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Estimation of ultimate modulus of elasticity and Poisson ratio of normal concrete

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Abstract

The relationship between compressive strength of concrete and modulus of elasticity, and between compressive strength of concrete and Poisson ratio was investigated in this study by using the cement hydration equation. The hydration criteria of modulus of elasticity and Poisson ratio of concrete are established hydration criteria and for this reason the cement hydration equation can be used for calculating their accurate values up to the final hydration. Because of the applicability of cement hydration equation to the hydration criteria of modulus of elasticity, Poisson ratio and compressive strength of mortar and concrete, there is a linear relationship between any two of them according to the results of previous study. If examined mortar and concrete were prepared using the same cement. Hence, the ''method of two points proportionality'' can be used to estimate ultimate modulus of elasticity and Poisson ratio using only two measured values of these properties at ages $\leqslant$ 28 days (e.g. at 7 and 28 days) and the hydration criterion of mortar's compressive strength.

1. Introduction

The modulus of elasticity and Poisson ratio of concrete are fundamental parameters necessary in structural analysis for the determination of the strain distributions and displacements, especially when the design is based on elasticity considerations. According to Euro Code 2 the tendency is to design at ultimate state and verify at serviceability one. Static modulus of elasticity of concrete (E_c) is defined as the slope of the (σ–ε) relationship at origin, according to ASTM C 318–92 (i.e., the ratio of the stress to the corresponding elastic strain) and Poisson ratio (ν_c) as the ratio of transverse to axial strain of concrete.

According to the results given in the literature and in design standards [1,2], the modulus of elasticity and Poisson ratio are estimated from compressive strength of concrete, which increases with age. The values of modulus of elasticity of concrete (E_c) and of Poisson ratio of concrete (ν_c) depend on the values of modulus of elasticity of paste (E_P), of Poisson ratio of paste (ν_P), of modulus of elasticity of aggregates (E_A) and of Poisson ratio of aggregates (ν_A) [3]. Since the microstructure of the paste changes with age, the values of E_P and ν_P in concrete increase with an increase in compressive strength of concrete. Several investigators and standards have proposed empirical equations for the estimation of E_c and ν_c on the basis of their correlation to compressive strength [1–8]. There is an agreement on the increasing of modulus of elasticity and Poisson ratio with the increase in compressive strength of concrete, but there is no agreement on the precise form of the relation of E_c and ν_c to compressive strength. The equations published do not include the whole hydration period, and for this reason ignore the values of E_c and ν_c at later ages, where compressive strength significantly increases [8–12], as shown in Fig. 1. Ultimate mechanical properties, such as compressive strength, flexural strength, modulus of elasticity and Poisson ratio are key parameters for evaluating the strength and durability of the structure and making decisions about its life duration.

In the present study, the relationship between compressive strength of concrete and modulus of elasticity, and between compressive strength of concrete and Poisson ratio was investigated using the cement hydration equation [13]. This equation enables the prediction of the final rate of hydration at the age of 15 years of the cement used, when the hydration ends. For this purpose

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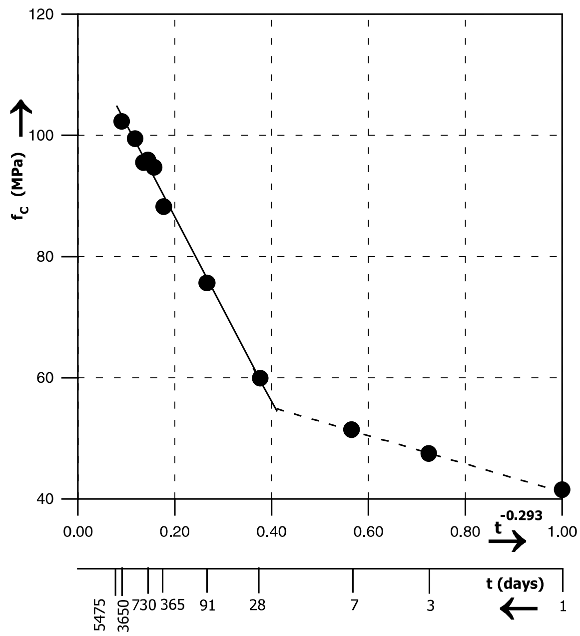

Fig. 1. Compressive strength of concrete prepared with Portland cement I as a function of hydration time. Compressive strength of 100×200-mm cores taken at ages ranging from 1 day to 10 years from 2.0×1.2×1.35-m thick concrete blocks cast, cured, and left exposed to elements in Toronto, Canada. $W/C = 0.27$ (measured values taken from Malhotra [12]).

the measured values of the hydration criteria of E_c and v_c of three concrete mixtures prepared with different cements (Tables 1–3) were analyzed with the aid of the cement hydration equation, in order to estimate the ultimate modulus of elasticity and Poisson ratio of concrete.

