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CDM model for residual strength of concrete under cyclic compression

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Abstract

An elasto-damage model developed recently for predicting response of concrete subjected to fatigue loading (Int. J. Damage Mech. 9(1) (2000) 57), is extended to predict the residual strength of concrete subjected to initial damage resulting from the application of a known number of stress cycles. Experimental corroboration of results is established by subjecting 75×150 mm cylinders of a high quality, pre-packaged repair concrete to damage due to a specified number of cycles in axial compression followed by loading to failure to record the residual strength.

Keywords: Concrete; Cyclic compression; Model; Residual strength

1. Introduction

With greater interest in service life prediction of structures undergoing damage due to repeated mechanical or environmental loading coupled with the effect of aggressive species diffusing through concrete, it is becoming important to develop analytical models that simulate the growth of damage and predict the residual strength of the structural component under such loadings.

Gettu et al. [2] have presented an interesting experimental study that demonstrates the influence of damage due to monotonic and cyclic compression on the tensile strength of concrete measured in the direction of maximum damage. The tests were conducted on high strength, silica-fume admixed concrete and it was concluded that damage in high strength concrete evolves at a much slower rate than that in normal strength concrete—which is intuitively appealing in that this allows the higher strengths to be attained prior to rupture of the concrete.

In order to develop an analytical predictor model for residual strength, use has been made of a continuum damage mechanics (CDM) formulation as presented initially by Suaris et al. [3] for monotonic and cyclic behavior of concrete in which the elastic potential is introduced in terms of principal stresses and a damage dependent compliance tensor. The evolution of damage is calculated by tracking the movement of the loading surface (LS) in its approach towards the bounding surface (BS), a concept originally introduced in elasto-plasticity by Dafalias and Popov [4]. The limit fracture surface (LFS) (which sets the threshold of damage), the LS, and the BS are all expressed in terms of the strain energy release rate vector R_i. Al-Gadhib et al. [5] have extended the Suaris stress control model, making provisions for strain-control under proportional triaxial loading and incorporating it into a finite element code.

A further evolution in CDM modeling has been provided by Chow and Wang [6,7] through the introduction of a damage-effect tensor M, allowing for constitutive equations for anisotropic damage to be developed, using the hypothesis of elastic strain energy equivalence. The concept of the M-tensor is an attractive one which has been applied to predict damage and behavior of metals. It is only recently that state-of-the-art constitutive equations expressed in terms of the M-tensor for concrete have been developed by Khan et al. [8] where essential features of concrete such as degradation of elastic properties, strain softening, gain in strength under confinement and different behavior in tension and compression have been captured effectively.

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This paper is an extension of the damage model presented by Al-Gadhib et al. [1] for predicting fatigue life of concrete under compression to include predictions for residual strength of concrete subject to a known number of stress cycles less than the fatigue life. Predictions of damage resulting from repeated loading are ensured by introducing the concept of a moving LFS, constrained by a bound on its size. The bound parameter is calibrated by using S–N data presented by Su and Hsu [9] for both low-cycle ($1-10^3$ cycles) and high-cycle ($>10^3$ cycles) experimentation. Although the experimental verification of the model has been limited to prediction of residual strength in the direction of the previously applied cyclic loading, it can be extended to include other modes of loading consequent to the initial cyclic loading.

2. Damage effect tensor versus effective compliance matrix

Damage variable may be considered as an internal state variable which characterizes the irreversible deterioration of a material point in accordance with the thermodynamic formulation [10]. Based on the theory of isotropic CDM, the effective Cauchy stress tensor $\underset{\sim}{\bar{\sigma}}$ is related to the usual Cauchy stress tensor $\underset{\sim}{\sigma}$ by

$$\underset{\sim}{\bar{\sigma}} = \left(\frac{1}{1-\omega}\right)\underset{\sim}{\sigma} \tag{1}$$

where ω is a scalar measure for damage and $0 \leqslant \omega \leqslant 1$. However in general, the internal state variable may be portrayed through a damage effect tensor, as introduced by Leckie and Onat [10]. For anisotropic damage, the effective stress can be expressed in a generalized form as

$$\underset{\sim}{\bar{\sigma}} = M(\omega) : \underset{\sim}{\sigma} \tag{2}$$

where the symbol (:) means tensorial product contracted on two indices and $M(\omega)$, known as damage-effect tensor, is a linear symmetric operator represented by a fourth order tensor. There are many possible forms of the generalized damage-effect tensor M_{ij}. However, one obvious criterion for development of such a damage-effect tensor is that it should reduce to a scalar for isotropic damage. This reduction should be possible not only in a principal coordinate system but also in any coordinate system. There are many possible forms of the generalized damage-effect tensor which obey the stated criterion, with some of these forms defined by Chow and Wang [6,7] for analysis of metals. However, as far as concrete behavior is concerned, one form of the M-tensor which satisfies the stipulated criterion has been introduced by Khan et al. [8] and takes the following form:

$$M_{ij} = \begin{bmatrix} \frac{(1-\beta\omega_1)}{(1-\beta\omega_2)(1-\beta\omega_3)} & 0 & 0 \\ 0 & \frac{(1-\beta\omega_2)}{(1-\beta\omega_3)(1-\beta\omega_1)} & 0 \\ 0 & 0 & \frac{(1-\beta\omega_3)}{(1-\beta\omega_1)(1-\beta\omega_2)} \end{bmatrix} \tag{3}$$

where ω_i, $i = 1, 2, 3$, are the principal damage components. The parameter β is introduced as a calibration parameter and which is obtained by matching experimentally measured peak strengths for various stress paths. It may be noted that for isotropic damage, $\omega_1 = \omega_2 = \omega_3 = \omega$ and Eq. (3) readily reduces to a scalar.

