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Design of flat slabs for punching – European and North American practices

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Abstract

Punching is a critical design case for reinforced concrete flat slabs but the provisions for punching shear design and detailing of the shear reinforcement differ considerably among the various European and American design codes. Therefore the thickness of the slab or the amount or distribution of shear reinforcement may vary between different countries. The punching shear capacity of concrete, the punching shear resistance with shear reinforcement and the relevant detailing provisions of four European, two American codes and the CEB-FIP Model Code will be compared. The provisions have been compared by analysing flat slabs with typical dimensions and reinforcement ratios. The possibilities and limitations of each code and the consequences in practice will be demonstrated, using the flat slab of an office building as an example. With regard to practical aspects, the importance of shear reinforcement that can easily be installed will be emphasised.

Keywords: Design codes; Moment transfer; Punching shear resistance; Shear reinforcement; Integrity reinforcement

1. Introduction

Flat slabs are being used mainly in office buildings due to reduced formwork costs, fast execution and easy installation. The reduced storey height and consequently the reduced height of the building result in lower overall construction and maintenance costs.

The design of reinforced concrete flat slabs is governed by deflection or punching. There are many theories about the slab-column connection and many tests have been conducted, but the design rules differ considerably around the world. The calculated punching shear capacity and relevant reinforcement detailing depend much more on the code applied than the flexural reinforcement for example. Therefore the thickness of the slab or the amount or distribution of the shear reinforcement may vary between different countries.

This paper will analyse and compare the provisions for punching with respect to the punching shear capacity of concrete, the punching shear resistance with shear reinforcement and the relevant detailing of the reinforcement specified in

Germany	DIN 1045	[1]
	E DIN 1045-1	[2]
Eurocode	EC 2	[3]
UK	BS 8110	[4]
USA	ACI 318	[5]
Canada	CSA A23.3	[6]
Model Code	CEB-FIP 1990	[7].

EC 2 will be applied together with the German application document. All the above codes have been applied to the flat slab of a typical office building to demonstrate the possibilities and limitations of the different rules.

2. Critical perimeter, consideration of moment transfer

The punching shear resistance is calculated on a critical perimeter u which is located between 0.5 and $2d$ from the face of the column (d: effective depth). Fig. 1 shows the location of the critical perimeter for an interior column with rectangular cross-section $c_1 \cdot c_2$. Several

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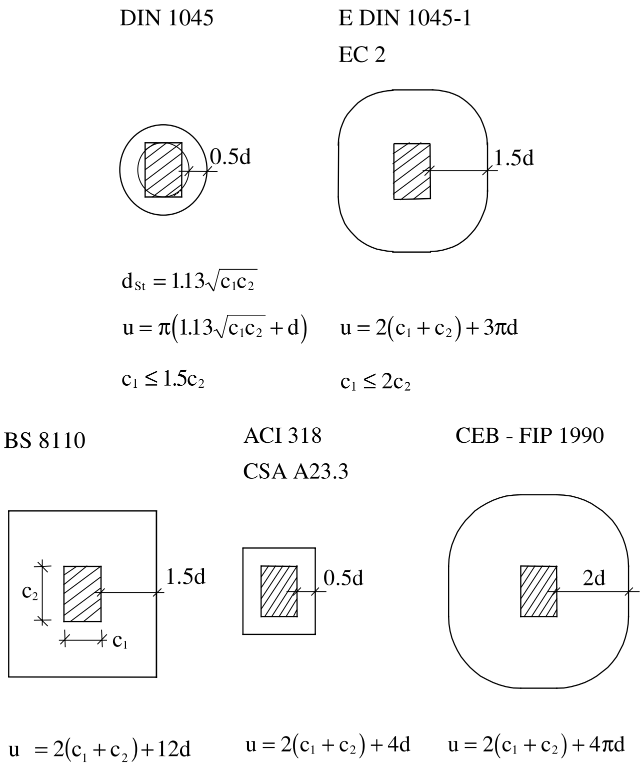

Fig. 1. Interior column: critical perimeter.

codes require that the shear resistance outside the shear reinforced zone be checked at successive perimeters u_2, u_3.

Due to moment transfer, the distribution of the shear force is not uniform. The increase in shear may be calculated using either fixed coefficients or equations which take the specific eccentricity into account as specified below for interior columns.

2.1. DIN 1045

The effect of moment transfer can be ignored if the spans of a panel do not differ by more than 33%.

