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A probabilistic method of testing for the assessment of deterioration and explosive spalling of high strength concrete beams in flexure at high temperature

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Abstract

In the fire community there are a number of misconceptions, which have been inherited from previous work as far back as the early 1900. These have arisen from deterministic experiments by respectable research workers, which have been dogmatically followed over the years. So we have inherited a number of misconceptions as standard fire tests to simulate an actual fire. Vague terms such as spalling has been frequently been used which does not differentiate between violent failure termed explosive spalling and breaking of surface concrete. These two distinct phenomena are the result of quite different effects.

Since a number of factors influence these type of failures under elevated temperature a non-deterministic series of tests on different types of high strength concrete have been carried out at City University, London. A number of factors known to affect explosive spalling have been selected at three different levels and the results have been examined statistically to assess the significance of the individual and interacting factors. The paper describes the approach and some of the results.

Keywords: Deterioration; Explosive spalling; Concrete beams

1. Introduction

The main aim of the project was to investigate the deterioration and explosive spalling of different types of concrete tested in flexure during fire and to find methods of alleviating this phenomenon. The work was carried out under a research contract with the Engineering and Physical Science Research Council (EPSRC) with support from the Health and Safety Executive (H&SE) and with co-operation from the Building Research Establishment.

The overall objectives of the research project were:

1. Establish the main factors affecting the susceptibility to deterioration and explosive spalling of different kinds of plain and reinforced concrete simply supported beams in flexure. All the tests were carried out on mature concrete of at least 90 days old. The factors examined were:
   (a) curing regime prior to heat testing;
   (b) rate of heating;
   (c) loading prior to heating;
   (d) water cement ratio;
   (e) type of aggregate;
   (f) polypropylene fibres;
2. This was intended to give a clearer understanding of how the factors affecting the stochastic process of explosive spalling influence this phenomenon. These same factors were used to study deterioration of concrete at high temperature.

Practical types of high strength concrete and previously used concrete were utilised for the tests. Hence a limestone concrete was used similar to one used by Building Research Establishment for their high temperature tests on limestone concrete columns [1,2]. A part limestone and part lightweight aggregate concrete termed modified concrete was based on that used on the Hibernia offshore platform [3]. A lightweight aggregate concrete was also tested with similar aggregate for the Hibernia concrete.

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Previous experience with explosive [4–8] spalling indicated that this sudden violent failure was a stochastic process and hence a probabilistic method of experimentation was adopted using fractionals factorial. Five factors suspected of influencing the behaviour of plain concrete beams at high temperature were investigated, each factor being tested at three different levels, so that non-linear effects could be studied. A series of tests using reinforcement in similar concrete beams was tested under identical conditions, since concrete beams are normally reinforced in practise. Polypropylene fibres were also used for a third series of tests on concrete beams maintained at 100% RH to study any improvements that may occur in the behaviour of concrete at high temperatures. This high humidity curing on fibre concrete is a more stringent condition than other forms of curing at high temperature.

2. Factors investigated

The high temperature tests have been carried out in a purpose built testing rig [8,9] on 27 (×3) plain, fibre and reinforced concrete beams of similar overall dimensions and a further 9(×3) control beams tested cold to failure. Three additional beams with embedded thermocouples have been tested to obtain thermal response data for the three different rates of heating and the three different types of aggregates.

The design of the experimental programme has been based on the fractional factorial method of analysis on factors A–E shown and described in Table 1. All the factors have been investigated at three levels, so that the non-linear variations in individual and interacting factors can be determined.

In the initial tests on plain concrete beams, a limestone beam cured at 65% RH exploded and this was selected as the high curing level for this test series, the other two levels being 45% and 0% [7]. Since this was the only beam, which exploded in this series of tests, the plain concrete test series was repeated at RH of nominally 100%, 65% and 45%. This was also the curing regime for the reinforced concrete beams.