This assumed form of M_{ij} has the constraint condition that principal axes of stress coincide with the principal damage directions. A more general form of M_{ij} valid for nonproportional loading has been presented by Chen and Chow [11] where the principal directions of stress do not necessarily coincide with those of damage.

For undamaged state, the linear elastic constitutive relation is

$$\underset{\sim}{\varepsilon}^{e} = C : \underset{\sim}{\sigma} \tag{4}$$

where $\underset{\sim}{\varepsilon}^{e}$ and $\underset{\sim}{\sigma}$ are the elastic strain and stress tensor in the principal coordinate system and C is the elastic compliance tensor given by

$$[C] = \frac{1}{E_0}\begin{bmatrix} 1 & -v & -v \\ -v & 1 & -v \\ -v & -v & 1 \end{bmatrix} \tag{5}$$

in which E_0, v are the initial elastic modulus and Poisson ratio of the material, respectively. However, if the material is in a damaged state, then the elasto-damage constitutive equation can be written as

$$\underset{\sim}{\varepsilon} = \overline{C} : \underset{\sim}{\sigma} \tag{6}$$

where $\overline{C}$ is the effective compliance matrix and $\underset{\sim}{\varepsilon}$ is the elasto-damage strain tensor. In the hypothesis of elastic energy equivalence stated by Sidoroff [12], the complementary elastic energy for a damaged material is the same in form as that for an undamaged material, except that the Cauchy stress σ is replaced by the Cauchy effective stress $\underset{\sim}{\bar{\sigma}}$ in the energy formulation. Accordingly, the complementary energy per unit volume $\rho\Lambda$ (ρ = material mass density) for undamaged and damaged states may be written as

$$\rho\Lambda(\sigma, 0) = \tfrac{1}{2}\underset{\sim}{\sigma}^{T}\underset{\sim}{\varepsilon}^{e} = \tfrac{1}{2}\underset{\sim}{\sigma}^{T} : C : \underset{\sim}{\sigma} \tag{7}$$

$$\begin{aligned} \rho\Lambda(\sigma, \omega) &= \tfrac{1}{2}\underset{\sim}{\bar{\sigma}} : C : \underset{\sim}{\bar{\sigma}} \\ &= \tfrac{1}{2}\underset{\sim}{\sigma}^{T} : (M^{T} : C : M) : \underset{\sim}{\sigma} \\ &= \tfrac{1}{2}\underset{\sim}{\sigma}^{T} : \overline{C} : \underset{\sim}{\sigma} \end{aligned} \tag{8}$$

where

$$\overline{C} = M^{T} : C : M \tag{9}$$

Upon substitution of (3) and (5) into (9), one may write the components of the effective compliance matrix explicitly as

$$\begin{aligned}
\overline{C}_{11} &= \frac{(1-\beta\omega_1)^2}{E_0(1-\beta\omega_2)^2(1-\beta\omega_3)^2} \\
\overline{C}_{22} &= \frac{(1-\beta\omega_2)^2}{E_0(1-\beta\omega_3)^2(1-\beta\omega_1)^2} \\
\overline{C}_{33} &= \frac{(1-\beta\omega_3)^2}{E_0(1-\beta\omega_1)^2(1-\beta\omega_2)^2} \\
\overline{C}_{12} &= \overline{C}_{21} = \frac{-\nu}{E_0(1-\beta\omega_3)^2} \\
\overline{C}_{13} &= \overline{C}_{31} = \frac{-\nu}{E_0(1-\beta\omega_2)^2} \\
\overline{C}_{23} &= \overline{C}_{32} = \frac{-\nu}{E_0(1-\beta\omega_1)^2}
\end{aligned} \tag{10}$$

From (10) it is obvious that the thermodynamic constraint requirement $E_i\nu_{ji} = E_j\nu_{ji}$ is satisfied.

3. Damage evolution law

In order to construct a rational model accounting for damage growth, concepts are borrowed from incremental theory of plasticity in general and the BS plasticity model in particular as introduced by Dafalias and Popov [4]. The plasticity BS model requires the definition of multiple surfaces in stress space. However, the fundamental surfaces in the present work are best described in strain-energy release space, as elucidated by Suaris et al. [3] and given by

$$f = (R_iR_i)^{1/2} - R_c/b = 0 \tag{11}$$

$$F = (\overline{R}_i\overline{R}_i)^{1/2} - R_c = 0 \tag{12}$$

$$f_0 = (R_iR_i)^{1/2} - R_0 = 0 \tag{13}$$

where f is the LS, F is the BS, and f_0 is a LFS as shown in Fig. 1. The loading function surface f is defined in terms of thermodynamic-force conjugates, R_i, where

$$R_i = \rho\frac{\partial\Lambda}{\partial\omega_i}(\sigma_{ij}, \omega_i) \tag{14}$$