2. Cement hydration equation

The cement hydration equation has the general form of Eq. (1) and was discussed in previous studies [13,14].

$$K = K_\infty \pm b * t^{-p} \tag{1}$$

where K is the hydration criterion examined; K_∞ is a constant, which represents the intersection of the straight line of Eq. (1) with the ordinate; b is the slope of the straight line of Eq. (1); t is the hydration time expressed in days ($t \gg 0$); p is the hydration number, which is a constant for a given cement or for a major individual phase and specifies the rectilinear course of hydration in the t^{-p} scale during the whole hydration period.

When the hydration criterion of heat of hydration (Q) is used it takes the form $Q = Q_\infty - b * t^{-p}$ and constitutes the equation that gives quantitatively the course of the chemical reaction of cement with water [13] and the values calculated by using the cement hydration equation are almost equal to the correct ones. An example is given in Ref. [15], where the surface area of C_3S, calculated by using the hydration equation defined on the basis of the measured values of surface area, determined at ages $\leqslant 400$ days (Fig. 2), equals at final hydration (15 years) to that determined on the basis of crystal structure calculations based on stoichiometry. Further, the straight line defined by performing linear regression over the value pairs of two established hydration criteria, calculated with the aid of cement hydration equation, appears a correlation coefficient close to 1 [14], as also shown in Tables 1–3. Hence, this straight line constitutes a precise relationship between the hydration criteria correlated.

The final hydration time was proposed in a previous study [14] to be equal to 15 years. This proposal was made on the basis of long-term data concerning heat of hydration [17] (obtained at ages ranging from 3 days to 6.5 years for cement pastes with $W/C = 0.6$ and 0.8 and from 3 days to 13 years for pastes with $W/C = 0.4$) analyzed with the aid of cement hydration equation. According to these analyses the heats of hydration calculated for the 0.4 water to cement ratio mixture were equal to 119.43, 111.92, 100.18 and 86.92, and 119.48, 111.96, 100.34 and 87.13 cal/g at the age of 13 and 15 years for cement type III, I, II and IV (ASTM C 150), respectively. Where, the cement types III, I, II and IV correspond to early strength, ordinary, moderate heat of hydration and low heat of hydration of Portland cement. When a water to cement ratio of 0.8 was used the respective heats of hydration at 13 and 15 years were 132.61, 126.19, 112.45 and 97.97, and 132.67, 126.27, 112.63 and 98.23 cal/g. Further the hydration at low water to cement ratio, as for example at $W/C = 0.27$ (Fig. 1), appears also a small difference between the rate of hydration at 13 and the respective one at 15 years. The compressive strength of concrete, calculated with the aid of cement hydration equation (Fig. 1), appears the values of 104.28 and 104.80 MPa at 13 and 15 years, respectively. These results show small differences between the rate of hydration calculated at the ages of 13 and 15 years for the same mixture. It could be said therefore that hydration practically ends at the age of 15 years irrespective of the cement type or the water to cement ratio used ranging from 0.27 to 0.8.

Further, according to recent results given elsewhere [15], it was found that the hydration of C_3S and β-C_2S ended at the age of 15 years when a water to solid ratio of 0.7 was used. Concretely, this study found that the surface area in the paste hydration of C_3S and β-C_2S, determined on the basis of crystal structure calculations based on their stoichiometry, equals to that determined on the basis of water vapor adsorption at the age of 15 years. The stoichiometry of calcium silicate hydrates was also identified at this age.

Since all constituents of Portland cement hydrate simultaneously at independent and at equal fractional

Table 1
Hydration equation of compressive strength of Mortar (f_M), wet screening concrete (f_{cS}), and modulus of elasticity of mortar (E_M) and concrete (E_S and E_D) (measured values taken from Vilardell et al. [19])