2.2. E DIN 1045-1, EC 2

The shear force per unit of the perimeter v_{Sd} is calculated using a coefficient

$$v_{Sd} = V_{Sd}\beta/u, \tag{1}$$

where V_{Sd} is the design shear force transferred to column

$\beta = 1.05$ (E DIN 1045-1),

$\beta = 1.15$ (EC 2).

2.3. BS 8110

The effective shear force is taken from either:

$$V_{eff} = 1.15V_t, \tag{2a}$$

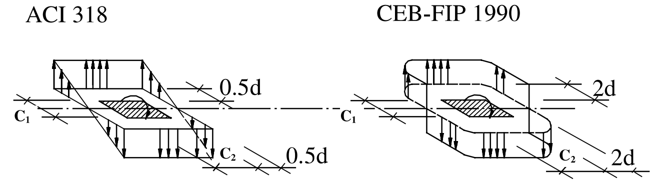

Fig. 2. Shear stresses due to a moment.

$$V_{eff} = V_t\left(1 + \frac{1.5M_t}{V_t x}\right), \tag{2b}$$

where V_t is the design shear force transferred to column; M_t is the design moment transferred to column; x is the length of the side of the perimeter considered parallel to the axis of bending.

2.4. ACI 318, CSA A23.3, CEB-FIP 1990

The value of the moment to be transferred is given by specific equations. ACI 318 and CSA A23.3 assume the shear due to the moment to vary linearly, CEB-FIP 1990 assumes the shear distribution shown by Fig. 2. The shear stresses due to the shear force and the moment are added.

3. Punching shear resistance

The punching shear resistance of concrete depends on its strength. Most design methods take into account the flexural reinforcement ratio ρ_l and a factor related to the depth of the slab. The maximum punching shear resistance with shear reinforcement is expressed as a multiple of the punching shear resistance without shear reinforcement or is calculated as the punching resistance of concrete at the perimeter of the column.

The punching shear resistance is expressed by:

V_{Rd1}: provided by the concrete,

V_{Rd2}: maximum punching shear resistance with shear reinforcement.

All codes, with the exception of DIN 1045, use partial safety factors. Table 1 shows that they are all different. Therefore, the comparison is based on characteristic loads. For simplification, a common safety factor γ_F for permanent (G_k) and imposed load (Q_k) has been chosen for each code. The respective punching shear resistance V_{Rd}/γ_F will be compared with each other and with the shear force V_{Sd}/γ_F calculated in the final example.

The equations given in the codes have been rewritten using the critical perimeter given in Fig. 1, assuming

$$d = 0.85h, \quad \alpha = (c_1 + c_2)/2h$$

and the material data of C35 (cube) or C30 (cylinder). For reasons of simplification, the size factor refers to

Table 1
Partial safety factors

	Loading			Material	
	Permanent γ_G	Imposed γ_Q	Common γ_F	Concrete γ_c	Reinforcement γ_s
E DIN 1045-1	1.35	1.50	1.40	1.50	1.15
EC 2	1.35	1.50	1.40	1.50	1.15
BS 8110	1.40	1.60	1.47	1.50 Shear: 1.25	1.05
ACI 318	1.40	1.70	1.50	$1/0.70 = 1.43$ Shear: $1/0.85 = 1.18$	$1/0.90 = 1.11$
CSA A23.3	1.25	1.50	1.33	$1/0.60 = 1.67$	$1/0.85 = 1.18$
CEB-FIP 1990	1.35	1.50	1.40	1.50	1.15

$d = 250$ mm, which means only a minor inaccuracy for an overall slab thickness h between 250 and 300 mm.

3.1. DIN 1045

$$Q_{r1} = 2.48\sqrt{\rho_l}(1 + 1.33\alpha)h^2, \tag{3a}$$

$$Q_{r2} \approx 1.4Q_{r1}, \tag{3b}$$

Q_{r1} and Q_{r2} correspond to $V_{\mathrm{Rd1}}/\gamma_F$ and $V_{\mathrm{Rd2}}/\gamma_F$, ρ_l is expressed as %.

