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Cement & Concrete Composites 27 (2005) 413–420
Cement & Concrete Composites
www.elsevier.com/locate/cemconcomp

Shear strengthening of RC deep beams using externally bonded FRP systems

M.R. Islam [a,*], M.A. Mansur [b], M. Maalej [b]

[a] Department of Civil Engineering, Chittagong University of Engineering and Technology, Chittagong 4349, Bangladesh
[b] Department of Civil Engineering, National University of Singapore, 1 Engineering Drive 2, Singapore 117576, Singapore

Received 15 October 2002; accepted 27 April 2004

Abstract

According to the available methods of analysis and design for reinforced concrete deep beams, addition of web reinforcement beyond the minimum amount provides only a marginal strength gain, if it does at all. This casts serious doubts on the feasibility and extent of strengthening by placing external reinforcement in the web, whenever such a need arises. This study therefore explores the prospect of strengthening structurally deficient deep beams by using an externally bonded fibre reinforced polymer (FRP) system. Six identical beams were fabricated and tested to failure for this purpose. One of these beams was tested in its virgin condition to serve as reference, while the remaining five beams were tested after being strengthened using carbon fibre wrap, strip or grids. The results of these tests are presented and discussed in this paper. Test results have shown that the use of a bonded FRP system leads to a much slower growth of the critical diagonal cracks and enhances the load-carrying capacity of the beam to a level quite sufficient to meet most of the practical upgrading requirements.

Keywords: Concrete; Deep beams; FRP; Reinforced concrete; Shear; Strengthening; Web reinforcement

1. Introduction

High strength non-metallic fibres, such as carbon, glass and aramid fibres, encapsulated in a polymer matrix in the form of wires, bars, strands or grids have shown great potentials as reinforcement for concrete, particularly where durability is of main concern. It is commonly known as fibre reinforced polymer or, in short, FRP. Despite being a recent development, numerous investigations have already been reported in the literature on various aspects of its structural use [1]. One area where FRP can play a major role is in strengthening and retrofitting of degraded or strength deficient structures already in existence. By virtue of its lightweight, extraordinarily high strength and high corrosion resistance, FRP presents an attractive material for structural rehabilitation. Moreover, being available in the form of thin sheets, such a system makes very little change to the dimension of the existing member.

The use of FRP materials in shear strengthening of reinforced concrete shallow beams has been investigated quite extensively in the past [2–9]. The results of these investigations have demonstrated the effectiveness of different forms of FRP systems, namely sheet, strip and wrap to achieve the desired effects. However, to the best knowledge of the authors, no such strengthening attempt has yet been reported for reinforced concrete deep beams, a fairly common structural element in tall buildings, offshore structures and in foundation systems. The investigation reported in this paper is therefore directed towards making an assessment of different FRP systems for strengthening structurally deficient reinforced concrete deep beams.

* Corresponding author. Tel.: +88 31 714948; fax: +88 31 714910.
E-mail address: mdrislam@hotmail.com (M.R. Islam).

0958-9465/$ - see front matter © 2004 Published by Elsevier Ltd.
doi:10.1016/j.cemconcomp.2004.04.002

Deep beams usually fail in shear. Therefore, when strengthening of such a beam using an externally bonded FRP system is considered, the system may be regarded as additional web reinforcement, but fixed externally. In order to check the effectiveness of increasing the web reinforcement in boosting the shear capacity, a hypothetical beam, which will be taken as the reference beam in the subsequent test programme is considered for analysis. The dimensions and reinforcement details of the beam are shown in Fig. 1. The material properties used are those reported subsequently for the reference beam.

Keeping everything else constant, ultimate strength analysis has been carried out for this beam to investigate the effect of web reinforcement. The methods used are those given by ACI Code [10], Kong et al. [11], CIRIA Guide 2 [12], Tan et al. [13], Mau and Hsu [14], and Subedi [15]. In the analysis, the material safety factors included in different methods were considered as unity.

The results presented in Fig. 2 convey two important facts. Firstly, for a particular web reinforcement ratio, ultimate loads predicted by various methods vary widely. Secondly, except for the method proposed by Subedi [15], an increase in web reinforcement especially beyond 0.2% in each direction (horizontal and vertical), hardly makes any difference in ultimate load carrying capacity of the beam. The above evaluation serves as a deterring factor trying out externally bonded FRP system to make any useful gain in ultimate strength. Yet, this method of strengthening has been attempted to see whether or not the above theoretical assessment is valid.