3. Fractional factorial method of experimentation

Fractional factorial experimentation is a well-established method based on statistical analysis [10–14]. A defined fraction of the full factorial (the principal block) has been selected to assess the influence of each of the factors and the important interacting factors on the overall average effect. One of the advantages [15,16] of this experimental method is the enormous amount of saving in time and cost compared to performing the full factorial experiments which would require $3^5 = 243$ tests for a series investigating five factors at 3 levels. Although three factor and higher interactions are aliased for a 1/9

Table 1
Factors and levels

Factors for concrete/RC	Level	Description
A. Curing [a]	0	Nominally 100% RH (65%)
	1	Nominally 65% RH (45%)
	2	Nominally 45% RH (0%)
B. Heating rate (A)[b] (Fig. 1)	0	Low 700 °C in 180 min
	1	Medium 700 °C in 70 min
	2	High 700 °C in 40 min
C. Loading (B)[b]	0	0% of load capacity at 20 °C
	1	10% of load capacity at 20 °C
	2	20% of load capacity at 20 °C
D. Water/cement ratio (C)[b]	0	0.25
	1	0.35
	2	0.50
E. Aggregate (D)[b]	0	Light weight (Lytag, *L*)
	1	Modified (Mixed *N* and *L*, 45% Lytag replacement of limestone by volume)
	2	Normal weight (Limestone, N)
E[b] (6 mm micro-fibres)	0 F1[b]	1 kg monofilament pp. fibre/m^3
	1 F2[b]	2 kg/m^3
	2 F3[b]	3 kg/m^3

[a] Curing regime constant at 100% RH for fibre concrete series.
[b] Factors for the fibre concrete test series shown in brackets (pp = polypropylene).

fraction (i.e. 27 tests) used in these tests, and cannot be determined, some of the two factor interactions and all the single factor effects can be safely assessed [15,16].

This method of testing is unlike classical one factor at a time testing which cannot discriminate between factors interacting simultaneously. Also classical testing assumes certainties in testing conditions, whereas factorial experimentation allows the error inherent in any test due to instrumentation, material uncertainties, or even human errors to be estimated. The variation of a change in level of each factor from the overall average result is estimated and the variances can then be compared to the error factor variance. This enables one to estimate percent probability of the variation being due to the error. If the probability is less than 10%, then the factor is said to be significant to >90% in influencing deterioration or explosive spalling depending on the result selected. Average values are closer to the truth than individual and this technique relies on average effects.

4. Fractional factorial procedure

The mechanical procedure for the analyses for explosive potential is shown in the Appendix A for a series of 27 tests on concrete beams. The criterion for explosive potential was estimated by subtracting the weight of the intact part of the beam after exploding from the original weight. Thus a value of 0 in the last column of Table A.1 indicates no explosion took place and a large value indicates a more violent failure. Table A.1 shows the 27 tests with the various factors at the three levels in the columns and the factor levels of individual tests in the rows. The remainder of the tables shows how the analysis of variance is carried out and Table A.6 shows how the significance of the factors are estimated.

Table A.1 can also be used for other criteria and the results measured inserted in the last column. So for deterioration, expansions of the beams were used as the criterion and an analysis of variance would then be carried out on these new values. Similarly these 27 tests can also be used over and over again for other criteria such as deflections, permeability or any other measurement that has been taken during the testing. This is another advantage of adopting this method of testing as the same tests can be used to assess more than one effect, if sufficient measurements are recorded.

5. Results

An analysis of variance (ANOVA) performed on the factors investigated revealed that a change in factor level affected deterioration to the percent significance in Table 2, and explosive spalling to the percent significance in Table 3. This analysis has not been shown in the appendix, but the same principles apply as for explosive spalling.

The values in Tables 2 and 3 denote that when each of the factors are tested simultaneously during heating, the factors have a significant effect on deterioration or explosive spalling if the values are greater than 95%. This also implies that there is only a 5% chance that the effect is due to the inherent error in the experimentation.

6. Conclusions

Explosive spalling and deterioration of the different types of concrete were influenced by the factors shown in Tables 2 and 3. It will be observed that deterioration and explosive spalling were not affected in the same manner, or with the same percent significance.

