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Fatigue strength of steel fibre reinforced concrete in flexure

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Abstract

The paper presents a study on the fatigue strength of steel fibre reinforced concrete (SFRC). An experimental programme was conducted to obtain the fatigue-lives of SFRC at various stress levels and stress ratios. Sixty seven SFRC beam specimens of size $500 \times 100 \times 100$ mm were tested under four-point flexural fatigue loading. Fifty four static flexural tests were also conducted to determine the static flexural strength of SFRC prior to fatigue testing. The specimens incorporated 1.5% volume fraction of corrugated steel fibres of size $0.6 \times 2.0 \times 30$ mm. Concept of equivalent fatigue-life, reported for plain concrete in literature, is applied to SFRC to incorporate the effects of stress level S, stress ratio R and survival probability L_R into the fatigue equation. The results indicate that the statistical distribution of equivalent fatigue-life of SFRC is in agreement with the two-parameter Weibull distribution. The coefficients of the fatigue equation have been determined corresponding to different survival probabilities so as to predict the flexural fatigue strength of SFRC for the desired level of survival probability.

Keywords: Fatigue; Equivalent fatigue-life; Steel fibre reinforced concrete; Stress level; Stress ratio; Flexure

1. Introduction

Considerable interest has developed in the fatigue strength of concrete members in recent years. There are several reasons for this. Firstly, the use of high strength materials require that the concrete members perform satisfactorily under high stress levels. Hence, the study of the effects of repeated loads on bridge slabs and crane beams is a matter of concern. Secondly, different concrete systems such as prestressed concrete railroad ties and continuously reinforced concrete pavement slabs are often used. The use of these systems demand a high performance product with an assured fatigue-life. Thirdly, there is a new recognition of the effects of repeated loading on a member, even if it does not cause a fatigue failure. There may be inclined cracks in the prestressed concrete beams at lower loads due to fatigue loading and the static load carrying capacity of the component material may be altered.

Many researchers carried out laboratory fatigue experiments to investigate the fatigue behaviour of plain as well as steel fibre reinforced concrete since Feret's pioneer tests [1]. Oh, [16,21] studied the distribution of flexural fatigue-life of plain concrete for various stress levels and found that it follows the two-parameter Weibull distribution. Following equations have been used in the past by the researchers to study the fatigue of concrete:

$$S = a - b\log(N) \quad (1)$$

$$S = 1 - b(1 - R)\log(N) \quad \text{for } 0 \leqslant R \leqslant 1 \quad (2)$$

$$S = C_1(N)^{-C_2} \quad (3)$$

where a, b, C_1 and C_2 are experimental coefficients. Many researchers [2,9,14] used Eq. (1) which represents the relationship between stress level S, and number of cycles to failure N. Eq. (2) used by researchers [12–14,16] is a modified form of the Eq. (1) which takes into account the effect of minimum fatigue stress (f_{min}) in the form of stress ratio R, ($R = f_{min}/f_{max}$). Aas-Jakobsen [7] obtained the value of b in Eq. (2) equal to 0.064 for compression fatigue of concrete. However, Tepfers et al. [12] recommended the value of b as 0.0685. Oh [16]

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Nomenclature

S	stress level $= f_{max}/f_{min}$
R	stress ratio $= f_{min}/f_{max}$
f_r	static flexural stress
f_{max}	maximum fatigue stress
f_{min}	minimum fatigue stress
L_R	reliability function or survival probability
N	fatigue-life or number of cycles to failure
EN	equivalent fatigue-life
n	specific value of the random variable
α	shape parameter of the Weibull distribution
u	characteristic life
CV	coefficient of variation of the data sample under consideration
μ	mean value of the data sample under consideration
$T(\ )$	gamma function
C_C	correlation coefficient

tested the Eq. (2) for flexural fatigue of plain concrete and obtained the value of b as 0.0690. Eq. (3) relates the stress level S to the number of cycles for failure N by a power relation. It has been used by pavement researchers such as Vesic and Saxena [6].