In this study, six identical deep beams were constructed to represent the structurally deficient beam. One of these beams served as the reference, while the remaining beams were strengthened in shear by using three different FRP systems-strips, grids and fibre wraps, and tested to failure. The main objectives include finding out whether strengthening of deep beams in shear with externally bonded FRP reinforcement is possible or not and finding out the effectiveness of different types of FRP systems. The results of these tests are presented and discussed in this paper.

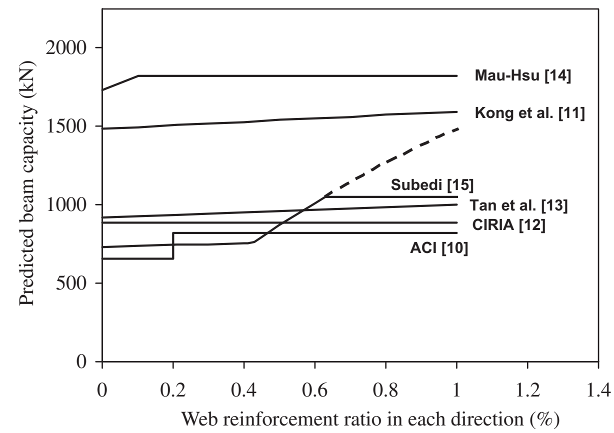

Fig. 2. Prediction of ultimate load with increase in web reinforcement.

2. Test programme

2.1. Details of the structurally deficient beam

The six deep beams constructed were identical in every respect. Each beam was 2000 mm long with a rectangular cross-section, 120 mm wide and 800 mm in overall depth (h). As shown in Fig. 1, the flexural reinforcement consisted of 2T25 and 2T20 deformed bars of yield strength 500 MPa and 543 MPa, respectively. These bars were placed in two layers and were welded to 16 mm thick steel plates at both ends to provide the necessary anchorage. The shear reinforcement consisted of two layers of welded wire fabrics having 100 mm square openings. The wire diameter was 4 mm (yield strength of 553 MPa). This gives rise to a web steel ratio of 0.2% in each horizontal and vertical direction, the minimum ratio specified in the CEB-FIP Model Code [16] and Canadian Code [17]. The minimum web reinforcement requirements by ACI Code [10] and CIRIA Guide 2 [12] are also close to this value. A 20 mm thick and 150 mm long steel plate was used at each loading and reaction points covering the full width of the beams. Additional reinforcement cages were provided at these points, as shown, to prevent premature local failure.

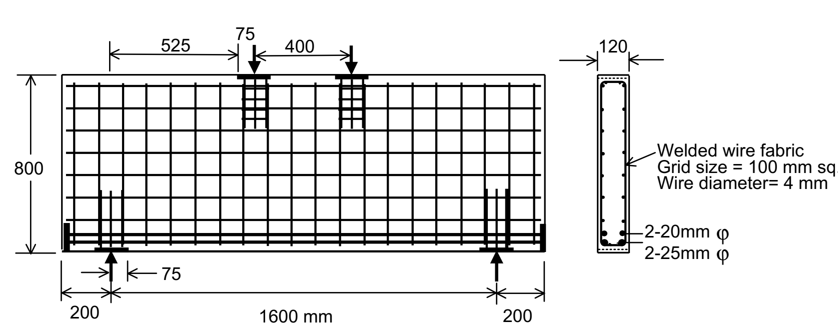

Fig. 1. Internal reinforcement details.

The beams were cast from a single supply of ready mix concrete. The cube compressive strength, cylinder compressive strength and cylinder splitting tensile strength of the concrete used were 41.9 MPa, 37.8 MPa and 3.73 MPa, respectively. The modulus of rupture was 3.83 MPa.

2.2. Strengthening materials and methods

Three different FRP systems have been used to strengthen the basic beams. These are: fibre wrap (MBrace 130), strips (SIKA) and grids (NEFMAC), all made up of carbon fibre. The mechanical properties of the strengthening materials and epoxy resins, as reported by the manufacturers are shown in Tables 1 and 2, respectively. Instead of providing these systems throughout the beam surface, strengthening was carried out only in the shear span across the potential failure plane.