3.2. E DIN 1045-1

$$\frac{V_{\mathrm{Rd1}}}{\gamma_F} = 3.44(100\rho_l)^{1/3}(1 + 0.50\alpha)h^2, \tag{4a}$$

$$V_{\mathrm{Rd2}} = 1.7V_{\mathrm{Rd1}}. \tag{4b}$$

3.3. EC 2

$$\frac{V_{\mathrm{Rd1}}}{\gamma_F} = 2.20(1.2 + 40\rho_l)(1 + 0.50\alpha)h^2, \tag{5a}$$

$$V_{\mathrm{Rd2}} = 1.6V_{\mathrm{Rd1}}. \tag{5b}$$

3.4. BS 8110

$$\frac{V_{\mathrm{Rd1}}}{\gamma_F} = 4.71(100\rho_l)^{1/3}(1 + 0.39\alpha)h^2, \tag{6a}$$

$$V_{\mathrm{Rd2}} = 2.0V_{\mathrm{Rd1}} \leqslant 10.95\gamma_F\alpha h^2. \tag{6b}$$

3.5. ACI 318

$$\frac{V_{\mathrm{Rd1}}}{\gamma_F} = 2.85(1 + 1.18\alpha)h^2, \tag{7a}$$

$$V_{\mathrm{Rd2}} = 1.5V_{\mathrm{Rd1}}. \tag{7b}$$

3.6. CSA A23.3

Corresponds in principle to the American code.

3.7. CEB-FIP 1990

$$\frac{V_{\mathrm{Rd1}}}{\gamma_F} = 4.59(100\rho_l)^{1/3}(1 + 0.37\alpha)h^2, \tag{8a}$$

$$V_{\mathrm{Rd2}} = 12.82\gamma_F\alpha h^2. \tag{8b}$$

Table 2 and Fig. 3 show the results for $\alpha = 1.0$ (thin column), $\alpha = 2.0$ (thick column), $0.5\% \leqslant \rho_l \leqslant 1.5\%$.

The calculated punching shear capacities are substantially different as are the effects of column thickness and reinforcement ratio.

An increased thickness of the column is most effective for DIN 1045, ACI 318, CSA A23.3 where the critical perimeter is $0.5d$ from the face of the column. Hence the punching shear capacity increases by 57% and 54%, respectively, if the column dimensions are doubled to a thickness twice that of the slab ($\alpha = 2$). The punching shear resistance specified in the other codes where the critical perimeter is 1.5 and $2d$ from the face of the column increases by 33% or 27%, respectively. Kordina and Noelting [8] have recalculated several tests and pointed out consistent predictions where the critical perimeter is $1.5d$ from the face of the column. This has been confirmed recently by the *fib* Task Group 4.3 for the critical perimeter given by CEB-FIP 1990 at a distance $2d$ from the face of the column [9].

The effect of the flexural reinforcement ratio on the shear capacity is greatest in the case of DIN 1045. If the reinforcement ratio is doubled, the shear capacity increases by 41% according to DIN 1045 compared with only 14% according to EC 2. The American and Canadian codes do not account for the reinforcement ratio.

With shear reinforcement, the punching shear capacity can be increased by:

$\approx 40\%$	according to DIN 1045
50%	according to ACI 318, CSA A23.3
60%	according to EC 2
70%	according to E DIN 1045-1
$> 70\%$	according to BS 8110, CEB-FIP 1990.

A check at the perimeter of the column for the maximum punching shear capacity V_{Rd2} is required by

Table 2
Interior column: punching shear resistance

	α	$V_{Rd1}/\gamma_F h^2$ (N/mm²)			$V_{Rd2}/\gamma_F h^2$ (N/mm²)		
		ρ_l (%)			ρ_l (%)		
		0.5	1.0	1.5	0.5	1.0	1.5
DIN 1045	1.0	4.09	5.78	7.08	5.66	8.00	9.80
	2.0	6.42	9.08	11.12	8.88	12.56	15.39
E DIN 1045-1	1.0	4.10	5.16	5.91	6.96	8.77	10.04
	2.0	5.46	6.88	7.88	9.28	11.70	13.39
EC 2	1.0	4.62	5.28	5.94	7.39	8.45	9.50
	2.0	6.16	7.04	7.92	9.86	11.26	12.67
BS 8110	1.0	5.20	6.55	7.49	10.40	10.95	10.95
	2.0	6.65	8.38	9.60	13.30	16.76	19.20
ACI 318	1.0	6.22	6.22	6.22	9.33	9.33	9.33
	2.0	9.58	9.58	9.58	14.37	14.37	14.37
CSA A23.3	1.0	6.24	6.24	6.24	9.36	9.36	9.36
	2.0	9.61	9.61	9.61	14.42	14.42	14.42
CEB-FIP 1990	1.0	5.00	6.30	7.21	12.82	12.82	12.82
	2.0	6.36	8.02	9.18	25.64	25.64	25.64