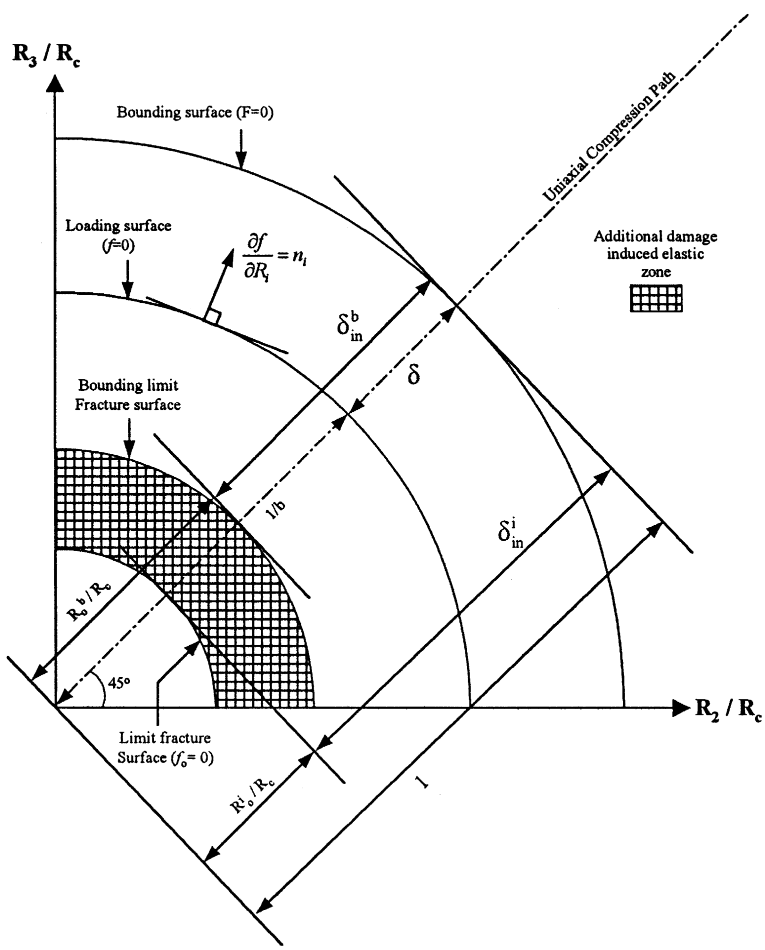

Fig. 1. Limit fracture, loading and bounding surfaces.

$\overline{R}$ is an image point on $F = 0$ associated with a given point R_i on $f = 0$ defined by a mapping rule

$$\overline{R}_i = bR_i \tag{15}$$

$$b = R_c/(R_iR_i)^{1/2} \tag{16}$$

with the mapping parameter b ranging from an initial value of ∞ to a limiting value of 1 on growth of LS to eventual coalescence with BS. R_c, critical energy release rate, is a parameter of the model and is calibrated to the standard uniaxial compression test. R_0 is the size of the LFS, assumed to be constant for the case of monotonic loading and varying with the magnitude of damage for the case of fatigue loading. The LFS defines a threshold in R-space beyond which there is an onset of damage.

The damage growth is determined from the LS $f = 0$ where the damage increment vector is assumed to be coaxial with the gradient of f which itself is also the unit vector n_i to the LS as shown in Fig. 1. Therefore, the principal damage components may be written as

$$\mathrm{d}\omega_i = \mathrm{d}\lambda \frac{\partial f}{\partial R_i} = \mathrm{d}\lambda n_i \tag{17}$$

with $k = R_c/b$, equation of LS becomes

$$f(R_i, k) = (R_iR_i)^{1/2} - k(\bar{\omega}_p) = 0 \tag{18}$$

where $\bar{\omega}_p$ is the norm of the accumulated damage and whose increment is defined by

$$\mathrm{d}\bar{\omega}_p = [\mathrm{d}\omega_i \mathrm{d}\omega_1]^{1/2} \tag{19}$$

It can be shown readily from (17) and (19) that the scalar magnitude of $\mathrm{d}\bar{\omega}_p = \mathrm{d}\lambda$.

The satisfaction of the consistency condition $\mathrm{d}f = 0$ yields

$$\frac{\partial f}{\partial R_i}\mathrm{d}R_i + \frac{\partial f}{\partial k}\mathrm{d}k = 0 \tag{20}$$

From (14) one may write

$$\mathrm{d}R_i = \frac{\partial R_i}{\partial \sigma_k}\mathrm{d}\sigma_k + \frac{\partial R_i}{\partial \omega_j}\mathrm{d}\omega_j \tag{21}$$

Also from (18), the incremental increase in the LS size may be written as

$$\mathrm{d}k = \frac{\mathrm{d}k}{\mathrm{d}\bar{\omega}_p}\mathrm{d}\bar{\omega}_p = \frac{\mathrm{d}k}{\mathrm{d}\bar{\omega}_p}\mathrm{d}\lambda \tag{22}$$

Introducing $H = \mathrm{d}k/\mathrm{d}\bar{\omega}_p$ = damage modulus, it can be measured experimentally in a uniaxial compression test, and the same form assumed for a more general stress path. In the present work, H is expressed as a function of the distance between the loading and the BS and given by Suaris et al. [3].

$$H = \frac{D\delta}{\langle \delta_{\mathrm{in}} - \delta \rangle} \tag{23}$$

where D is a material parameter which needs to be calibrated in accordance with concrete strength. The role of D is to essentially simulate the post peak $\sigma - \varepsilon$ response. $< >$ are Macaulay brackets that set the quantity within to zero if the argument is negative. The normalized distance δ between the loading and BS is given by

$$\delta = 1 - \frac{1}{b} \tag{24}$$

The $\delta = \delta_{\mathrm{in}}$ corresponds to R_0 when the LS first crosses the LFS (Fig. 1). Substitution of (21) and (22) into (20) and solving for $\mathrm{d}\lambda$ and then substituting the results into (19) yields

$$\mathrm{d}\omega_k = \left[\frac{\frac{\partial f}{\partial R_j}\frac{\partial R_j}{\partial \sigma_s}\mathrm{d}\sigma_s}{H - \frac{\partial f}{\partial R_n}\frac{\partial R_n}{\partial \omega_m}\frac{\partial f}{\partial R_m}}\right]\frac{\partial f}{\partial R_k} \tag{25}$$

Eq. (25) is convenient for stress control. However, for strain control one may adopt the strain energy density for damaged material ρW defined as

$$\rho W(\varepsilon_i, \omega_k) = \frac{1}{2}\underset{\sim}{\sigma}^{\mathrm{T}}(\varepsilon_{\mathrm{i}}, \omega_{\mathrm{k}}) : \underset{\sim}{\varepsilon} \tag{26}$$