Table 3
Significance for explosive spalling for the plain and RC beams

Factor	Explosive spalling of beams cured at 85, 65 and 45 % RH	
	Plain concrete	RC
A (curing)	–	>90
B (heating)	>95	≫99
C (loading)	<95	–
D (W/C)	–	>95
E (aggregate)	–	≫95

Table 2
Significance for deterioration for the plain, reinforced and fibre concrete beams

Factor	Deterioration of beams cured at an RH%			
	65, 45 and 0 RH%	85, 65 and 45 RH%		
	Plain concrete	Plain concrete	RC	Fibre concrete
A (curing)	–	–	>90	Not applicable
B (heating)	>99	95	≫99	<90
C (loading)	>90	–	–	–
D (W/C)	>95	–	>95	≫95
E (aggregate)	≫99	>99	≫95	<90
F (fibre)				–

6.1. Deterioration

The manner in which the factor levels influenced deterioration together with the percent significance is indicated in Fig. 1 and described in Table 4.

The plain concrete beams initially tested after curing the mixes at 65%. 45% and 0% RH confirmed the order of deterioration for the aggregate, heating rate and W/C factors in Table 4. However only the heating rate and the aggregate factors were significant for the concrete at the higher RH. A change in curing did not affect deterioration of concrete but there was a significant difference in deterioration for the reinforced concrete cured from around 60% to 85% RH.

For the fibre concrete the only significant factor, which affected deterioration was W/C with a maximum at 0.35, slightly tailing away at 0.25 and at a faster rate towards 0.5. This was confirmed for the plain concrete series tested at the lower humidity. For the reinforced concrete, this trend was reversed indicating a minimum deterioration at a W/C of 0.35 with the 0.25 and 0.5 W/C increasing equally in deterioration. This apparent anomaly may be accounted for in that the 0.35 mixes had a higher mortar fraction than the other mixes making the 0.35 concrete more compact and impermeable as subsequently indicated by the capillary rise tests, thus creating more damage to the fibre and plain concrete. The steel in the under-reinforced beams spanned the cracks at elevated temperature and appeared to make no difference to deterioration.

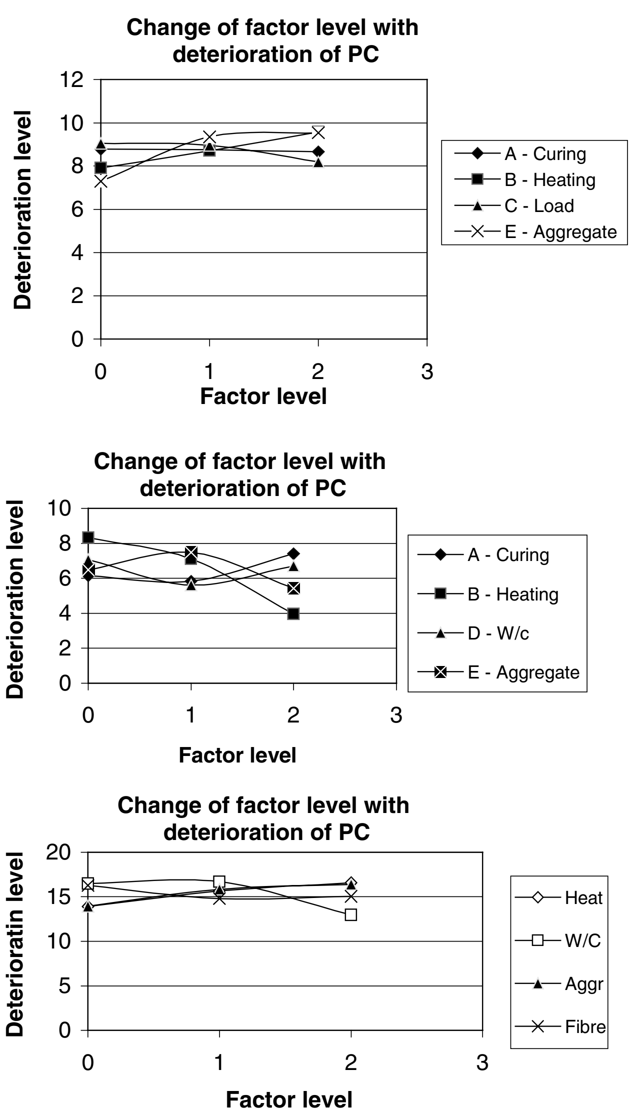

Fig. 1. Change of factor level with deterioration of reinforced concrete (RC) plain concrete (PC) and fibre concrete (FC) beams.