Shi et al. [22] introduced the concept of equivalent fatigue life (EN), in an attempt to incorporate the effect of stress ratio and survival probability into the Eq. (3). They modified the Eq. (3) by replacing N with EN to get a new equation as follows:

$$S = C_1(\mathrm{EN})^{-C_2} \quad (4)$$

The coefficients C_1 and C_2 of Eq. (4) were obtained for plain concrete for different survival probabilities. It is also shown that the statistical description of equivalent fatigue-life follows the two-parameter Weibull distribution [22].

The studies on steel fibre reinforced concrete (SFRC) were mainly confined to the determination of its flexural fatigue endurance limit for different type/volume fraction/aspect ratio of fibres [10,17–20]. Yin and Hsu [23] studied the fatigue behaviour of steel fibre reinforced concrete under uniaxial and biaxial compression and observed that the *S–N* curves can be approximated by two straight lines connected by a curved knee instead of a single straight line.

2. Research significance

The work of Shi et al. [22] provides a good opportunity to apply the concept of equivalent fatigue-life to the data of steel fibre reinforced concrete subjected to flexural fatigue loading and to examine the two-parameter Weibull distribution to describe the distribution of equivalent fatigue-life of SFRC. It is also proposed to obtain the coefficients of the fatigue equation (4) corresponding to different reliabilities/survival probabilities, making the equation thus applicable to SFRC.

3. Experimental programme

The concrete mix used for casting the test specimens is shown in Table 1. Ordinary Portland Cement, crushed stone coarse aggregates (maximum size 12 mm) and Badarpur sand were used. The materials used conformed to relevant Indian Standard specifications. Corrugated steel fibres of size $30 \times 2.0 \times 0.6$ mm at 1.5% volume fraction were incorporated in the concrete. The specimens used for flexural fatigue tests as well as static flexure tests were fibre concrete beams of size $100 \times 100 \times 500$ mm. The specimens were cast in 14 batches, each batch consisting of nine standard flexural test specimens and four $150 \times 150 \times 150$ mm cubes. The cube specimens were used to determine the 28 days compressive strength for each batch. The mixing was done in a rotary mixer and the fibres were gradually sprinkled into the drum by hand. The specimens were cured for 60 days to avoid a possible strength increase during fatigue tests. The specimens were removed from the curing bath and kept in the laboratory conditions till testing. The 28 days average compressive strength of the mix was 46.34 MPa.

Four beams from each batch were tested in a 100 kN *INSTRON* closed loop universal testing machine to determine the mean static flexural strength (f_r). The beams were simply supported on a span of 450 mm and loaded at third points. The average static flexural strength of the mix was 8.76 MPa. The fatigue tests were conducted on a 100 kN *MTS* closed loop electrohydraulic universal testing machine. The span/points of loading in the fatigue tests were kept the same as for the static tests.

Table 1
Mixture proportion

Water/cement ratio	Sand/cement ratio	Coarse aggregate/cement ratio
0.46	1.52	1.88

Flexural fatigue tests were conducted at different stress levels 'S' ($S = f_{max}/f_r, f_{max}$ = maximum fatigue *stress*, f_r = static flexural stress), ranging from 0.90 to 0.60 and at two different stress ratios 'R' ($R = f_{min}/f_{max}, f_{min}$ = minimum fatigue stress) i.e. 0.10, and 0.30. Constant-amplitude sinusoidal loads were applied at a frequency of 12 Hz. The number of cycles to failure for each specimen under different load conditions were noted as fatigue-life N. With the decrease in the stress level, the number of cycles to failure of the specimens went on increasing. Since fatigue testing is a very time consuming and expensive process, an upper limit of the number of cycles i.e. 2 million cycles, to be applied was selected. The test was terminated as and when the failure of the specimen occurred or this upper limit was reached whichever was earlier.

