'_s = \varepsilon_u (1 - d'/c) \tag{14}$$

from equilibrium $C = T$

$$C = C_w + C_f + C_s. \tag{15}$$

$$C_w = \gamma_f f'_c b_w \beta_{fc}. \tag{16}$$

$$C_f = \gamma_f f'_c (b_f - b_w) h_f. \tag{17}$$

$$C_s = f'_s A'_s - \gamma \gamma_f A'_s f'_c,$$

$$\gamma = 1 \text{ for } a > d', \quad \gamma = 0 \text{ for } a \leqslant d'. \tag{18}$$

$$T = T_s + T_f. \tag{19}$$

$$T_s = A_s f_y. \tag{20}$$

$$T_f = \sigma_t b_w (h - c). \tag{21}$$

Ultimate strength of underreinforced concrete T-beams is calculated as follows:

$$M_u = (A_s f_y - A_{sf} f_y - f'_s A'_s)(d - a/2) + A'_s f'_s (d - d') + A_{sf} f_y (d - h_f/2) + \sigma_t b_w (h - c) \times (h + c - a)/2 - \gamma \gamma_f A'_s f'_s (d - d'). \tag{22}$$

Ultimate strength of overreinforced concrete T-beams is calculated as follows:

$$M_u = \gamma_f f'_c b_w a(d - a/2) + \gamma_f f'_c (b_f - b_w) h_f (d - h_f/2) + A'_s f'_s (d - d') - \gamma \gamma_f f'_c \times A'_s (d - d') + \sigma_t b_w (h - c) \times (h + c - a)/2. \tag{23}$$

The calculated stress block parameters (γ_f, β_f and ε_u), theoretical moments and the ratio of M_{exp}/M_{th} are shown in Table 2.

5. Discussion of results

The average value of M_{exp}/M_{th} for beams in group two (underreinforced concrete T-beams) is 1.035, which is approximately equal to 1.024 for beams in group one (overreinforced concrete T-beams), that means the contribution of steel fibers to the tensile flexural strength is fairly accurately represented by the proposed equations and gives approximately the same ductility.

Table 2
Stress block parameters and theoretical moments

Beam group	γ_f	β_f	ε_u	σ_t (MPa)	M_{th} (kN m)	M_{exp}/M_{th}
Group one						
G10	0.850	0.850	0.00300	–	41.41	1.025
G11	0.861	0.870	0.00337	1.37	43.75	1.040
G12	0.872	0.890	0.00375	2.73	45.77	1.042
G13	0.880	0.900	0.00400	4.10	47.46	0.989
Group two						
G20	0.850	0.850	0.00300	–	42.90	1.011
G21	0.861	0.870	0.00337	1.37	43.86	1.045
G22	0.872	0.890	0.00375	2.73	44.87	1.042
G23	0.880	0.900	0.00400	4.10	45.90	1.043

In conventionally reinforced concrete members, sufficient ductility could be achieved by making the tension steel yield before the crushing of concrete. Sometimes it may be economical to use more amounts of tension steel than allowed by codes which leads to better utilization of concrete section strength. This may be feasible if the concrete ductility is improved [12]. One method to attain this is by using randomly oriented discrete steel fibers [13,14]. The use of fiber reinforced concrete in compression zone of flexural members will increase the ductility and serve that purpose [12].

6. Conclusion

The following are the main conclusions of the results.

1. The addition of steel fibers causes an increase in ultimate strength for both over and underreinforced concrete T-beams.
2. A negligible increase in moment capacity between under and overreinforced concrete T-beams has been observed, that means the overreinforced concrete beams may be used instead of underreinforced concrete beams by using steel fibers in compression zone. Moreover, this is more economical.
3. Theoretical equations are developed to calculate the ultimate strength of reinforced concrete T-beams taking into account the contribution of steel fibers. They show good agreement when compared with the experimental results.
4. The addition of steel fibers reduces the width of the crack and crack propagation is reduced during incremental loading.

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