The constitutive equation defined by (6) may be inverted and expressed as

$$\sigma_i = \overline{D}_{ij}\varepsilon_j \tag{27}$$

where $\overline{D}_{ij}$ are the components of the stiffness matrix defined by the inverse of the effective compliance matrix such that $\overline{D} = [\overline{C}]^{-1}$. Using (27) into (26) yields

$$\rho W(\varepsilon_i, \omega_k) = \frac{1}{2}\{\underset{\sim}{\varepsilon}\}^{\mathrm{T}} : \overline{D} : \{\underset{\sim}{\varepsilon}\} \tag{28}$$

The energy released rate vector R_i may now be expressed alternative to (14) as

$$R_j = -\rho\frac{\partial W(\varepsilon_i, \omega_k)}{\partial \omega_j} \tag{29}$$

whose increment may be written as

$$\mathrm{d}R_i = \frac{\partial R_i}{\partial \omega_j}\mathrm{d}\omega_j + \frac{\partial R_i}{\partial \varepsilon_l}\mathrm{d}\varepsilon_l \tag{30}$$

Employing the same procedure as that used for derivation of Eq. (25), except using Eq. (30) instead of Eq. (21) leads to the increment of damage

$$\mathrm{d}\omega_i = \left[\frac{\frac{\partial f}{\partial R_j}\frac{\partial R_j}{\partial \varepsilon_k}\mathrm{d}\varepsilon_k}{H - \frac{\partial f}{\partial R_n}\frac{\partial R_n}{\partial \omega_m}\frac{\partial f}{\partial R_m}}\right]\frac{\partial f}{\partial R_i} \tag{31}$$

which is convenient for the case of strain control.

4. Incremental stress–strain relations

The incremental form of the elasto-damage constitutive equations of (6) may be expressed in indicial notation as

$$d\varepsilon_i = \overline{C}_{ij} d\sigma_j + \sigma_l \frac{\partial \overline{C}_{il}}{\partial \omega_k} d\omega_k \tag{32}$$

which upon substitution of (25) yields

$$d\varepsilon_i = \left[\overline{C}_{ij} + \sigma_l \frac{\partial \overline{C}_{il}}{\partial \omega_k} \left(\frac{\frac{\partial f}{\partial R_k} \frac{\partial f}{\partial R_s} \frac{\partial R_s}{\partial \sigma_j}}{H - \frac{\partial f}{\partial R_n} \frac{\partial R_n}{\partial \omega_m} \frac{\partial f}{\partial R_m}} \right) \right] d\sigma_j \tag{33}$$

The incremental form of Eq. (27) in indicial notation may be written as

$$d\sigma_i = \overline{D}_{ij} d\varepsilon_j + \varepsilon_l \frac{\partial \overline{D}_{il}}{\partial \omega_k} d\omega_k \tag{34}$$

which upon substitution of (31) results in

$$d\sigma_i = \left[\overline{D}_{ij} + \varepsilon_l \frac{\partial \overline{D}_{il}}{\partial \omega_k} \left(\frac{\frac{\partial f}{\partial R_k} \frac{\partial f}{\partial R_s} \frac{\partial R_s}{\partial \varepsilon_j}}{H - \frac{\partial f}{\partial R_n} \frac{\partial R_n}{\partial \omega_m} \frac{\partial f}{\partial R_m}} \right) \right] d\varepsilon_j \tag{35}$$

The term within the square parenthesis in Eq. (35) may be interpreted as being the elasto-damage stiffness D_{ij}^{ed}.

4.1. Uniaxial compression–stress control

For uniaxial compression, the Cauchy stress tensor in the principal coordinate system reduces to a diagonal matrix or a stress vector given by

$$[-\sigma, 0, 0] \tag{36}$$

The complementary energy density $\rho\Lambda$ of Eq. (25) takes the following form in indicial or in matrix notation

$$\rho\Lambda = \frac{1}{2}\sigma_i \overline{C}_{ij} \sigma_j = \frac{1}{2}\{\sigma\}^{\text{T}}[\overline{C}]\{\sigma\} \tag{37}$$

Substituting Eqs. (10) and (36) into (37), one obtains

$$\rho\Lambda = \frac{\sigma^2 (1-\beta\omega_1)^2}{2E_0(1-\beta\omega_2)^2(1-\beta\omega_3)^2} \tag{38}$$

Differentiating (38) with respect to ω_i and substituting into (14), yields

$$R_1 = \frac{-\beta\sigma^2(1-\beta\omega_1)}{E_0(1-\beta\omega_2)^2(1-\beta\omega_3)^2} = 0 \quad (\text{since } R_i \not< 0) \tag{39a}$$

$$R_2 = \frac{\beta\sigma^2(1-\beta\omega_1)^2}{E_0(1-\beta\omega_2)^3(1-\beta\omega_3)^2} \tag{39b}$$

$$R_3 = \frac{\beta\sigma^2(1-\beta\omega_1)^2}{E_0(1-\beta\omega_2)^2(1-\beta\omega_3)^3} \tag{39c}$$

From symmetry, $\omega_2 = \omega_3 = \omega$ and $\omega_1 = 0$ (by virtue of Eq. (39a). Thus,

$$R_2 = R_3 = \frac{\beta\sigma^2}{E_0(1-\beta\omega)^5} \tag{40}$$

and the LS of Eq. (11) becomes

$$f = [R_2^2 + R_3^2]^{1/2} - R_c/b = 0 \tag{41}$$

whose gradient may be expressed as

$$\frac{\partial f}{\partial R_i} = \left[0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \tag{42}$$

Differentiating R_j with respect to ω_i and σ_k and substituting the results along with (42) into (25) yields $d\omega_i = 0$ and $d\omega_2 = d\omega_3 = d\omega$ given by

$$d\omega = \frac{2\sigma\beta d\sigma/E_0(1-\beta\omega)^5}{H - [5\beta^2\sigma^2/E_0(1-\beta\omega)^6]} \tag{43}$$

Differentiating $\overline{C}_{ij}$ of Eq. (10) with respect to ω_k and substituting the results along with (10) and (43) into (32), one obtains

$$d\varepsilon_1 = \left\{ \frac{1}{E_0(1-\beta\omega)^4} + \frac{8\beta^2\sigma^2/E_0(1-\beta\omega)^{10}}{H - [5\beta^2\sigma^2/E_0(1-\beta\omega)^6]} \right\} d\sigma \tag{44}$$

as the elasto-damage constitutive equation for uniaxial compression with the damage parameter ω being obtained by the accumulation of its increment defined in (43).