6.2. Explosive spalling

The manner in which the factor levels influenced explosive spalling together with the percent significance is indicated in Fig. 2 and described in Table 5.

The following behaviour of the plain and reinforced concrete beams was observed.

- Curing conditions did not appear to affect explosive spalling in plain concrete beams, but the reinforced concrete beams were more susceptible to explosive spalling between 85% and 65% and reduced considerably at 45%.
- Heating rate increased the chances of explosive spalling from the medium to high rate of heating for both plain and reinforced concrete, no explosions taking place at the low rate.
- Loading decreased the chance of explosive spalling to plain concrete, but did not have any effect on reinforced concrete.
- A W/C ratio of 0.35 appeared to be the most susceptible for reinforced concrete to explode, with the susceptibility decreasing towards W/C's of 0.25 and 0.5. For plain concrete a W/C of 0.35 and 0.25 had a greater possibility of explosion than that with a W/C of 0.5, but this was however, not statistically significant.
- Reinforced limestone concrete had a greater inclination to fail violently. This reduced with modified and then slightly increased with LWA reinforced concrete. The aggregate type did not significantly affect plain concrete, although the modified mixes appear to be the least affected.
- No explosions took place on any of the fibre concrete beams even though they were cured at a nominal RH of 100% prior to testing.

Some identical concrete mixes for the plain and reinforced concrete beams failed explosively at the high rate of heating. However, since different factors affected explosive spalling for the plain and reinforced concrete,

Table 4
Increase in deterioration for plain, reinforced and fibre concrete

Factor	Deterioration increase for beams cured at a RH%			
	65, 45 and 0	85, 65 and 45		
	Plain concrete	Plain concrete	RC	Fibre concrete
A (curing)	–	–	60–85% RH[a]	Not applicable
B (heating)	High to low rate	High to low rate	Low to high rate	–
C (loading)	Linear increase	–	–	–
D (W/C)	Maximum at 0.35	–	Minimum at 0.35	Maximum at 0.35
E (aggregate)	LWA to limestone	LWA to limestone	Maximum Modified	
F (fibre)				

[a] A slight increase in deterioration at 45% RH.