4. Analysis of the fatigue test data

The results of the fatigue tests as obtained in this investigation are given in Table 2. Large variability was observed in the fatigue-life data, particularly at stress level at stress levels 0.80, 0.75, 0.70 and 0.65. It was difficult to carry out analysis of the data in such a case. This difficulty was resolved by merging the original data at stress levels 0.80 and 0.75; and 0.70 and 0.65 into two groups and using the average value of the stress levels i.e. 0.775 (average of stress levels 0.80 and 0.75) and 0.675 (average of stress levels 0.70 and 0.65) for each group in the analysis of the data.

Table 2
Laboratory fatigue-life data (number of cycles to failure, N, in ascending order) for steel fibrous concrete ($\mathbf{V}_f = 1.5\%$)

Stress ratio 'R'	Stress level 'S'				
	0.90	0.85	0.775[a]	0.675[a]	0.60
0.10	24	264	37[b]	1706[b]	124,390[c]
	50	300	5742	47,268	2,000,000[d]
	62	300	7360	51,680	
	78	350	8300	65,960	
	100	384	9160	81,910	
	3312[b]	516	13,104	132,860	
		564	14,420	189,820	
		1200	16,960	206,880	
		1240	23,316	280,910	
		1452	35,260	375,540	
		1560	51,120	585,750	
		2328	75,900	920,160	
		2580	90,540	1,238,370	
		3150			
		47,364[b]			
0.30	212	308[b]	272[b]		
	368	6080	80,960		
	430	9600	100,140		
	810	10,276	156,760		
	1280	32,796	313,680		
	16,200[b]	45.580	550,400		

[a] Obtained by merging data at stress levels 0.80 and 0.75, 0.70 and 0.65, respectively.
[b] Rejected as outliers by Chauvenet's criterion, not included in analysis.
[c] As the specimen failed at a low number of cycles, hence rejected.
[d] Specimen did not show any cracks, test terminated at 2×10^6 cycles, treated as run out, not included in analysis.

Some data points in Table 2 may be treated as outliers. Chauvenet's criterion [11] was applied to data points at each stress level and points meeting this criterion were excluded from further analysis.

To carry out the analysis when both stress level S and stress ratio R are variables, concept of equivalent fatigue-life was introduced by Shi et al. [22]. They defined equivalent fatigue-life as follows:

$$\mathrm{EN} = (N)^{1-R} \tag{5}$$

Eq. (5) was obtained from Eqs. (1) and (2). The fatigue-life data as given in Table 2 has both S and R as variables. It is difficult to carry out the statistical analysis in such a case. Shi et al. [22] observed that "all the fatigue-life data at a certain stress level S with different R values can be transformed to have a common equivalent fatigue-life EN by applying Eq. (5). The analysis of the fatigue-life N with two variables S and R now becomes an analysis of the equivalent fatigue-life EN with one variable S". Equivalent fatigue-lives for SFRC at different stress level as obtained by Eq. (5) are summarised in Table 3.

Table 3
Equivalent fatigue-life data (EN, in ascending order) for SFRC, ($V_f = 1.5\%$)

Stress level 'S'			
0.90	0.85	0.775	0.675
17	151	2416	16,111
34	170	2727	17,458
41	170	3021	21,745
43	195	3165	26,424
50	212	3366	40,837
63	276	3679	56,300
63	299	4332	60,834
70	445	5078	80,114
109	591	5534	104,038
150	608	6404	155,218
	613	7039	233,065
	643	8529	304,485
	701	10,434	
	748	12,375	
	1072	17,287	
	1176	24,673	
	1408	28,917	
	1449		
	1824		

4.1. Graphical method of analysis

A number of mathematical probability models have been employed for the statistical description of fatigue data. One of the popular models has been the logarithmic-normal (lognormal) distribution function [4]. However, it was pointed out by Gumble [5] that the hazard function or risk function for lognormal distribution decreases with increasing life. The Weibull distribution has an increasing hazard function with time and is most commonly used for describing the fatigue data these days.