	Hydration time (days)							
	7	14	28	90	180	365	730	5475
Mortar's compressive strength (f_M)								
f_{Mmeas} (MPa)	22.8	–	41.4	52.6	60.8	–	–	–
Hydration equation: $f_M = 132.7817 - 140.9103 * t^{-0.128}$ (MPa), $t \geqslant 7$, $r = 0.99903$, $s = 0.887$ (MPa)								
f_{Mcalc} (MPa)	22.94	32.27	40.80	53.57	60.29	66.56	72.19	85.96
Compressive strength of wet-screening concrete (f_{cS})								
f_{cSmeas} (MPa)	23.7	–	45.0	50.7	56.9	–	–	–
Hydration equation: $f_{cS} = 121.0629 - 124.9333 * t^{-0.128}$ (MPa), $t \geqslant 7$, $r = 0.99998$, $s = 0.164$ (MPa)								
f_{cScalc} (MPa)	23.68	31.95	39.51	50.82	56.80	62.36	67.34	79.55
Modulus of elasticity of mortar (E_M)								
E_{Mmeas} (GPa)	19.6	–	23.8	28.2	30.7	–	–	–
Hydration equation: $E_M = 51.7736 - 41.7691 * t^{-0.128}$ (GPa), $t \geqslant 7$, $r = 0.9942$, $s = 0.643$ (GPa)								
E_{Mcalc} (GPa)	19.21	21.98	24.51	28.29	30.29	32.14	33.81	37.89
Modulus of elasticity of wet-screening concrete (E_S)								
E_{Smeas} (GPa)	24.8	–	34.5	35.1	37.2	–	–	–
Hydration equation: $E_S = 61.4252 - 46.967 * t^{-0.128}$ (GPa), $t \geqslant 7$, $r = 0.99994$, $s = 0.10$ (GPa)								
E_{Scalc} (GPa)	24.81	27.92	30.77	35.02	37.26	39.36	41.23	45.82
Modulus of elasticity of dam concrete (E_D)								
E_{Dmeas} (GPa)	30.3	–	37.3	43.0	42.2	–	–	–
Hydration equation: $E_D = 75.5866 - 58.24 * t^{-0.128}$ (GPa), $t \geqslant 7$, $r = 0.99933$, $s = 0.329$ (GPa)								
E_{Dcalc} (GPa)	30.19	34.04	37.57	42.84	45.63	48.22	50.54	56.23

Relationship between compressive strength of mortar (f_M) and concrete (f_{cS}). Linear regression over the value pairs f_{Mcalc}–f_{cScalc}

Straight line: $f_M = 3.187 + 0.8902 * f_{cScalc}$ (MPa), $r = 0.99999$, $s = 0.1109$ (MPa)

Because the correlation coefficient is equal to about 1.0, there is a linear relationship between compressive strength of mortar and concrete

Relationship between mortar' s compressive strength (f_M) and modulus of elasticity E_M, E_S and E_D. Linear regression over the value pairs E_{Mcalc}–f_{Mcalc}, E_{Scalc}–f_{Mcalc} and E_{Dcalc}–f_{Mcalc}

Straight line: $E_M = 12.4147 + 0.2964 * f_{Mcalc}$ (GPa), $r = 1.0$, $s = 0.0035$ (GPa)

Straight line: $E_S = 17.1629 + 0.3334 * f_{Mcalc}$ (GPa), $r = 1.0$, $s = 0.0034$ (GPa)

Straight line: $E_D = 20.7074 + 0.4133 * f_{Mcalc}$ (GPa), $r = 1.0$, $s = 0.0048$ (GPa)

Because the correlation coefficient is equal to 1.0, there is a linear relationship between mortar's compressive strength and modulus of elasticity E_M, E_S and E_D, respectively

Relationship between compressive strength of wet-screening concrete (f_{cS}) and modulus of elasticity E_M, E_S and E_D. Linear regression over the value pairs E_{Mcalc}–f_{cScalc}, E_{Scalc}–f_{cScalc} and E_{Dcalc}–f_{cScalc}

Straight line: $E_M = 11.2987 + 0.3343 * f_{cScalc}$ (GPa), $r = 1.0$, $s = 0.0037$ (GPa)

Straight line: $E_S = 15.9075 + 0.376 * f_{cScalc}$ (GPa), $r = 1.0$, $s = 0.0039$ (GPa)

Straight line: $E_D = 19.1513 + 0.4661 * f_{cScalc}$ (GPa), $r = 1.0$, $s = 0.0023$ (GPa)

Because the correlation coefficient is equal to 1.0, there is a linear relationship between compressive strength of wet-screening concrete and modulus of elasticity E_M, E_S and E_D, respectively

rate during the whole hydration period, equality (2) is consistent [15] at all ages. C_3S and β-C_2S hydrated completely at the age of 15 years when a W/C of 0.7 was used (in this case degree of hydration (α) is equal to 1.0). C_3A, C_4AF and Portland cement were therefore also hydrated completely at the same age according to equality (2). The hydration of Portland cement ended at the age of 15 years when a water to cement ratio of 0.7 was used. Because the differences between heat of hydration calculated at the age of 13 and 15 years were almost equal to each other for water to cement ratios of 0.4 and 0.8, it turns out that the hydration also ends at the age of 15 years for mixtures with water to cement ratio of 0.4.

$$\alpha = \alpha_{PC} = \alpha_{C_3S} = \alpha_{C_2S} = \alpha_{C_3A} = \alpha_{C_4AF} \quad (2)$$