4.2. Uniaxial compression–strain control

For uniaxial compression, the strain energy density ρW for damaged material may be expressed from Eqs. (26), (27) and (10) as

$$\rho W(\varepsilon_i, \omega_k) = \frac{1}{2}\sigma_1\varepsilon_1 = \frac{(\varepsilon_1)^2(E_0/2)(1-\beta\omega_2)^2(1-\beta\omega_3)^2}{(1-\beta\omega_1)^2} \tag{45}$$

Note that for uniaxial compression, $\sigma_2 = \sigma_3 = 0$ while $\varepsilon_2 = \varepsilon_3 \neq 0$.

Differentiating (45) with respect to ω_i and substituting into (29) yields

$$R_1 = 0 \tag{46a}$$

$$R_2 = \frac{\beta\varepsilon_1^2 E_0(1-\beta\omega_2)(1-\beta\omega_3)^2}{(1-\beta\omega_1)^2} \tag{46b}$$

$$R_3 = \frac{\beta\varepsilon_1^2 E_0(1-\beta\omega_2)^2(1-\beta\omega_3)}{(1-\beta\omega_1)^2} \tag{46c}$$

Again from symmetry $\omega_2 = \omega_3 = \omega$ and also $\omega_1 = 0$. Differentiating R_j with respect to ω_i and ε_k and substituting the results along with (42) into (31) yields $d\omega_1 = 0$ and $d\omega_2 = d\omega_3 = d\omega$ given by

$$d\omega = \frac{2\beta\varepsilon_1 E_0(1-\beta\omega)^3 d\varepsilon_1}{H + 3\beta^2\varepsilon_1^2 E_0(1-\beta\omega)^2} \tag{47}$$

Differentiating $\overline{D}_{ij}$ with respect to ω_k and substituting the results along with (47) into (34) yield

$$d\sigma = \left\{ E_0(1-\beta\omega)^4 - \frac{8\beta^2\varepsilon_1^2 E_0(1-\beta\omega)^6}{H + 3\beta^2\varepsilon_1^2 E_0(1-\beta\omega)^2} \right\} d\varepsilon_1 \tag{48}$$

5. Model for cyclic loading

The distinguishing feature of cyclic loading from monotonic loading is a rational updating of the threshold of damage with increasing number of stress cycles. For monotonic loading, the threshold of damage is identified by the LFS $f_0 = 0$ and whose size is $R_0 = R_0^{\rm i}$, as shown in Fig. 1. However, for cyclic loading R_0 is hypothesized to change and increase with each successive cycle and is represented as $R_0 = R_0(\beta\bar{\omega})$, where $\beta\bar{\omega} = \beta(\omega_i\omega_i)^{1/2}$ is the magnitude of the effective damage vector $\beta\omega_i$.

The LFS is the surface beyond which the material behaves inelastically due to initiation or propagation of crack damage. The size of the surface R_0 is a function of the amount of the accumulated damage. Different functional forms describing the movement of the surface $f_0 = 0$ were considered and that of an elliptical form was found to successfully predict the experimental results for cyclic loading in compression [13]. The form of the surface in $R_0 - \beta\bar{\omega}$ space may be expressed as

$$\frac{(R_0 - R_0^{\rm i})^2}{(R_0^{\rm b} - R_0^{\rm i})^2} + \frac{(\beta\bar{\omega} - \beta\bar{\omega}_{\rm b})^2}{(\beta\bar{\omega}_i - \beta\bar{\omega}_{\rm b})^2} = 1 \tag{49}$$

The parameters $R_0^{\rm i}$ and $\beta\bar{\omega}_i$ correspond to the initial size of the LFS and the associated damage, respectively, with $R_0^{\rm b}$ and $\beta\bar{\omega}_{\rm b}$ corresponding to the bound or the limiting size of the LFS and the associated damage, respectively, as shown in Fig. 2.

The function in Eq. (49) represents an ellipse in $R_0 - \beta\bar{\omega}$ space, where R_0 grows monotonically as damage $\beta\bar{\omega}$ increases until it reaches a limiting value$R_0^{\rm b}$, which is the size of the bounding LFS, as shown in Fig. 1. At this stage the damage will reach $\beta\bar{\omega}_{\rm b}$, as shown in Fig. 2. It is emphasized that the LFS may reach its BS whilst the LS $f = 0$ may still be remote from its own conjugate BS $F = 0$. Consequently, further damage is deemed to occur at a fixed size of LFS ($R_0^{\rm b}$) until damage reaches its limiting value $\beta\bar{\omega}_{\max}$ and the LS $f = 0$ reaches the BS $F = 0$, defining incipient failure. The experimental results of Suaris et al. [3] indicate that crack initiation in compression occurs at about 40% of the peak stress for the particular case of concrete