The survivorship function of the two-parameter Weibull distribution can be written as follows [3,8,16, 22]:

$$L_R(n) = \exp\left[-\left(\frac{n}{u}\right)^{\alpha}\right] \tag{6}$$

in which n = specific value of the random variable; α = shape parameter or Weibull slope at stress level S; u = scale parameter or characteristic life at stress level S.

Taking the logarithm twice of both sides of Eq. (6):

$$\ln\left[\ln\left(\frac{1}{L_R}\right)\right] = \alpha\ln(n) - \alpha\ln(u) \tag{7}$$

This equation can be used to verify whether the statistical distribution of equivalent fatigue-life of SFRC, at a given stress level S, follows the two-parameter Weibull distribution. The equivalent fatigue-life data at a given stress level must be first arranged in ascending order to obtain a graph from Eq. (9) and the empirical survivorship function can be calculated from the following relation [8,11,16,21]:

$$L_R = 1 - \frac{i}{k+1} \tag{8}$$

in which i = failure order number and k = number of equivalent fatigue data or sample size at a given stress

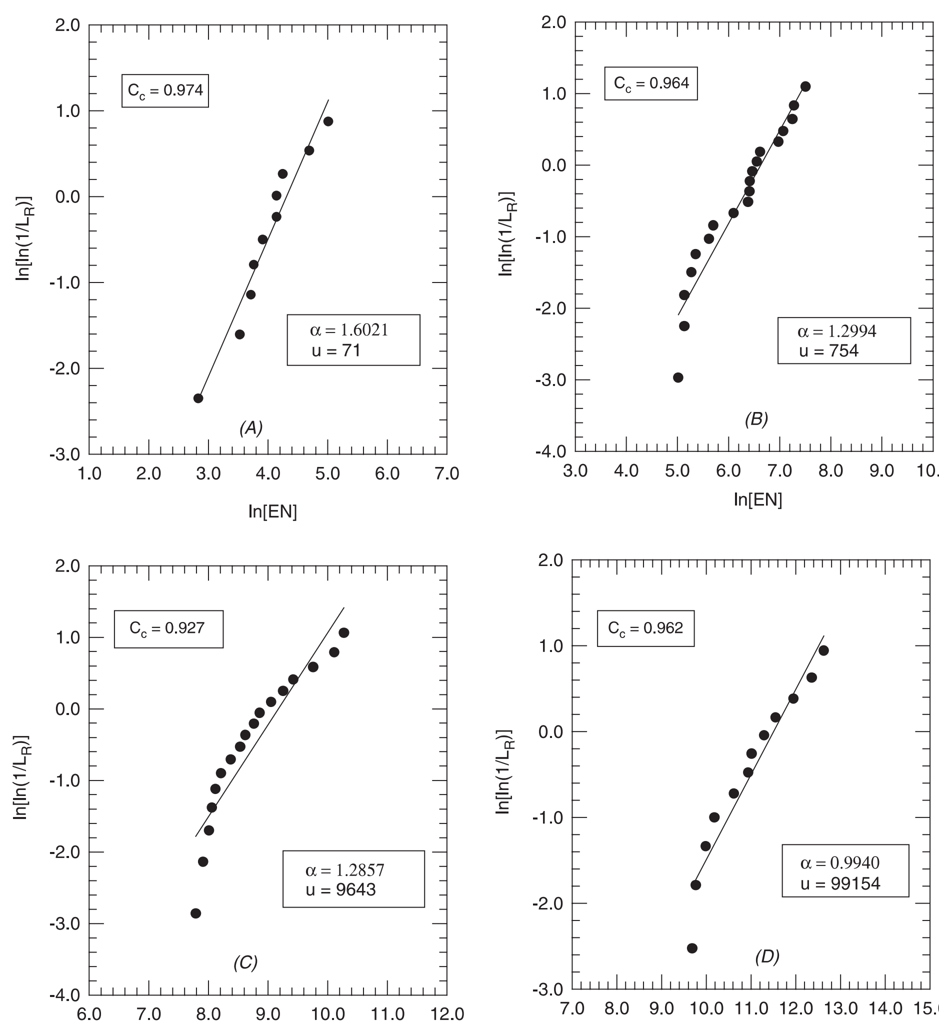

Fig. 1. Graphical analysis of equivalent fatigue-life data for SFRC ($V_f = 1.5\%$): (A) $S = 0.90$; (B) $S = 0.85$; (C) $S = 0.775$; (D) $S = 0.675$.

level S. A graph is plotted between $\ln[\ln(1/L_R)]$ and $\ln(EN)$, and if a linear trend is observed for the equivalent fatigue-life data at a given stress level S, it can be assumed that the two-parameter Weibull distribution is a reasonable assumption for the statistical description of equivalent fatigue-life data at that stress level. The parameter α and u can be directly obtained from the graph.

Fig. 1 shows the plots of the equivalent fatigue-life data at $S = 0.90$, 0.85, 0.775 and 0.675. It can be observed that the data points fall approximately along a straight line, which indicates that the two-parameter Weibull distribution is a reasonable assumption for the distribution equivalent-fatigue-life of SFRC at these stress levels. The corresponding values of the correlation coefficient (C_C) are 0.974, 0.964, 0.927 and 0.962, respectively as shown in Fig. 1. Table 4 presents the basic calculations to plot the equivalent fatigue-life data of SFRC at stress level $S = 0.90$. The estimated parameters for the equivalent fatigue-life data of SFRC, at different stress levels are shown in Fig. 1 and Table 5.