However, taking into consideration the ultimate heats of hydration reported earlier, calculated at 0.4 and 0.8 water to cement ratio, it turns out that although the hydration has practically ended at 15 years a portion of cement remained unhydrated when a water to cement ratio of 0.4 was used. At this water to cement ratio the degree of hydration equals to about 0.89–0.9. A fraction of about 10.0–11.0% remains unhydrated at the final hydration time. This fraction, appearing as unhydrated cement cores, was usually observed in old concrete prepared with water to cement ratio less than 0.70. At water to cement ratio greater than 0.7, unhydrated

Table 2
Hydration equation of compressive strength (f_c) and modulus of elasticity (E_c) of concrete (measured values taken from Naik et al. [6])

	Hydration time (days)							
	1	7	28	91	180	365	730	5475
Compressive strength (f_c)								
f_{cmeas} (MPa)	9.7	32.6	44.6	52.9	–	63.3	–	–
Hydration equation: $f_c = 176.9895 - 162.1497 * t^{-0.06}$ (MPa), $t \geqslant 1$, $r = 0.9997$, $s = 0.399$ (MPa)								
f_{Mcalc} (MPa)	–	32.71	44.22	53.29	58.25	63.18	67.82	80.25
Modulus of elasticity (E_c)								
E_{cmeas} (GPa)	11.7	27.4	31.1	38.0	–	42.4	–	–
Hydration equation: $E_c = 117.173 - 105.2175 * t^{-0.06}$ (GPa), $t \geqslant 1$, $r = 0.99807$, $s = 1.03$ (GPa)								
E_{ccalc} (GPa)	11.96	23.55	31.02	36.90	40.12	43.32	46.33	54.40

Correlation between the calculated values of the hydration criteria E_{ccalc} and f_{ccalc}

Relationship between compressive strength and modulus of elasticity of concrete. Linear regression over the value pairs E_{ccalc}–f_{ccalc}

Straight line: $E_c = 2.3237 + 0.6489 * f_{ccalc}$ (GPa), $r = 1.0$, $s = 0.0024$ (GPa)

Because the correlation coefficient is equal to 1.0, there is a linear relationship between compressive strength and modulus of elasticity of concrete

Table 3
Hydration equation of compressive strength (f_c), modulus of elasticity (E_c) and Poisson ratio (ν_c) of concrete (Portland cement I, Cement 340 kg/m^3, $W/C = 0.5$, Grading B32 according to DIN 1025)

	Hydration time (days)							
	7	14	28	90	180	365	730	5475
Concrete compressive strength (f_c)								
f_{cmeas} (MPa)	21.8	26.0	30.4	34.6	36.8	37.5	–	–
Hydration equation: $f_c = 41.8672 - 43.576 * t^{-0.394}$ (MPa), $t \geqslant 7$, $r = 0.9989$, $s = 0.336$ (MPa)								
f_{ccalc} (MPa)	21.62	26.46	30.14	34.47	36.23	37.60	38.62	40.40
Modulus of elasticity (E_c)								
E_{cmeas} (GPa)	34.0	38.8	41.2	44.6	45.6	47.8	–	–
Hydration equation: $E_c = 51.1983 - 37.1522 * t^{-0.394}$ (GPa), $t \geqslant 7$, $r = 0.9993$, $s = 0.267$ (GPa)								
E_{ccalc} (GPa)	33.94	38.07	41.20	44.89	46.39	47.56	48.43	49.95
Poisson ratio (ν_c)								
ν_{cmeas}	0.226	0.254	0.279	0.301	0.309	0.328	–	–
Hydration equation: $\nu_c = 0.3515 - 0.2726 * t^{-0.394}$, $t \geqslant 7$, $r = 0.9975$, $s = 0.0032$								
ν_{calc}	0.2248	0.2551	0.2781	0.3052	0.3162	0.3248	0.3312	0.3423

Correlation between the calculated values of the hydration criteria E_{ccalc}, f_{ccalc} and ν_{ccalc}

Relationship between compressive strength (f_c) and modulus of elasticity (E_c) of concrete. Linear regression over the value pairs E_{ccalc}–f_{ccalc}

Straight line: $E_c = 15.5142 + 0.8523 * f_{ccalc}$ (GPa), $r = 1.0$, $s = 0.0028$ (GPa)

Because the correlation coefficient is equal to 1.0, there is a linear relationship between compressive strength and modulus of elasticity of concrete

Relationship between compressive strength (f_c) and Poisson ratio (ν_c) of concrete. Linear regression over the value pairs ν_{ccalc}–f_{ccalc}

Straight line: $\nu_c = 0.0895 + 0.0063 * f_{ccalc}$, $r = 1.0$, $s = 0.00002$

Because the correlation coefficient is equal to 1.0, there is a linear relationship between compressive strength and Poisson ratio of concrete

cement cores are also expected to be present when curing temperatures are higher than 5 °C. These cores mainly consist of unhydrated β-C_2S, because this compound hydrates completely at 5 °C [15].