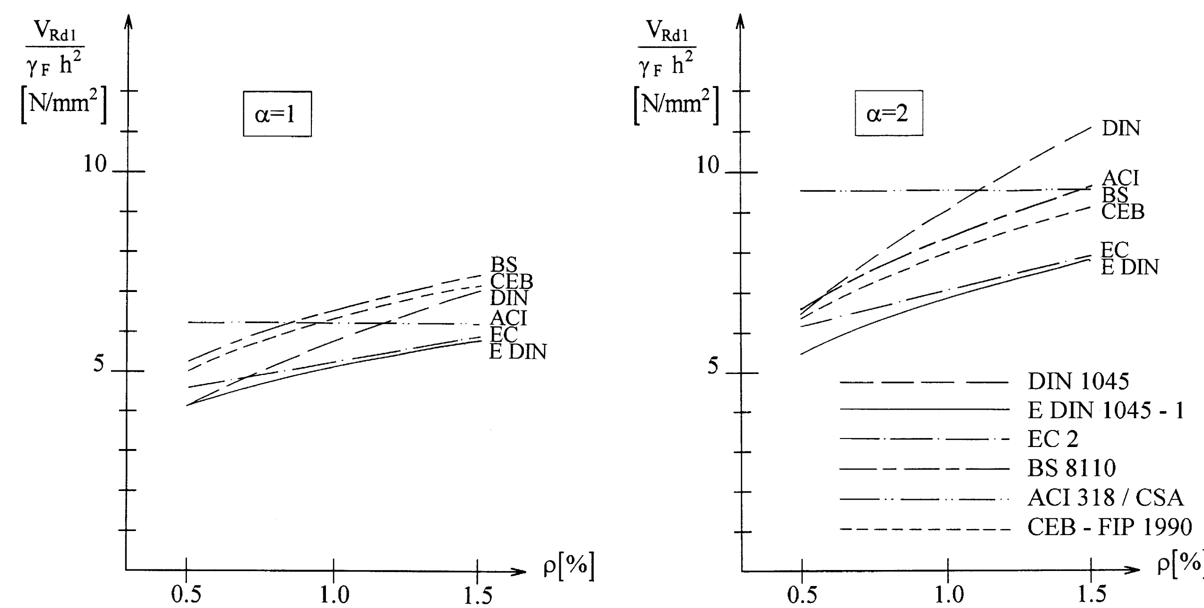

Fig. 3. Interior column: punching shear resistance without shear reinforcement.

BS 8110 and CEB-FIP 1990. This may limit the punching shear capacity of thin columns. The usefulness of the high punching shear capacities allowed by BS 8110 and CEB-FIP 1990 for slabs with shear reinforcement depends on whether it is practical to install the required shear reinforcement.

4. Punching shear resistance with shear reinforcement

The rules governing the combined punching shear resistance of concrete and shear reinforcement vary considerably. Some simply add the values, others reduce the concrete component or the efficiency of the shear reinforcement. Several codes require extending the shear reinforcement until a perimeter is reached which no longer needs shear reinforcement. Consequently, both the shear reinforcement and the shear reinforced zone are increased.

4.1. DIN 1045

Where the shear force is higher than the punching shear capacity of the concrete, the shear reinforcement has to take 75% of the shear force, arranged in two perimeters.

4.2. E DIN 1045-1

The punching shear capacity has to be checked at perimeters spaced at $0.75d$ intervals, starting at $0.5d$ from the face of the column, Fig. 4. The entire punching shear capacity of the concrete is taken into consideration, but, as it refers to the unit length, the contribution varies with the relevant perimeter. Therefore a substantial amount of the shear reinforcement has to be provided on the first perimeter u_1. At the perimeter $u_a - 1.5d$ from the outermost shear reinforcement – the punching shear resistance per unit length is gradually reduced to the shear capacity of continuously supported slabs. For slabs with $d \leqslant 400$ mm the stress in the links is limited to $f_{yd} \approx 300$ N/mm^2.

4.3. EC 2

Simple addition of the shear capacity of the concrete and the shear reinforcement.