Beam B1 was strengthened with fibre wrap (MBrace). Two layers of such wrap were applied, one over the other, in the clear shear span. Each wrap was continuous at the bottom face of the beam and terminated close to the top of the beam on both faces, as shown in Fig. 3(a). In beam B2, 1.2 mm thick and 50 mm wide isolated FRP strips (SIKA system) of 450 mm, 600 mm and 700 mm in lengths were bonded perpendicular to the diagonal connecting the loading and the support points in a symmetrical manner with the 700 mm strip at the middle (Fig. 3(b)). The method of surface preparation, type of adhesive and the method of fixing followed for a particular FRP system were those recommended by the respective manufacturer.

Beams B3, B4 and B5 were strengthened by 500 mm long and 300 mm wide FRP grid system with 50 mm square openings. The parameters investigated in these beams were the size of the grid bars and orientation of the grid system, and the method of bonding. In order to accommodate these variables, the two shear spans for each beam were provided with different details, as can be seen in Fig. 3, with the intention of generating two data from the same beam. Three different grid systems, designated Grid 1, Grid 2 and Grid 3, depending on the sectional area of the grid bar of 6.6 mm^2, 17.5 mm^2 and 39.2 mm^2, respectively, were employed. Grid 2 and Grid 3 were bonded to the concrete surface by placing them within the preformed matching grooves, about 3 mm in depth, except for beam B5 where Grid 3 was bonded directly to the concrete surface in one of the shear spans. Also, no grooving was used for Grid 1 as the grid bars were much thinner. The strengthening process was carried out at least after 28 days of casting when the surface was fully air-dried in the laboratory. Different grid systems and their orientation for each beam are shown in Fig. 3.

2.3. Instrumentation and test procedure

The beams were simply supported over a span l of 1600 mm and tested in flexure under two symmetrical point loads, thus giving a l/h ratio of 2.0 and an a/h ratio of 0.75, where a is the shear span (see Fig. 1). The load was applied using a servo controlled Instron hydraulic actuator with a maximum capacity of 2000 kN. All beams were instrumented by electrical resistance strain gauges for measuring strains in the main tensile steel at mid-span, strains in the orthogonal internal web reinforcement at the middle of the shear span at mid-depth and strains in the strengthening materials at different critical locations. Mid-span deflection of the beams

Table 1
Properties of strengthening materials

CFRP material	Width (mm)	Thickness (mm)	Sectional area[a] (mm^2)	Tensile strength (MPa)	Tensile modulus of elasticity (GPa)	Failure strain (%)
Wrap	–	0.165	–	3480	230	1.5
Strip	50	1.2	60.0	3050	165	1.7
Grid 1[b]	–	–	6.6	1200	100	1.2
Grid 2[b]	–	–	17.5	1200	100	1.2
Grid 3[b]	–	–	39.2	1200	100	1.2

[a] Areas shown are sectional areas of one strip or of one bar of the grid.
[b] Grid size was 50 mm for all grid systems.

Table 2
Properties of epoxies

Epoxy type		Used for	Density (kg/l)	Adhesive tensile strength (MPa)	Modulus of elasticity (GPa)	Shear strength (MPa)
Type 1	Primer	Wrap	1.10	12	0.72	–
	Saturant	Wrap	0.98	54	3.03	–
Type 2		Strip, grid	1.77	33	12.80	15