Further, according to the results given elsewhere [9,13,14,18], the final rate of hydration can be determined by using measured values obtained at ages $\leqslant 28$ days, ranging from 3 to 28 days or from 3 to 60 (or 90) days. That happens because the extrapolated values predicted by the cement hydration equation defined on the basis of measured values at early ages, are equal to the values obtained by the cement hydration equation when measured values at early and later ages are used. For example, using the hydration equation of heat of hydration $Q = 124.2 - 105.837 * bt^{-0.547}$, defined on the basis of measured values ranging from 7 to 365 days [14], the extrapolated value of heat of hydration at 2372 days (6.5 years) equals to 122.64 cal/g. This is almost

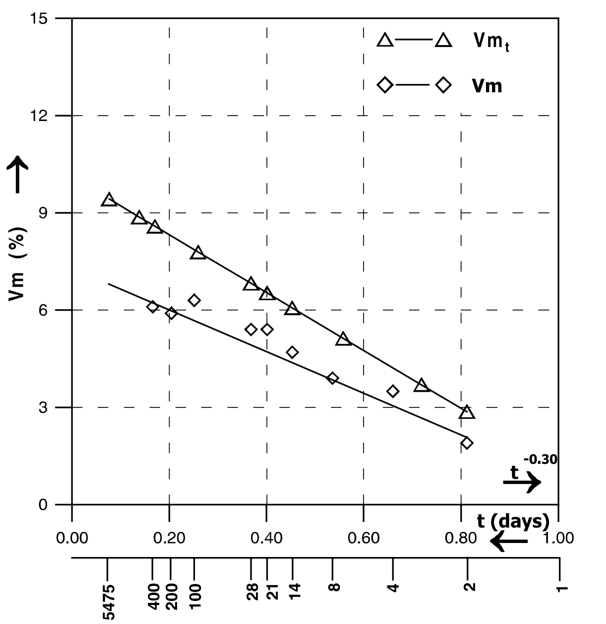

Fig. 2. Surface area (Vm) of C_3S as a function of hydration time (measured values of Vm were taken from Kantro et al. [16], and that of calculated values of Vm_t from Ref. [15]).

equal to the value $Q = 122.75$ cal/g obtained by the hydration equation $Q = 124.27 - 106.53 * bt^{-0.547}$, which was defined on the basis of measured values ranging from 3 to 2372 days. Both values are very close to the value $Q = 123.1$ cal/g, measured at the age of 6.5 years [14]. In the present study the cement hydration equation was set up using 6 test ages (at 7, 14, 28, 90, 180 and 365 days) as proposed in the literature [14]. According to Ref. [14], greater accuracy can be achieved if the concrete testing is carried out at test ages $\geqslant 6$ days.

The cement hydration equation $K = K_\infty \pm b * t^{-p}$ takes the following forms (Eqs. (3)–(7)) using the hydration criteria of compressive strength of mortar (f_M), of compressive strength of concrete (f_c), of modulus of elasticity of mortar (E_M) or concrete (E_c) and of Poisson ratio of concrete (v_c):

$$f_M = f_{M\infty} - bt^{-p} \quad (3)$$

$$f_c = f_{c\infty} - bt^{-p} \quad (4)$$

$$E_M = E_{M\infty} - bt^{-p} \quad (5)$$

$$E_c = E_{c\infty} - bt^{-p} \quad (6)$$

$$v_c = v_{c\infty} - bt^{-p} \quad (7)$$

where f_M, f_c, E_M, E_c and v_c are the hydration criteria examined; $f_{M\infty}$, $f_{c\infty}$, $E_{M\infty}$, $E_{c\infty}$ and $v_{c\infty}$ are constants, which represent the intersection of the straight line of Eqs. (3)–(7), respectively, with the ordinate; b is the slope of the straight line of Eqs. (3)–(7), respectively; t is the hydration time expressed in days ($t \gg 0$); p is the hydration number, depending only on the chemical and mineralogical composition of the cement used. All the established hydration criteria determined in the hydration of given cement (e.g. compressive strength of paste, mortar and concrete etc.) have the same hydration number [13,14].

Further, the value of hydration number (p) remains the same irrespective of the hydration criterion used [13,14] (i.e. curing temperature (5–60 °C), fineness of cement, ratio of W/C, and the aggregate grading curve or aggregate quality, when the hydration criterion of compressive strength of mortar or concrete is examined).

The procedure for the evaluation of hydration number p, for setting up the hydration equation and for defining the straight line of the correlation between any two hydration criteria of a given cement used, was discussed in previous study [13].