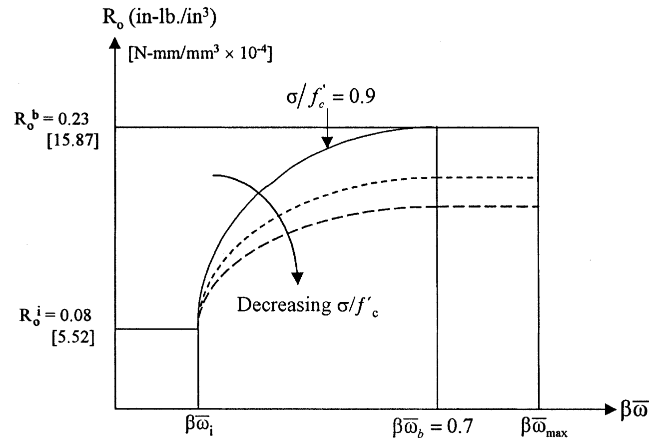

Fig. 2. Size evolution of limit fracture surface in $R_0 - \bar{\omega}$ space.

strength $f_c' = 5600$ psi (38.6 MPa), assuming an inherent initial damage component $\beta\omega_i = 0.05$. The strain energy release rate components corresponding to this initial damage and stress level are used in Eq. (13) in order to determine the initial size of the LFS $R_0^{\rm i}$. However, the initial size $R_0^{\rm i}$ tends to increase with concrete strength as damage is initiated at higher levels of threshold stress.

Using $\beta\bar{\omega}_i = \sqrt{2}\beta\omega_i = 0.0707$ into Eq. (49) yields the size of the LFS R_0 as a function of $\bar{\omega}$

$$R_0 = R_0^{\rm i} + \left\{ (R_0^{\rm b} - R_0^{\rm i})^2 \left[1 - \frac{(\beta\bar{\omega} - \beta\bar{\omega}_{\rm b})^2}{(0.0707 - \beta\bar{\omega}_{\rm b})^2} \right] \right\}^{1/2} \tag{50}$$

Eq. (50) is a two parameter model in terms of $R_0^{\rm b}$ and $\beta\bar{\omega}_{\rm b}$ for describing the size of the LFS in $R_0 - \beta\bar{\omega}$ space. These two parameters remain to be calibrated in accordance with phenomenological data available from cyclic loading of concrete in uniaxial compression.

6. Calibration of model parameters

The developed elasto-damage incremental laws, given by Eqs. (33) and (35), and the associated damage evolution described by Eqs. (25) and (31) reflect a general form, valid for any monotonic loading state. Certain standard tests are utilized to calibrate the model.

In the model predicting monotonic response, there are basically four parameters that need to be calibrated using experimental data. These parameters are $R_0^{\rm i}$ (or $\delta_{\rm in}$), β, $R_{\rm c}$, and D.

The parameter $R_0^{\rm i}$ defines the initiation of microcracking which occurs at about 40% of the peak stress under uniaxial compressive loading for concrete with $f_c' = 5600$ psi (38.6 MPa). This initiation of damage has been noted to vary with the compressive strength [2] where the damage threshold has been noted to be almost 60% of the peak stress for concrete with $f'_c = 11,000$ psi (75.9 MPa). For concrete strength in the range of $3000 \leqslant f'_c \leqslant 7000$ psi (20.7 MPa $\leqslant f'_c \leqslant 48.3$ MPa), a median value of $R_0^i = 0.08$ in-lb/cu-in (5.52×10^{-4} N mm/m^3) was computed; whereas for the range $7000 < f'_c \leqslant 9700$ psi (48.3 MPa $< f'_c \leqslant 66.9$ MPa), R_0^i was fixed at 0.16 in-lb/cu.in (1.103×10^{-3} N mm/m^3). Once R_0^i is determined, then the corresponding δ_{in} can be obtained from Eqs. (16) and (24).

The most important and critical model parameter is β which controls the damage growth rate and influences the pre-peak behavior as well as the level at which the peak stress is attained. Effect of cyclic loading is taken care of by introducing β_{dyn} which takes into account the additional 17% strength gain due to cycling of load as reported by Su and Hsu [9]. Thus, the behavior of concretes of varying compressive strength and cyclic loading is simulated by different values of β. The variation of β (referred to as β_{stat} for monotonic loading and β_{dyn} for cyclic loading) as a function of concrete strength f'_c is shown in Fig. 3, where the higher the concrete strength, the lower is the value of β. This variation guarantees a slower accumulation of damage in order to attain levels of stress commensurate with increasing concrete strength.

The parameter R_c is the critical energy release rate and is the magnitude of the energy release rate vector R_i when the LS $f = 0$ reaches the BS $F = 0$. Just as R_0 is a function of concrete strength, R_c must also vary with f'_c. However, the introduction of the scaling parameter β obviates this requirement and R_c is chosen to be fixed at 1.29 in-lb/cu-in (8.896×10^{-3} N mm/m^3).

The parameter D controls the softening phase of material response in $\sigma - \varepsilon$ space. In order to simulate sharper softening gradients as depicted by concretes of increasing brittleness and higher strength [14], the variation of D with concrete strength was adopted as shown in Fig. 3.

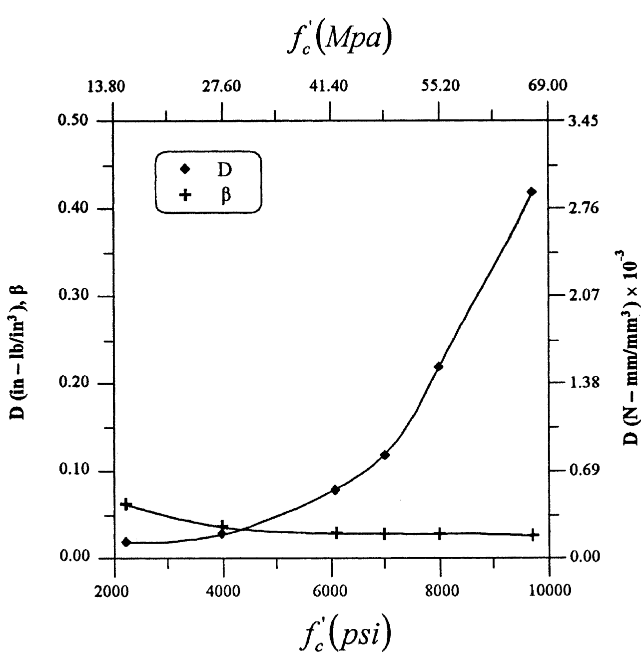

Fig. 3. Variation of model parameters β_{st} and D with compressive strength f'_c.