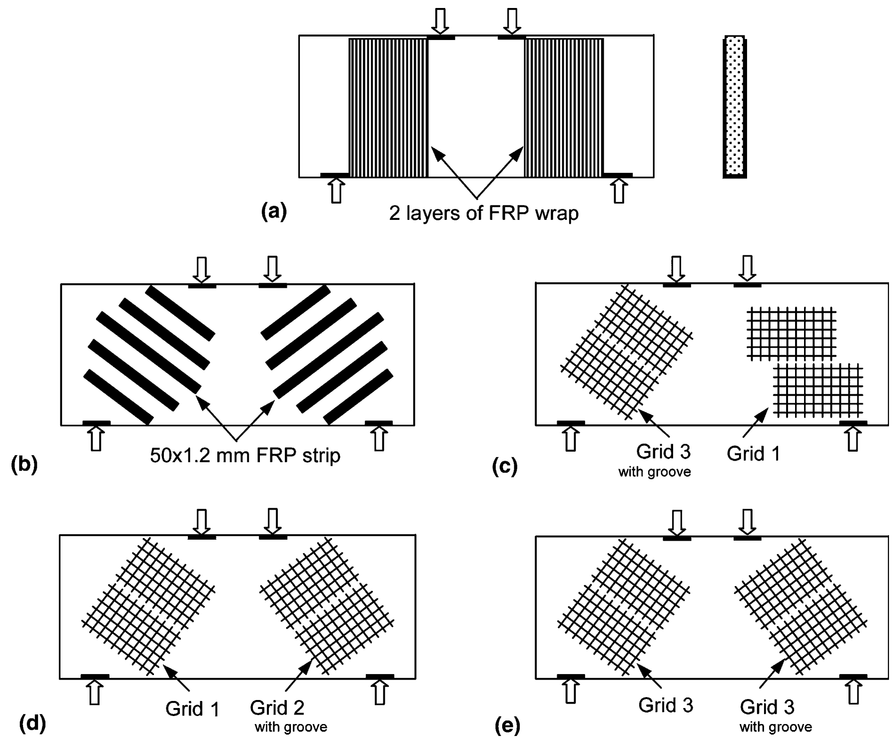

Fig. 3. Arrangement of externally bonded FRP systems: (a) beam B1, (b) beam B2, (c) beam B3, (d) beam B4 and (e) beam B5.

was measured by LVDTs. A hand held microscope with a resolution of 0.02 mm was used to measure the width of cracks. The load was applied monotonically up to failure.

3. Test results and discussion

3.1. Load deflection behaviour

The load vs. mid-span deflections of the beams plotted in Fig. 4 can be seen to be typical of those generally observed for a deep beam with a small *a*/*d* ratio. All beams demonstrated a nearly linear response up to about 80% of the ultimate load. However, as expected, strengthening by external bonding of different FRP systems resulted in an increase in stiffness, the highest increase being exhibited by beams B3 and B5 containing Grid 1 in normal orientation and Grid 3 in diagonal direction, respectively.

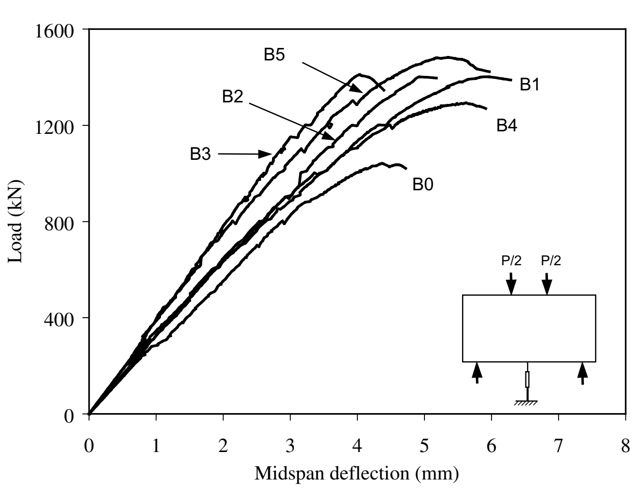

Fig. 4. Load-deflection behaviour.

3.2. Cracking behaviour

The loads corresponding to the first appearance of flexural and diagonal cracking are presented in Table 3. The first crack appeared at about the mid-span, followed by the formation of new cracks progressively towards the supports. Diagonal cracks formed next, which were originated at about quarter depth from the beam soffit and then propagated downwards to the inner end of the support and upwards to the flexural compression zone. Once formed, the width of the diagonal cracks exceeded that of the flexural ones. With increasing load, the existing diagonal cracks developed further and additional diagonal cracks appeared in succession forming a cracking pattern in the strengthened beams almost identical to that in the parent beam B0, as can be seen in Fig. 5.