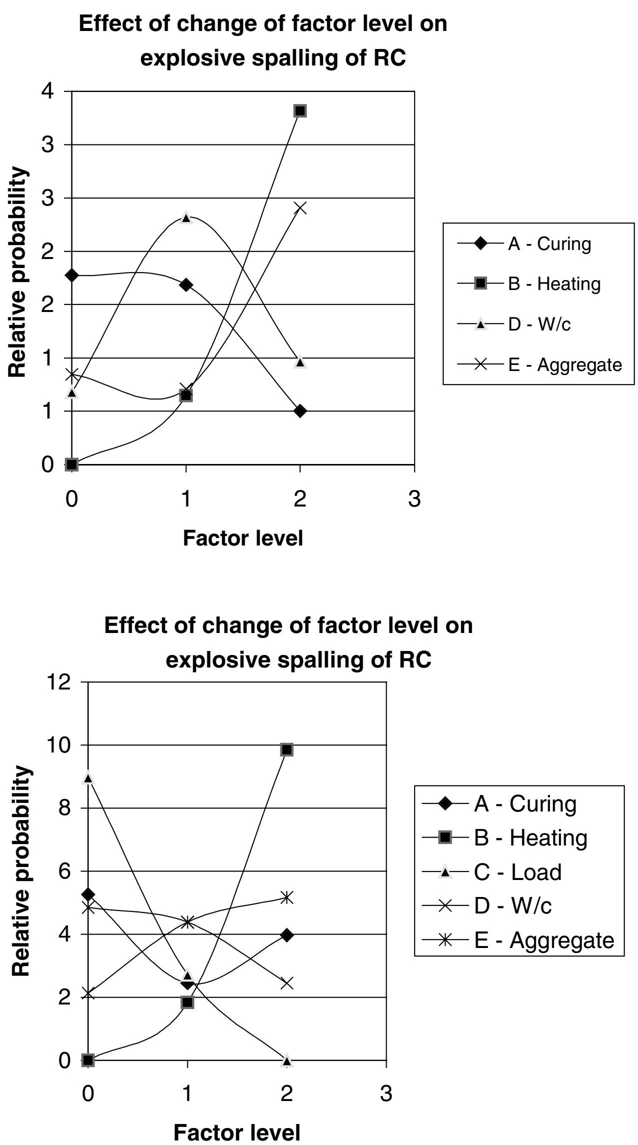

Fig. 2. Change of factor level with explosive spalling of reinforced concrete (RC) and plain concrete (PC) beams (Note: The probability scale for the PC is three times that for RC and the factor level scale is 0.9 that of for RC).

some of the former with identical mixes to the reinforced concrete beams exploded but not the latter and vice versa. The factors influencing explosive spalling in the two identical mixes are shown in Tables 3 and 4. Only one explosion occurred for the plain concrete cured at the lower RH. This occurred on a limestone concrete beam with a W/C of 0.35, cured at 65% heated at the high rate with no load applied.

Table 5
Increase in explosive spalling for plain and reinforced concrete

Factor	Increase in explosive spalling	
	Plain concrete	RC
A (Curing)	–	60–85% RH[a]
B (Heating)	Medium to high[a]	Medium to high[a]
C (Loading)	High to low[a]	–
D (W/C)	–	Maximum at 0.35
E (Aggregate)	–	Modified to limestone (slight increase to LWA)

[a] Note no explosions occurred at the low rate of heating and at high load.

7. Remarks

The effect of the salient factors influencing deterioration and explosive spalling acting simultaneously during heating has been established by the use of the fractional factorial method of experimentation. The ANOVA has helped to distinguish clearly which factor change in level statistically influences to a significant degree, these phenomena and the percentage probability of these influences. Previous research conducted by the author and others using classical one factor at a time testing techniques was unable to distinguish the difference in flexural behaviour of plain and reinforced concrete using identical mixes at elevated temperature as observed above.

Factors, which influence beams adversely during heating and/or loading causing initial cracking, are unlikely to fail violently. Concrete with W/C ratio of 0.35, which were less liable to deteriorate were more susceptible to explosive spalling than concrete with

W/Cs of 0.25 and 0.5. The reason for this anomaly was that the concrete with a W/C of 0.35 had the lowest surface permeability.

The use of the polypropylene micro-fibres used in the tests prevented explosive spalling occurring. The fibres were thought in the past to melt during heating, thus relieving the pore pressures during heating. This may be one reason, but another reason discovered in the tests above is that the fibres have incompatible movements with the mortar matrix producing micro-cracks long before melting. These micro-cracks also help relieve pore pressures.

Some tests but not reported here where a percentage of plastic nodules replaced some of the aggregates were more susceptible to explosive spalling [17]. This was confirmed by tests carried out in Hokkaido recently on polypropylene nodules and is to be published shortly in a Ph.D. thesis. Other tests carried out recently by others [18], have also indicated that some polypropylene fibres are more effective than others at reducing explosive spalling under fuel fires, so care has to be exercised when recommending the use of fibres to alleviate these violent failures.

Explosive spalling is an entirely different mechanism to normal spalling. Normal spalling takes place when the ultimate tensile strength of the concrete at the surface of the concrete is reached during heating and the concrete cracks and finally falls off. This is a ductile failure as there is plenty of warning before ultimate failure with a crack first appearing before failure. Explosive spalling on the other hand is a brittle failure and occurs suddenly and violently. This is caused by the stresses and strains imposed on the gel structure creating high strain energies within the structure. The stresses are not necessarily tensile. They may be compressive or a combination of both, as restraint to movement of a member can increase the likelihood of explosive spalling. When these high strain energies are suddenly released due to a flaw appearing in the gel pore structure, the sudden release of energy creates the violent failure.