4.4. BS 8110

The shear stress has to be checked on subsequent perimeters until a perimeter is reached which does not need shear reinforcement. Up to $V_{Rd2} = 1.6V_{Rd1}$ the shear strength of the concrete and the shear reinforcement are added in full. For $1.6V_{Rd1} \leqslant V_{Rd2} \leqslant 2.0V_{Rd1}$, the amount of shear reinforcement increases substantially because its efficiency is reduced, see [10].

4.5. ACI 318

Only 50% of the punching shear capacity of the concrete is taken into account and the steel stress is limited to 6000 psi $\hat{=}$ 414 N/mm^2. The shear stress on a critical perimeter $0.5d$ from the outermost link is limited to value valid for beams which is only half the value for punching.

4.6. CSA A23.3

Corresponds to ACI in principle, however, the shear reinforcement is distributed uniformly around the column.

4.7. CEB-FIP 1990

75% of the punching shear capacity of the concrete is considered and the steel stress is limited to 300 N/mm^2. At a perimeter $2d$ from the outermost shear reinforcement all shear resistance has to be provided by the concrete.

Fig. 5 shows the shear reinforcement required for

$\alpha = 1, \quad \rho_l = 1\%.$

The punching shear resistance with shear reinforcement specified in BS 8110 and CEB-FIP 1990 has been limited to $V_{Rd2} = 1.6V_{Rd1}$ for better comparison. The yield stress of the reinforcement has been assumed to be $f_{yk} = 500$ N/mm^2.

Fig. 5 demonstrates the differences in shear reinforcement requirements among the different codes. The reason is on the one hand the use of different design models and on the other hand the different limits set on the steel stress.

A report on new developments of design models on punching of structural concrete slabs and a comparison of theoretical analyses and empirical code formulae with test results have been prepared recently by the *fib* Task Group 4.3 [9].

5. Detailing of shear reinforcement and integrity reinforcement

The following codes specify a minimum thickness for slabs with shear reinforcement:

DIN 1045	$h \geqslant 150$ mm
E DIN 1045-1, EC 2	$h \geqslant 200$ mm
CSA A23.3	$h \geqslant 300$ mm

Most codes specify links which have to be anchored around one layer of the top and one layer of the bottom reinforcement. Such reinforcement is difficult to install. To make this easier, single leg links are common in the UK [10]. Alternatively shear ladders of welded fabric as shown in Fig. 6 may be used. Although shear ladders do not enclose the tensile reinforcement, they are nonetheless quite effective, as recent tests have shown [11].

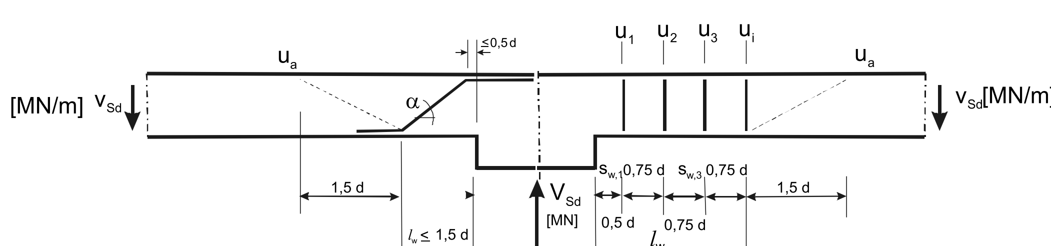

Fig. 4. Shear reinforcement at successive perimeters according to [2].

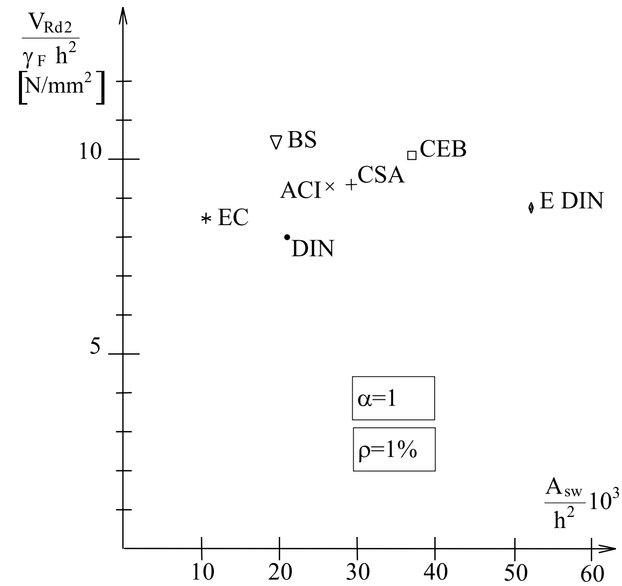

Fig. 5. Interior column: shear reinforcement.