In this study, all the major cracks were visually scrutinised and the maximum width of cracks was measured up to about 75% of the ultimate loads. The measured maximum crack widths are plotted against the applied

Table 3
Test results

Beam	First crack load		Ultimate load (kN)	Increase in ultimate load after strengthening (%)	Maximum crack width at service load of B0[a] (mm)
	Flexural (kN)	Diagonal (kN)			
B0	130	280	1040	–	0.62
B1	160	–	1402	35	0.19
B2	150	370	1400	35	0.19
B3	195	490	1410	36	0.15
B4	130	350	1292	24	0.28
B5	190	330	1482	43	0.25

[a] Service load of beam B0 is calculated by dividing the ultimate load of beam B0 by 1.6, which gives 650 kN.

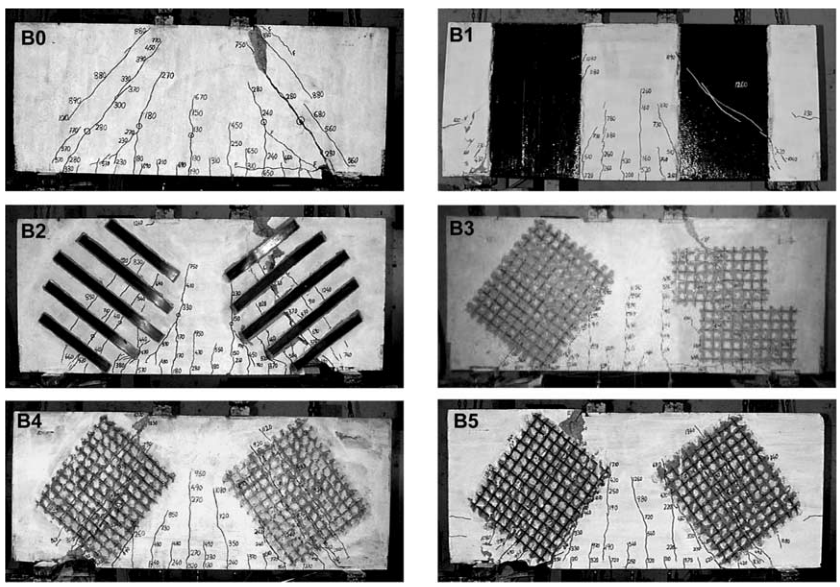

Fig. 5. Failure patterns of test beams.

load in Fig. 6. As mentioned earlier, diagonal cracks displayed the maximum widths shortly after their formation. For beam B1, measurement of diagonal crack widths was not possible as the fibre wrap concealed the cracks. Therefore, the curve shown in Fig. 6 for this beam represents the cracks that appeared outside the covered area. It may be seen in Fig. 6 that strengthening accomplished by externally bonded FRP systems, in general, provided restraint to the widening of diagonal cracks. At a load level corresponding to the calculated service load (experimental ultimate load/1.6) for the parent beam B0, strengthened beams displayed a maximum crack width about one-third of that in beam B0 (see Table 3), the smallest width being demonstrated by beam B3.

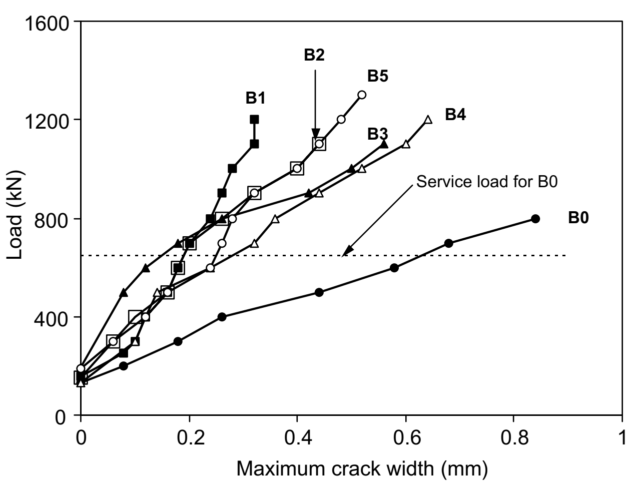

Fig. 6. Maximum crack widths at different loads.

3.3. Ultimate strength and mode of failure

All the beams failed in shear. However, the beams with FRP grids that were reinforced differently in the two shear spans could not be retested after failure on the weaker side.