3. Analyzing the data concerning modulus of elasticity and Poisson ratio according to Eqs. (3)–(6)

Based on the measured values of modulus of elasticity and Poisson ratio of concrete, the applicability of cement hydration equation to these hydration criteria was investigated (Tables 1–3). In order to correlate modulus of elasticity (E_c) and Poisson ratio (v_c) of concrete with mortar compressive strength (f_M) and concrete compressive strength (f_c), these hydration criteria were also analyzed (f_M in Table 1 and f_c in Tables 1–3). The procedure of analyzing the hydration criteria f_M, f_c, E_M and E_c in Table 1 is given below and this was also used for the analyses made in Tables 2 and 3.

3.1. Analyzing the hydration criteria f_M, f_c, E_M and E_c in Table 1

The measured values used in Table 1 were taken from Vilardell et al. [19]. The characteristics of the materials used are given in the publication of the authors. The same cement was used for the production of all mixtures. The measured values, presented in Table 1, were analyzed according to the following procedure:

3.1.1. Evaluating the hydration number (p) of cement used

Based on the measured values of mortar compressive strength (f_{Mmeas}) at the ages of 7, 28, 90 and 180 days the hydration number $p = 0.128$ was evaluated according to the procedure discussed in Ref. [14]. Since the hydration number remains the same when the cement used also remains the same and does not depend on the hydration criterion examined, the value $p = 0.128$ was used for setting up the hydration equations of the hydration criteria f_M, f_c, E_M and E_c.

3.1.2. Setting up the hydration equations of f_M, f_c, E_M and E_c

3.1.2.1. Hydration criterion of mortar compressive strength (f_M). Based on the hydration number $p = 0.128$ and the measured values of mortar compressive strength (f_{Mmeas}) presented in Table 1, the mortar's compressive strength equation was set up according to Eq. (3), by performing linear regression over the value pairs f_{Mmeas}–$t^{-0.128}$. Using the defined hydration equation $f_{\mathrm{M}} = 132.7817 - 140.9103 * t^{-0.128}$ (MPa), the values of mortar's compressive strength were calculated (f_{Mcalc}). These values of f_{Mcalc} are presented in Table 1.

3.1.2.2. Hydration criterion of compressive strength of wet screening concrete (f_{cS}). Based on the hydration number $p = 0.128$ and the measured values of compressive strength of wet screening concrete (f_{cSmeas}) presented in Table 1, the concrete's compressive strength equation was set up according to Eq. (4), by performing linear regression over the value pairs f_{cSmeas}–$t^{-0.128}$. Using the defined hydration equation $f_{\mathrm{cS}} = 121.0629 - 124.9333 * t^{-0.128}$ (MPa), the values of compressive strength of wet screening concrete were calculated (f_{cScalc}). These values of f_{cScalc} are presented in Table 1.

3.1.2.3. Hydration criterion of modulus of elasticity of mortar (E_M). Based on the hydration number $p = 0.128$ and the measured values of modulus of elasticity of mortar (E_{Mmeas}), presented in Table 1, the mortar's modulus of elasticity equation was set up according to Eq. (5), by performing linear regression over the value pairs E_{Mmeas}–$t^{-0.128}$. Using the defined hydration equation $E_{\mathrm{M}} = 51.7736 - 41.7691 * t^{-0.128}$ (GPa), the values of modulus of elasticity of mortar were calculated (E_{Mcalc}). These values of E_{Mcalc} are presented in Table 1.

3.1.2.4. Hydration criterion of modulus of elasticity of wet-screening concrete (E_S). Based on the hydration number $p = 0.128$ and the measured values of modulus of elasticity of wet-screening concrete (E_{Smeas}), presented in Table 1, the concrete' s modulus of elasticity equation was set up according to Eq. (6) (E_{S} instead of E_{c}), by performing linear regression over the value pairs E_{Smeas}–$t^{-0.128}$. Using the defined hydration equation $E_{\mathrm{S}} = 61.4252 - 46.967 * t^{-0.128}$ (GPa), the values of modulus of elasticity of wet-screening concrete were calculated (E_{Scalc}). These values of E_{Scalc} are presented in Table 1.

3.1.2.5. Hydration criterion of modulus of elasticity of dam concrete (E_D). Based on the hydration number $p = 0.128$ and the measured values of modulus of elasticity of dam concrete (E_{Dmeas}), presented in Table 1, dam concrete's modulus of elasticity equation was set up according to Eq. (6) (E_{D} instead of E_{c}), by performing linear regression over the value pairs E_{Dmeas}–$t^{-0.128}$. Using the defined hydration equation $E_{\mathrm{D}} = 75.5866 - 58.24 * t^{-0.128}$ (GPa), the values of modulus of elasticity of dam concrete were calculated (E_{Dcalc}). These values of E_{Dcalc} are presented in Table 1.