4.2. Parameters from the method of moments

The parameters of the Weibull distribution for the equivalent fatigue-life of SFRC at a given stress level can also be obtained by the method of moments using the following relations [8,15,16,21]:

$$\alpha = (\mathrm{CV})^{-1.08} \tag{9}$$

u can be estimated from the following equation

$$u = \frac{\mu}{T\left(\frac{1}{\alpha} + 1\right)} \tag{10}$$

where μ is the sample mean of the data at a given stress level and CV the coefficient of variation of the data, $T(\ )$ is the gamma function.

The equivalent fatigue-life data given in Table 3 has been analysed to obtain the distribution parameters by this method. The calculated parameters are listed in Table 5.

4.3. Parameters from the method of maximum likelihood estimate

The method of maximum likelihood estimate was also used for calculation of distribution parameters of the Weibull distribution for the equivalent fatigue-life data of SFRC at various stress levels [8,21]. The values of the parameters i.e. shape parameter and characteristic extreme life as estimated by this method are listed in Table 5.

It can be seen from Table 5 that all the three methods employed here to estimate the parameters of the Weibull distribution yield almost similar results except for the equivalent fatigue-life data at stress level $S = 0.775$ where the value of the shape parameter α obtained by the method of moments is much different from the one obtained by the other two methods. Hence, in this case, the average values of the parameters obtained by the graphical method and by the method of maximum likelihood estimate are used for further analysis.

Table 4
Equivalent fatigue-lives and empirical survivorship function for $S = 0.90$

i	EN_i	$L_R = 1 - \frac{i}{k+1}$	$\ln[\ln(1/L_R)]$	$\ln(EN_i)$
1	17	0.9091	−2.3507	2.8332
2	34	0.8182	−1.6062	3.5264
3	41	0.7273	−1.1444	3.7136
4	43	0.6364	−0.7942	3.7612
5	50	0.5455	−0.5008	3.9120
6	63	0.4545	−0.2376	4.1431
7	63	0.3636	+0.0116	4.1431
8	70	0.2727	+0.2619	4.2485
9	109	0.1818	+0.5335	4.6913
10	150	0.0909	+0.8746	5.0106