In the testing described, the design of the furnace creates a heating regime, which imposes lateral restraint in addition to local cool spots especially for the high heating rate. The stresses imposed by these effects and by pore pressures are locked in the pore gel structure and can produce quite high strain energies particularly in the more impermeable concrete with a tougher gel structure, which can therefore resist failure far longer. When a local defect releases this pent-up strain energy a violent failure of the gel structure occurs. This is the mechanism by which the higher strength more compact moist concrete heated at the faster heating rate in conjunction with the various imposed restraints created explosive spalling in our tests.

Appendix A

Table A.1 shows the fractional factorial table with the factors and their levels as indicated in the table. Each row represents a test and the last column shows the results, which in this case is the difference in weight of the intact parts of the concrete and the original weight. The loss in weight during heating was not significant in comparison to the concrete lost during the explosions, hence the 0's in the last column for the beams that did not explode.

Table A.2 shows the sum of nine test results of each individual factor at each level. (Level 0 for curing is C1, level 2 is C2 and level 3 is C3.) The sum Tot (S$Xijk$) is the sum of all the individual results and should be equal. This serves as a check.

Table A.3 shows the sum of three test results of each of the nine interacting factor levels. The sum of each of these 27 results Tot(Sum$XiXk$) is also equal to Tot (S$Xijk$) is the sum of all the individual results and should be equal, which also serves as a check.

Table A.4 shows the working for the sum of squares for individual factors.

The first row in Table A.4 above gives the average sum of squares over 9 tests of each individual factor (Sometimes called the crude average sum of squares) $\mathrm{SS}Xijk = \{(\mathrm{Sum}Xi)^2 + (\mathrm{Sum}Xj)^2 + (\mathrm{Sum}Xk)^2\}/9$. The second row or SSTot/27 is (sum of individual values) squared averaged over the 27 values.

The difference between these two values gives the deviance of each individual factor shown in the last row as Sa–Se. These values divided by the degree of freedom of each factor $(3 - 1 = 2)$ give the variances of the factors.

Table A.5 gives the sum of the squares of the interacting factors together with their deviance in the last row.

The deviance values S$a \times b$ of the interacting factors divided by the degree of freedom of each interacting factor (i.e. $6 - 2 = 4$) also gives the variances of the interacting factors and are also used in the table of significance shown in Table A.6.

The first two columns in Table A.6 show the source or individual and interacting factors and their degrees of freedom. The third column is the deviance derived from the tables above; the fourth column the variances. The fifth and sixth columns show the Fisher factors F and are the result of the variance of the individual and interacting factors divided by that of the error selected.

The degree of freedom (df) of the whole experiment is $27 - 1 = 26$. Since the sum of all the degrees of freedom of all the individual and interacting factors in column 2 of Table A.6 is 26, the degree of freedom of the error would be 0 and it would not be possible to estimate columns 5 and 6 as the divisor would be 0.

Table A.1
Fractional factorial analyses for explosions of plain concrete

Factors Test No.	Curing	Heating	Load	W/C	Aggreg	Weight lost (kg)
	A	B	C	D	E	
1	C1 = 45%	Low	0%	0.25	Light	0.00
2	C1	L	10%	0.50	Modified	0.00
3	C1	L	20%	0.35	Normal	0.00
4	C1	Medium	0	0.50	Normal	0.00
5	C1	M	10	0.35	Light	0.00
6	C1	M	20	0.25	Modified	0.00
7	C1	High	0	0.35	Modified	7.64
8	C1	H	10	0.25	Normal	8.14
9	C1	H	20	0.50	Light	0.00
10	C2 = 65%	L	0	0.35	Modified	0.00
11	C2	L	10	0.25	Normal	0.00
12	C2	L	20	0.50	Light	0.00
13	C2	M	0	0.25	Light	0.00
14	C2	M	10	0.50	Modified	0.00
15	C2	M	20	0.35	Normal	0.00
16	C2	H	0	0.50	Normal	7.36
17	C2	H	10	0.35	Light	0.00
18	C2	H	20	0.25	Modified	0.00
19	C3 = 85%	L	0	0.50	Normal	0.00
20	C3	L	10	0.35	Light	0.00
21	C3	L	20	0.25	Modified	0.00
22	C3	M	0	0.35	Modified	5.51
23	C3	M	10	0.25	Normal	0.00
24	C3	M	20	0.50	Light	0.00
25	C3	H	0	0.25	Light	6.40
26	C3	H	10	0.50	Modified	0.00
27	C3	H	20	0.35	Normal	0.00