3.1.2.6. Relationship between the calculated values of f_M, f_{cS}, E_M, E_S and E_D. Using the calculated values of the hydration criteria f_{M} f_{cS}, E_{M}, E_{S} and E_{D}, the relationship between any two of them was studied by performing linear regression over the value pairs f_{Mcalc}–f_{cScalc}, E_{Mcalc}–f_{Mcalc}, E_{Scalc}–f_{Mcalc}, E_{Dcalc}–f_{Mcalc}, E_{Mcalc}–f_{cScalc}, E_{Scalc}–f_{cScalc} and E_{Dcalc}–f_{cScalc}. The equations of the respective straight lines are presented in Table 1.

3.2. Analyzing the hydration criteria f_c and E_c in Table 2

The measured values used in Table 2 were taken from Naik et al. [6]. The characteristics of the materials used are given in the publication of the authors. The hydration number ($p = 0.06$) was evaluated using the values of compressive strength of concrete, f_{cmeas}, according to the procedure discussed in Ref. [13]. The measured values f_{cmeas} and E_{cmeas}, presented in Table 2, were analyzed according to the above procedure used by the analyses made in Table 1.

3.3. Analyzing the hydration criteria f_c, E_c and v_c in Table 3

In order to correlate the compressive strength of concrete with its modulus of elasticity and Poisson ratio, a concrete mixture was prepared using a type I Portland cement (according to ASTM C 150, corresponding to ordinary Portland cement). The aggregates used in this study were crushed gravel (4/32 in mm) and silica sand (0/4 in mm) having a grading curve equal to the B_{32} curve of DIN 1025. The gravel (4/32 in mm) was obtained by crashing natural gravel of 100–300 mm in diameter. The original rock was an amphibolite with compressive strength of 231.5 MPa and porosity with pores radius $r > 37.5$ (Å) of 0.006 $\mathrm{cm^3/g}$ (determined by using mercury intrusion porosimetry). The water to cement ratio used was equal to 0.5. The specimens, cylinder of $150 * 300$ in mm, were cured up to test age in a curing room with 95–98% RH at constant temperature of 20 °C. Compressive strength f_{c}, modulus of elasticity E_{c} and Poisson ratio v_{c} were determined according to ASTM specifications (ASTM C 873-99 and ASTM 469-94, respectively). The measured values obtained by the hydration criteria examined (f_{cmeas}, E_{cmeas} and v_{cmeas}), are presented in Table 3 and were analyzed according to the above procedure used by the analyses made in Table 1. The hydration number ($p = 0.394$) was evaluated using the values of f_{cmeas}, according to the procedure discussed in Ref. [13].

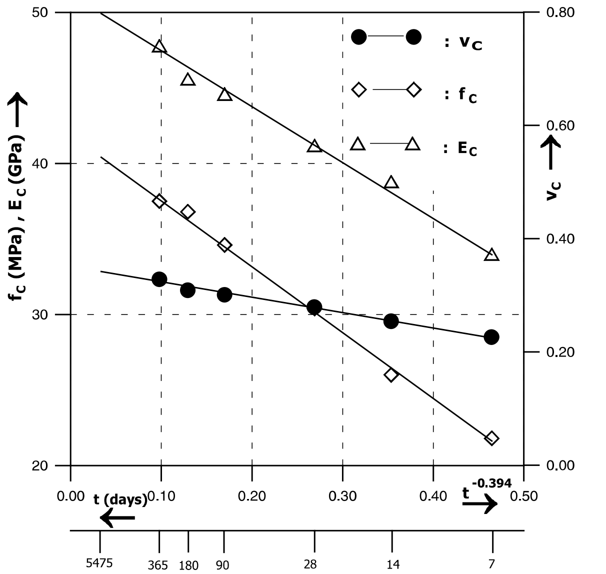

Fig. 3. Compressive strength (f_c), modulus of elasticity (E_c) and Poisson ratio (ν_c) of concrete as a function of hydration time (calculated and measured values taken from Table 3).

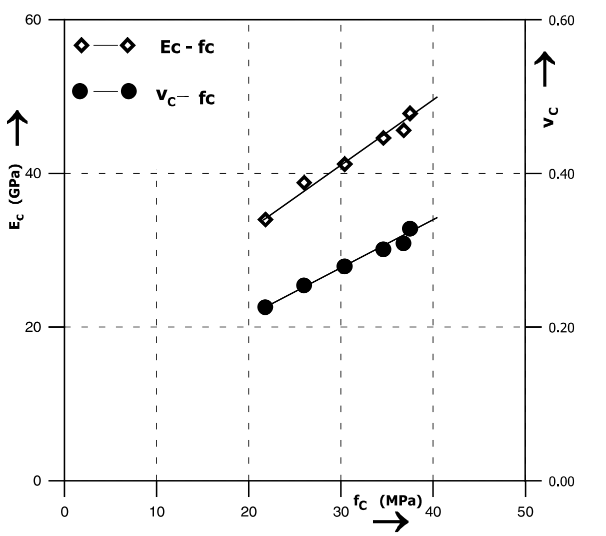

Fig. 4. Linear correlation between modulus of elasticity (E_c) and compressive strength (f_c) of concrete, and between Poisson ratio (ν_c) and compressive strength (f_c) of concrete (calculated and measured values taken from Table 3).