7. Model response

In order to simulate the response under monotonic loading, the strain control model of Section 4.2 was coded into a Fortran program. Input for four model parameters (R_0, R_c, β, D) was provided as a function of concrete strength.

7.1. Predictions for fatigue life

The general problem of response of elasto-damage material to a prescribed loading is highly nonlinear. The use of incremental forms for description of state variables such as damage ω_i is necessary in order to describe their evolution. A Fortran 77 program coding has been written, the flow chart of which is shown in Fig. 4. The coding computes cumulative damage as the number of cycles N is increased.

The flow chart consists of three main do-loops: the innermost loop accounts for computation of damage increment based on the values of damage and strain energy release rate of previous increment as an initial guess. The iterative procedure is implemented until convergence is attained in terms of a consistent set $\omega_i - R_i$. The intermediate loop is related to the incrementation of stress starting from zero until the prescribed stress level is reached, followed by unloading to the origin; the outermost do-loop monitors the movement of the LS as it approaches the BS.

The evolution of the LFS has been defined in terms of the parameters R_0^b and $\beta\bar{\omega}_b$. In general these parameters are noted to be functions of the maximum amplitude of cycling load σ/f'_c and uniaxial compressive strength f'_c. For the concrete strengths ranging from 4000 to 6100 psi (27.6–42.1 MPa), assuming $\beta\bar{\omega}_b = 0.7$ = constant for simplicity, and using the data as given by Su and Hsu [9] and assuming a similar variation of S–N curves for the entire range of f'_c considered, one obtains:

for $0.6 \leqslant \sigma/f'_c \leqslant 0.9$

$$\begin{aligned} R_0^b = {} & (2.98633\text{E}-08 \times f_c'^2 - 0.000233196 \times f'_c \\ & + 1.094711) \times \sigma/f'_c + (-2.05902\text{E}-08 \times f_c'^2 \\ & + 0.000243343 \times f'_c \\ & - 0.8807802)(f'_c \text{ in psi}, R_0^b \text{ in in-lb/cu-in}) \end{aligned}$$

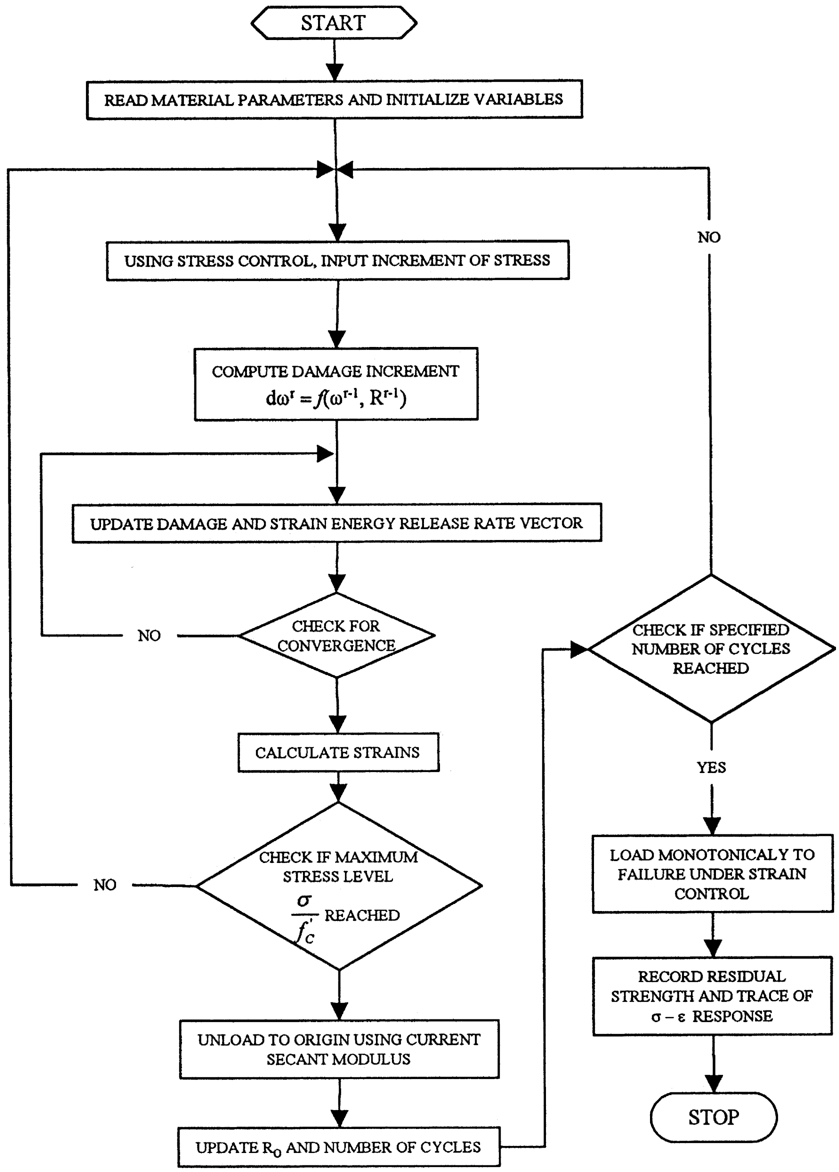

Fig. 4. Flow chart for code predicting cycles to failure.

for $\sigma/f_c' > 0.9$

$$R_0^b = (9.37396\mathrm{E}-05 \times f_c' - 0.04719945)$$

The movement of the LFS is noted to be a function of σ/f_c' . The elastic core as defined by $(R_iR_i)^{1/2} < R_0$ is constrained so as to decrease with decreasing amplitude of cycling stress, resulting in a corresponding increase in the damage growth zone that allows for greater accumulation of damage. This transition allows for failure to occur at a higher number of cycles commensurate with experimental findings when cycling at lower σ/f_c'. The elastic core evolution also implies that the system is rendered more elastic or flexible when cycled at high σ/f_c'.