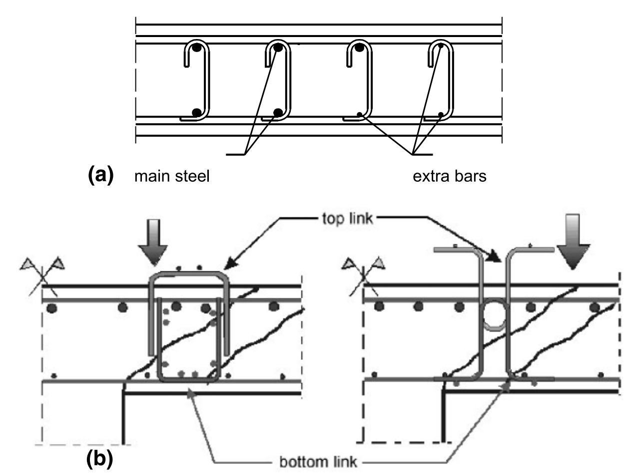

Fig. 6. Shear reinforcement (a) Single leg links according to [10]; (b) Welded fabric according to [11].

DIN 1045, E DIN 1045-1, EC 2, CEB-FIP 1990 also provide for bent-up bars.

Furthermore prefabricated shear elements such as shear studs have become common. The relevant design rules are presented in specific documents and in the American and Canadian codes as well. The requirements for shear stud reinforcement design and detailing also vary in different countries [12].

Some codes require an integrity reinforcement to be provided by continuous bottom bars passing through the column cage in order to prevent progressive collapse in the case of local punching. DIN 1045 requires such reinforcement only for slabs without shear reinforcement. E DIN 1045-1 and CEB-FIP 1990 require it in all cases, the amount is:

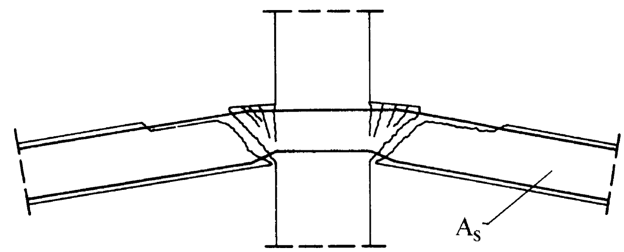

Fig. 7. Model of connection during punching failure.

$$A_s = \frac{V_{Sd}}{f_{yk}} \text{ (E DIN)}, \tag{9a}$$

$$A_s = \frac{V_{Sd}}{f_{yd}} \text{ (CEB)}. \tag{9b}$$

ACI 318 specifies that at least two of the bottom bars in each direction must pass through the column core. More detailed recommendations are given by ACI–ASCE Committee 352 in [13]. These recommendations are based on the model of a membrane at an angle of 30° to the horizontal, Fig. 7, and take at least double the permanent load into account. This results in nearly twice the amount of reinforcement required by E DIN 1045-1 and CEB-FIP 1990. CSA A23.3 refers to the same model, however without the partial safety factors:

$$A_s = \frac{2V_{se}}{f_{yk}} \text{ (CSA)}, \tag{9c}$$

where V_{se} is the shear force transferred to column.

A comparison of the amount of integrity reinforcement is given in the following example.

6. Example

The flat slab of a multi-storey office building presented in [14] will be calculated against punching at the interior column:

Spans	longitudinal 7.50 m
	transverse 7.00 and 5.00 m
Slab	$h = 280$ mm
	$d = 0.85h$
Interior column	$c_1 = c_2 = 400$ mm
	$\alpha = (c_1 + c_2)/2h = 1.43$
Concrete	C35 (cube)
	C30 (cylinder)
Reinforcement	$f_{yk} = 500$ N/mm^2
	$\rho_l = 1\%$ (flexural reinforcement)
Loading	permanent 9 kN/m^2
	imposed 5 kN/m^2
Shear force	$V_{Sd}/\gamma_F = 770$ KN (characteristic value)

Table 3
Example interior column: shear force, punching shear resistance, shear reinforcement, integrity reinforcement