% of ultimate load.

Table A.2
Main effects

	Main effect					Level
	Curing	Heating	Load	W/C	Aggregate	
	A	B	C	D	E	
SUM1 (*Xi*)	15.78	0.00	26.91	14.54	6.40	0
SUM2 (*Xj*)	7.36	5.51	8.14	13.15	13.15	1
SUM3 (*Xk*)	11.91	29.54	0.00	7.36	15.50	2
Tot (S*Xijk*)	35.05	35.05	35.05	35.05	35.05	

Table A.3
Interaction

Combination	AB	AC	BC	DE	CD	CE
XiYi 00	0.00	7.64	0.00	6.40	6.40	6.40
XiYj 01	0.00	8.14	0.00	0.00	13.15	13.15
XiYk 02	15.78	0.00	0.00	8.14	7.36	7.36
10	0.00	7.36	5.51	0.00	8.14	0.00
11	0.00	0.00	0.00	13.15	0.00	0.00
12	7.36	0.00	0.00	0.00	0.00	8.14
20	0.00	11.91	21.40	0.00	0.00	0.00
21	5.51	0.00	8.14	0.00	0.00	0.00
22	6.40	0.00	0.00	7.36	0.00	0.00
Tot (Sum *XiYk*)	35.05	35.05	35.05	35.05	35.05	35.05

Selecting interacting factor DE as the error, results in the values in column 5. Comparing these values with those in Fisher tables, which compare the value of the factor with that of the error, none of the factors are significant to more than 90%.

We can increase the sensitivity of the factor significance by increasing the degree of freedom of the error. In the 6th column we have taken all the interacting factors as contributing to the error with the result that now we can discriminate between the factors which are significant and those which are not as indicated in column 7.

Table A.4
Deviance of factors

	A	B	C	D	E
SS *XiXjXk*/9	49.447	100.330	87.823	48.723	50.459
SS Tot/27	45.500	45.500	45.500	45.500	45.500
S*a*–S*e*	3.947252	54.8301	42.32299	3.222541	4.959074

Note: S*a*–S*e* is equal to (SS*XiXjXk*/9–SS Tot/27) = Effect of factor X on O/A/average.

Table A.5
Deviance of interaction

	AB	AC	BC	DE	CD	CE
SS *XiYi*	374.50	320.65	554.58	334.31	334.31	334.31
SS *XiYk*/3–SS T/27	79.33	61.38	139.36	65.94	65.94	65.94
$Sa \times b = Sab - Sa - Sb$	20.56	15.11	42.21	57.76		

Table A.6
Demonstrating the derivation of significance

Source	df	S*a*–S*ab*	V	$V(a)/V(\mathrm{Er1})$	$V(a)/V(\mathrm{Er2})$	Significance (%)
A (curing)	2	3.95	1.97	0.14	0.23	–
B (heating)	2	54.83	27.42	1.90	3.23	>95
C (loading)	2	42.32	21.16	1.47	2.50	<95
D (W/C)	2	3.22	1.61	0.11	0.19	–
E (aggregate)	2	4.96	2.48	0.17	0.29	–
AB	4	20.56	5.14	0.36		
AC	4	15.11	3.78	0.26	Errors	
BC	4	42.21	10.55	0.73		
DE	4	57.76	14.44	Error		
Error	0	0				
(Error)1	4	57.76	14.44			
(Error)2	16	135.63	8.48			
(Errror)3	12	77.87	6.49			

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