Table 6
Calculated equivalent fatigue-lives corresponding to different reliabilities

L_R	$S = 0.90$	$S = 0.85$	$S = 0.775$	$S = 0.675$
0.95	12	83	998	5290
0.90	19	141	1730	10,680
0.80	29	245	301	22,215
0.70	38	346	4396	35,112
0.60	48	452	5785	49,855
0.50	57	565	7306	67,155

Table 5
Values of the Weibull parameters for equivalent fatigue-life, SFRC ($V_f = 1.5\%$)

	$S = 0.90$		$S = 0.85$		$S = 0.775$		$S = 0.675$	
	α	u	α	u	α	u	α	u
Graphical method	1.6021	71	1.2994	754	1.2857	9643	0.9940	99,154
Method of moments	1.7043	70	1.3791	735	1.1200	9112	1.0029	93,052
Method of maximum likelihood	1.7169	71	1.3892	736	1.3298	9694	1.0769	95,902
Average	1.6744	71	1.3559	741	1.3078[a]	9669[a]	1.0246	96,036

[a] Average of the values of the parameters obtained by the graphical method and the method of maximum likelihood estimate.

5. Goodness-of-fit tests

It has been shown in the preceding sections that the equivalent fatigue-life data of SFRC at a given stress level S, can be modelled by the two-parameter Weibull distribution. To further substantiate this, the χ^2-test and Kolmogorov–Smirnov test as described by Oh [21] were carried out on the equivalent fatigue-life data of SFRC at each stress level. These tests indicate that the two-parameter Weibull distribution is a valid model for the statistical distribution of equivalent fatigue-life of SFRC at 5% level of significance.