	β_1	β_2	$\beta_{1,2}V_{Sd}/\gamma_F$ (kN)	ρ_l (%)	V_{Rd1}/γ_F (kN)	V_{Rd2}/γ_F (kN)	A_{sw} (mm^2)	n	A_s (mm^2)
DIN 1045	1.00		770	1.0	564	780	2020	2	–
E DIN 1045-1	1.05		809	1.0	463	787			
				1.06	476	809	4920	4	2150
EC 2[a]	1.15		886	1.0	473	757			
BS 8110	1.15	1.13	870	1.0	575	1150	1630	3	–
ACI 318		1.16	893	–	600	900	2530	[b]	2 × 2 bars
CSA A23.3[c]		1.16	893	–	603	904	2780	3	3960
CEB-FIP 1990		1.13	870	1.0	552	1437	3200	3	2480

[a] $V_{Sd}/\gamma_F > V_{Rd2}/\gamma_F$: requirement not fulfilled.
[b] Two rows of closed links perpendicular to the column.
[c] $h \geqslant 300$ mm ignored.

Table 3 shows the shear force V_{Sd} which has been multiplied by β to take moment transfer into account (column 4). Because of the different span length in the transverse direction, the moment at the interior column is comparatively high. Therefore the calculated coefficients β_2 and the fixed coefficients β_1 are about the same. According to DIN 1045, the effect of moment transfer may be ignored as the spans do not differ by more than 33%.

The punching shear resistance V_{Rd2} specified in EC 2 is not sufficient – even if the reinforcement ratio is increased to 1.5%. A slight increase of the reinforcement ratio is required to comply with DIN 1045-1, DIN 1045, ACI 318, CSA A23.3 allowing a margin of 1–5%. The punching shear capacity according to BS 8110 and CEB-FIP 1990 is considerably higher, although a large increase in the shear reinforcement is required.

Shear reinforcement is generally required. The number of rows n of shear reinforcement depends on checking an outer perimeter. The total amount of shear reinforcement A_{sw} varies considerably among the different codes.

Great differences occur in the case of the amount of integrity reinforcement A_s as well. DIN 1045 requires this reinforcement only for slabs without shear reinforcement. According to EC 2 and BS 8110 no integrity reinforcement is required, while E DIN 1045-1 and CSA A23.3 require 2150 and 3960 mm^2, respectively.

7. Conclusions

This paper reveals considerable differences among seven different codes with respect to the punching shear capacity and to the amount and distribution of shear reinforcement and integrity reinforcement in reinforced concrete flat slabs. In all codes punching shear capacity calculations are based on a critical perimeter, which is located between 0.5 and $2d$ from the face of the column. The location of the critical perimeter is decisive for the increase of the punching shear capacity with an enlarged column. Except in the North American codes, the punching shear capacity depends on the flexural reinforcement ratio. However, the effect of the flexural reinforcement is quite different in each code.

The punching shear resistance with shear reinforcement depends on which design model has been used. Furthermore the stress limit in the steel is different in each code. Therefore, the shear reinforcement differs considerably with regard to its amount and extent. There is a substantial difference with regard to the amount of integrity reinforcement required to prevent progressive collapse – some codes do not even require this reinforcement.

The example set by the flat slab of an office building with typical dimensions demonstrates how much design and construction are governed by a design code. This is because different models are used in different design codes to describe the punching shear capacity and different limitations are put on the stress in the shear reinforcement. Because a typical range of span lengths, column dimensions and load intensities apply to flat slabs, the thickness, shear reinforcement and integrity reinforcement required in flat slab constructions should also be within a certain range.

The different code provisions have been derived from many tests which take into account the reinforcement practices common in the respective countries. This should be emphasised, as design and construction form an integral whole. A design code is a complex set of interrelated rules and guidelines that has been calibrated over time to give acceptable results. In the UK the high punching shear capacity is linked to the extent of the shear reinforcement. The comparatively high punching shear resistance without shear reinforcement provided by the North American codes should be seen together with the required integrity reinforcement. Therefore engineers should be very careful in applying individual provisions from different codes in a design.

Due consideration of practical aspects should be given as the safety of structures depends as well on the quality of execution – the skill of workmanship, the pressure of time and costs. This may be a point in favour of flat slabs without shear reinforcement. Otherwise, shear elements prefabricated from welded fabric which reduce the problems of installation or specific systems such as shear studs, may be of advantage.

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