The hydration diagrams of the hydration equations of f_c, E_c and ν_c, and the respective measured values of Table 3 were plotted in Fig. 3. Further, the straight lines of the correlation between compressive strength and modulus of elasticity of concrete and between compressive strength and Poisson ratio of concrete found in Table 3 as well as the respective measured values were also plotted in Fig. 4.

4. Discussion

1. From Tables 1–3 and Figs. 1–3 it turns out that cement hydration equation is applicable to the hydration criteria of modulus of elasticity and Poisson ratio of concrete. Hence, these hydration criteria can be considered as established hydration criteria, like that of compressive strength of paste, mortar and concrete, heat of hydration, non-evaporable water, surface area etc. This implies that the hydration equation of modulus of elasticity and Poisson ratio of concrete can be drawn up in order to calculate the values of these hydration criteria up to the age of 15 years, where the hydration ends.

2. The finding concerning the established hydration criteria of modulus of elasticity and Poisson ratio of concrete leads to the linear proportionality between any two established hydration criteria [13–15], which exists at all instances. This implies that there is a linear relationship between modulus of elasticity of concrete and mortar' s compressive strength evidenced in Table 1, between compressive strength and modulus of elasticity of concrete evidenced in Tables 1–3, and between Poisson ratio and compressive strength of concrete evidenced in Table 3.

3. This finding as well as the linear relationship between compressive strength of mortar and concrete (Table 1) lead to the possibility of estimating ultimate modulus of elasticity and Poisson ratio using the "method of two points proportionality", discussed in a previous study [18]. This method leads to reliable results and is useful because only simple measurements are needed. For this reason it can be used in each usual concrete laboratory. Further, this method can be also used to investigate systematically the effect of curing time, of water to cement ratio, of cement content, of cement composition, of pozzolan content, of aggregate type, of fine material content, of superplasticizer etc. on the modulus of elasticity and Poisson ratio of concrete.

4. Tables 1 and 3 show the magnitude of the difference between the values of modulus of elasticity and Poison ratio at 28 days and that by final hydration of concrete. The differences range up to about 75% for E_c and 32% for ν_c of the examined concrete. This implies that the elastic characteristics of concrete significantly increase at final hydration in respect to that determined at 28 days, and for this reason their ultimate values must be taken in account by the prediction of the displacements in structures.

5. From Tables 1 and 2 it is seen that compressive strength and modulus of elasticity of concrete appear at 7 days the values 23.7–24.8, and 32.7–23.6, respectively. The respective values at 7 days in Table 3 are 21.6–33.9. This implies that there is a difference of about 40%, between modulus of elasticity found in Tables 1, 2 and that of Table 3. A similar difference was also found by comparison of the values of modulus of elasticity of

Table 3 with that published by several investigators [10,20,21]. On the other hand, a smaller difference (of about 20%) is given by the comparison of modulus of elasticity found in Table 3 at the age of 7 days with that determined by Short and Kinniburgh [22] ($f_c = 22.0$ MPa, and $E_c = 24.0–29.0$ GPa). Further, the results of Table 3 (i.e. the values at 7 days of compressive strength, modulus of elasticity and of Poisson ratio of concrete) do agree with the results of Klink [7] who determined the actual modulus of elasticity at the age of 7 days. The difference between modulus of elasticity given in literature and that found in Table 3 by using aggregates with great stiffness remains to be investigated. An explanation can be given by the high incompressibility of gravel used, which improves the rigidity of concrete, but also contributes to a decrease of its strength [21]. This decrease in strength is due to the fact that, at high levels of stress, there is a concentration of stress at the aggregate–mortar interface due to the large difference in the elastic modulus of elasticity between the aggregate and the mortar [21].

5. Conclusions

The hydration criteria of modulus of elasticity and Poisson ratio of concrete are established hydration criteria, like that of compressive strength of paste, mortar and concrete, heat of hydration, non-evaporable water, surface area etc. Consequently, the cement hydration equation can be used for calculating the values of these hydration criteria up to the final hydration.

Since modulus of elasticity of concrete, Poisson ratio of concrete, and mortar compressive strength are established hydration criteria, there is a linear relationship between any two of them. The "method of two points proportionality" is applicable therefore for the estimation of ultimate modulus of elasticity and Poisson ratio of concrete.

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