7.2. Residual strength predictions

In order to predict the residual strength of concrete after it has been subjected to a number of load cycles, the cyclic loading model of Section 5 was coded into a FORTRAN program with four model parameters (R_0, R_c, β, D) and number of loading cycles provided as input. The structure and working of the program is almost similar to the one as discussed in previous section. The only difference is that first the concrete is subjected to a number of load cycles less than the fatigue life and then monotonically loaded to failure to determine the residual strength.

A series of experimental cyclic loading tests were conducted to compare the results with the model pre-

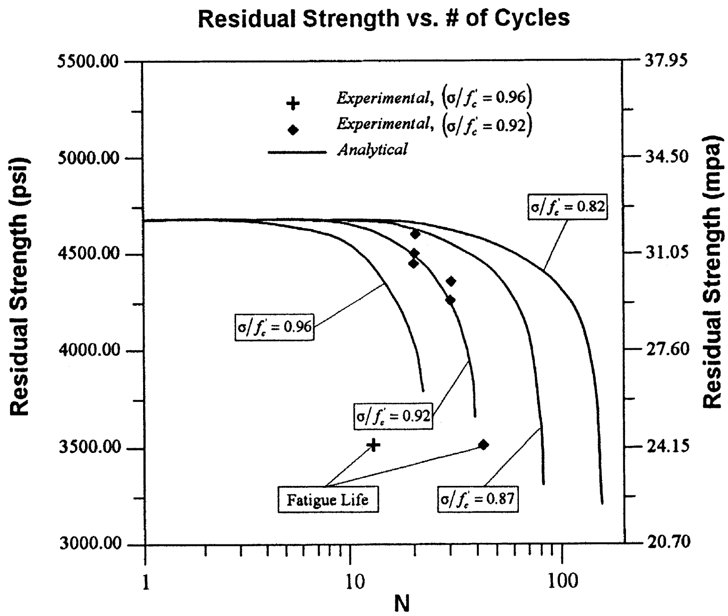

Fig. 5. Residual strength predictions.

dictions. Sixteen 75 × 150 mm cylinders were cast using a commercial grade cementitious repair material in order to have a uniform and consistent mix. The tests were conducted by sinusoidally cycling the load between preset limits. The maximum stress values investigated varied from 82% to 96% of the uniaxial compressive strength. High cycling stresses were chosen to ensure the failure of the specimen within about 400 cycles. The minimum stresses were set close to zero.

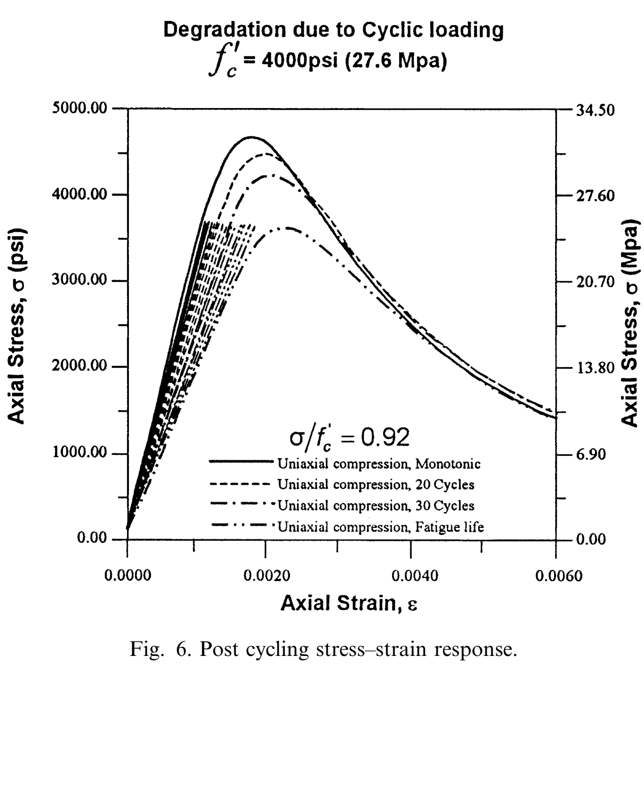

Fig. 6. Post cycling stress–strain response.

Residual strength predictions of the model are compared with the experimental data in Fig. 5. Residual strengths predicted experimentally and numerically for $\sigma/f'_c = 0.92$ seem to be in close agreement with each other.

Fig. 6 shows the analytical stress–strain curve for a cyclic test in which the residual strength is predicted after 20, 30 and 39 load cycles, superimposed on monotonic stress–strain curve. The monotonic stress–strain curve serves as the bound for cyclic loading tests and once the LS reaches the BS, failure occurs.

8. Conclusions

A recently derived CDM model for fatigue loading of concrete of varying compressive strengths has been adapted for use in the prediction of residual strength of concrete. The material is subjected to prior damage induced by loading cycles less than the fatigue life conjugate to the maximum amplitude of stress. Reasonably close correspondence is noted between theoretical predictions and experimental results for the residual strength of a high commercial grade repair concrete with prior damage sustained due to cyclic loading at $\sigma/f'_c = 0.92$.

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