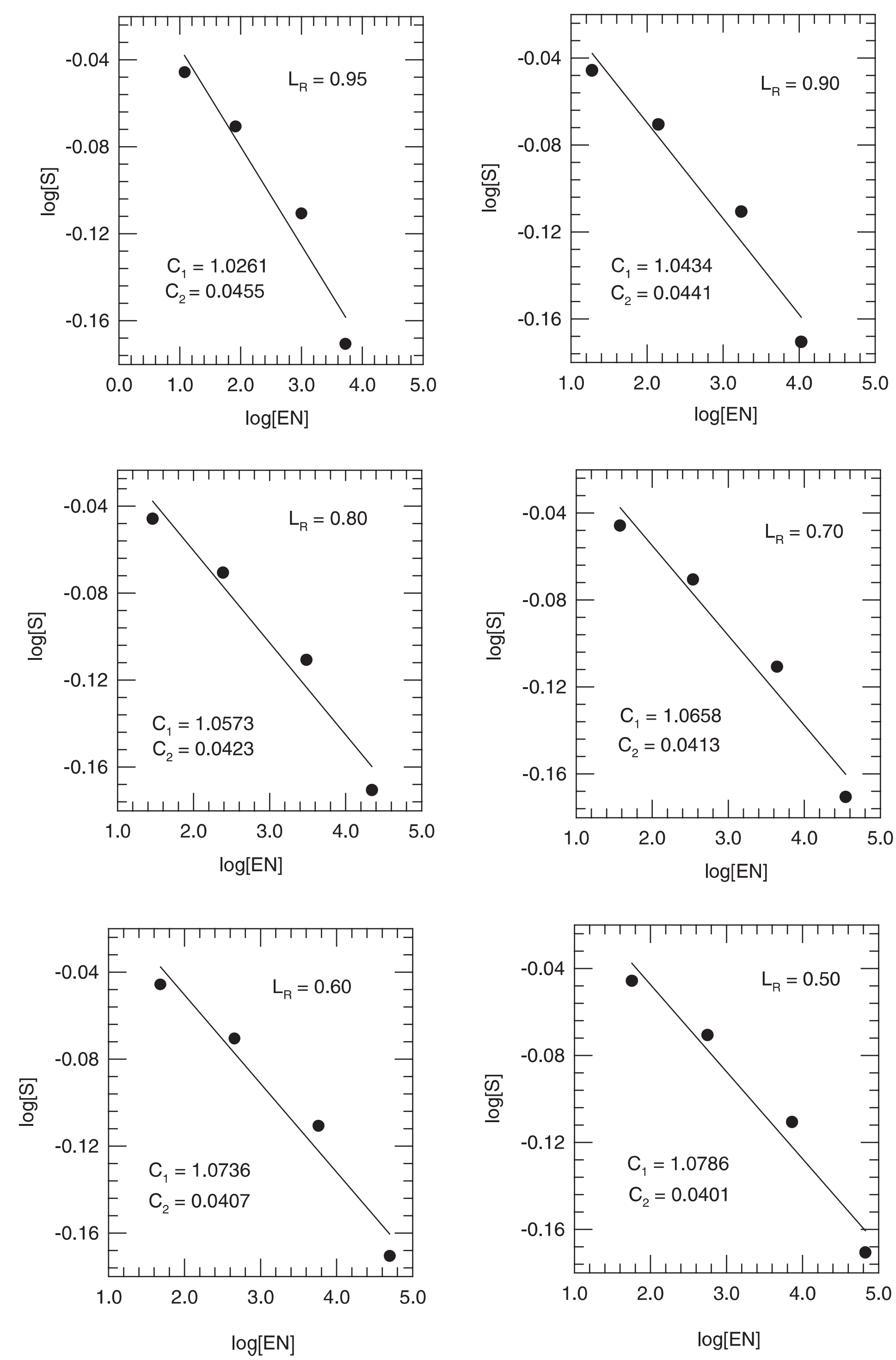

Fig. 2. Determination of coefficients of the fatigue equation for SFRC ($V_f = 1.5\%$).

Table 7
Coefficients C_1 and C_2 of the fatigue equation corresponding to different reliabilities

L_R	C_1	C_2
0.95	1.0261	0.0455
0.90	1.0434	0.0441
0.80	1.0573	0.0423
0.70	1.0658	0.0413
0.60	1.0736	0.0407
0.50	1.0786	0.0401

6. Coefficients of the fatigue equation for SFRC

It has been shown in the preceding sections that the equivalent fatigue life data of SFRC can be described by the two-parameter Weibull distribution. Therefore, it can be used to calculate the equivalent fatigue-lives corresponding to different survival probabilities. Using the average values of the parameters as obtained above by different methods, the calculated values of the equivalent fatigue-lives are listed in Table 6.

The coefficients C_1 and C_2 of the Eq. (4) can be obtained by regressing it against the data in Table 6.

Taking logs on both sides of the Eq. (4)

$$\log(S) = \log(C_1) - C_2 x \log(\mathrm{EN}) \tag{11}$$

Fig. 2 shows the results of the regression analysis. The coefficients thus obtained are listed in Fig. 2 and Table 7. These coefficients can be chosen for the desired level of the survival probability. Eq. (4) can be written in the following form:

$$S = C_1(N)^{-C_2(1-R)} \tag{12}$$

The Eq. (12) can thus be used by the design engineers to estimate the flexural fatigue strength of SFRC for the desired level of survival probability.

The results obtained are obviously applicable for SFRC with 1.5% fibre content and the type and the aspect ratio of the fibres used. The authors also tested SFRC with 1.0% and 0.5% fibre content. The results were analysed and it was observed that same general conclusions could be drawn as for as the parameters of the Weibull distribution and the coefficients of the fatigue equation are concerned. However, additional research work is required to be carried out to investigate as to how the fibre type and aspect ratio affect the parameters of the Weibull distribution and the coefficients of the fatigue equation.

7. Conclusion

The concept of equivalent fatigue-life as reported in literature, has been applied to SFRC. It has been shown that the statistical distribution of equivalent fatigue-life of steel fibre reinforced concrete at a given stress level, S approximately follows the two-parameter Weibull distribution. The parameters of the Weibull distribution for the equivalent fatigue-life are obtained by the graphical method, the method of moments and the method of maximum likelihood estimate. The coefficients of the fatigue equation have been determined for SFRC corresponding to different survival probabilities, thus incorporating the survival probabilities into the fatigue equation. The fatigue equation can be used for obtaining the flexural fatigue strength of steel fibre reinforced concrete for the desired level of survival probability. However, additional work is needed to determine as to how the fibre type and aspect ratio affect the results i.e. the parameters of the Weibull distribution and the coefficients of the fatigue equation.

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References

[1] Feret R. Etude experimentale du ciment arme. Grauthier-Villiers; 1906. [chapter 3].
[2] Kesler CE. Effect of speed of testing on flexural strength of plain concrete. In: HRB Proceedings, vol. 32. p. 1953.
[3] Weibull W. Fatigue testing and analysis of results. Oxford: Pergamon Press; 1961.
[4] ASTM. A guide for fatigue testing and the statistical analysis of fatigue data. ASTM Spec Tech Publ 1963;91-A.
[5] Gumble EJ. Parameters in the distribution of fatigue life. J Eng Mech, ASCE 1963;(October).
[6] Vesic AS, Saxena SK. Analysis of structural behaviour of road test rigid pavements. Highway Research Record, no. 291, 1969.
[7] Aas-Jakobsen K. Fatigue of concrete beams and columns. Bulletin No. 70-1, NTH Institute for Betonkonstruksjoner, Trondheim, September, 1970.
[8] Wirsching PH, Yao JTP. Statistical methods in structural fatigue. J Struct Div, Proc ASCE 1970;(June).
[9] Ballinger CA. Cumulative fatigue damage characteristics of plain concrete. Highway Research Record, no. 370, 1972.
[10] Batson G, Ball C, Bailey L, Lenders E, Hooks J. Flexural fatigue strength of steel fibre reinforced concrete beams. ACI J 1972;(November).
[11] Kennedy JB, Neville AM. Basic statistical methods for engineers and scientists. New York: A Dun-Donnelley Publishers; 1976.
[12] Tepfers R, Kutti T. Fatigue strength of plain, ordinary, and light weight concrete. ACI J 1979;(May).
[13] Tepfers R. Tensile fatigue strength of plain concrete. ACI J 1979;(August).
[14] Hsu TTC. Fatigue of plain concrete. ACI J 1981;(July–August).
[15] Wirsching PH, Yao JTP. Fatigue reliability-introduction. J Struct Div, Proc ASCE 1982;(January).
[16] Oh BH. Fatigue analysis of plain concrete in flexure. J Struct Eng 1986;112(2).

[17] Ramakrishnan V, Oberling G, Tatnall P. Flexural fatigue strength of steel fibre reinforced concrete. ACI Spec Publ 1987;SP-105-13.

[18] Tatro SB. Performance of steel fibre reinforced concrete using large aggregates. Transportation Research Record 1110, TRB Washington, 1987.

[19] Ramakrishnan V, Wu GY, Hosalli G. Flexural fatigue strength, endurance limit and impact strength of fibre reinforced concretes. Transportation Research Record 1226, TRB, Washington, 1989.

[20] Johnston CD, Zemp RW. Flexural fatigue performance of steel fibre reinforced concrete-influence of fibre content, aspect ratio and type. ACI Mater J 1991;(July–August).

[21] Oh BH. Fatigue-life distributions of concrete for various stress levels. ACI Mater J 1991;(March–April).

[22] Shi XP, Fwa TF, Tan SA. Flexural fatigue strength of plain concrete. ACI Mater J 1993;2(September–October).

[23] Yin W, Hsu TTC. Fatigue behaviour of steel fibre reinforced concrete in uniaxial and biaxial compression. ACI Mater J 1995;(January–February).