The parent beam B0 failed by crushing and shearing off of the concrete in the upper part of the strut like portion between the load and support as can be seen in Fig. 5. Beam B1, which was strengthened by fibre wrap, failed at a load 35% higher than the parent beam. Failure occurred by crushing of the concrete under the wrap. At failure, partial debonding of the fibre wrap was noted. Beam B2 also showed a 35% increase in ultimate strength. The failure of this beam was initiated due to the separation of the lower end of the bottom strip from the concrete surface. Shortly afterwards, the other strips gave way one after another in succession starting from the lower end, thus causing the complete failure in a manner almost identical to the parent beam. A closer observation revealed that the separation of the FRP strips was associated with concrete peel-off and not due to the interfacial bond failure (see Fig. 7).

In beams B3 and B4, failure occurred in the shear span strengthened with Grid 1 as the other side was strengthened with higher reinforcement provided by Grids 2 and 3, respectively. Normal orientation of grids in beam B3 showed better performance than diagonal placement in beam B4, both with respect to the maximum crack width at service load and ultimate strength, as can be noted from Table 3. The critical diagonal crack in beam B3 was not initially concentrated in a single crack especially at the mid-depth of the beam, rather it was somewhat distributed. Therefore, the maximum crack width for this beam remained small until about half of its full loading history. No bond failure at the end of the grid systems was observed in these beams up to the ultimate load. At the instant of final collapse beyond the ultimate, some grid bars ruptured. But most of them experienced splitting of the bar fibres and this was associated with debonding in a small distance on either side of the failure crack. This splitting and local debonding of grid bars might have been caused by the relative movement of the two parts of the beam on either side of the critical diagonal crack. The relative movement seemed not perpendicular to the crack and therefore, it created shear stresses in the grid bars that crossed the diagonal cracks in beam B4 for which these thin bars were much weak. Only the vertical bars crossing the diagonal crack in beam B3 did not show any splitting and it ruptured in direct tension, which indicates the higher effectiveness of the grid system in carrying the applied load in beam B3 than in beam B4.

Fig. 7. Separation of FRP strip in beam B2.

Beam B5 failed on the side in which Grid 3 was bonded without any grooving. Grooving can provide better bond between concrete and the grid incorporating more surface area for bonding. But the failure showed that the bond between the concrete and grid was good enough even without grooving as no debonding was observed. The cross-sectional area of the grid bar used in beam B5 is 5.94 times higher than that of Grid 1 used in beams B3 and B4. This much higher additional reinforcement precludes the formation of any critical diagonal crack within the region having bonded grids. Failure surface therefore bypassed the grid system and was associated with transverse splitting of the concrete attached to the grids. Being bonded with the highest amount of FRP, this beam attained the highest strength increment of 43% among the beams tested.

It may be noted that grid type of FRP in normal orientation performed better than wrap or strip. This becomes obvious when the results of beams B1, B2, and B3 with an additional web reinforcement ratio of 0.44%, 0.63% and 0.28%, respectively (calculated from the volume of additional web reinforcement divided by the volume of concrete in the shear span), as presented in Table 3, are compared. All these three beams attained almost the same strength though different amount of additional web reinforcements were involved with them. However, FRP wrap, strip and grid, all can be considered suitable for shear strengthening a deep beam.

3.4. Discussion on analytical predictions

Externally bonded FRP reinforcements can be regarded as similar to the conventional internal reinforcement by taking into account the behaviour of the bond between the FRP material and the concrete surface, and the property of the bonded surface. Usually, the full strength of the bonded FRP composites cannot be materialised due to their debonding type of failure. Therefore, using an effective strain value for the FRP material [8] or using a limiting shear stress for the bonded concrete [18] are the ways to incorporate the effect of the bonded reinforcement in any suitable strength prediction method for the structure.

The strength of the reference beam B0 was calculated by the methods of ACI Code [10], Kong et al. [11], CIRIA Guide 2 [12], Tan et al. [13], Mau and Hsu (explicit equation) [14], and Subedi [15]. Wide scattering is observed in the predictions giving the ratio of experimental to predicted ultimate loads of 1.27, 0.69, 1.17, 1.11, 0.57 and 1.40, respectively. Tan et al.'s equation predicts the strength safely and most closely among the methods considered.

ACI Code method is based on strut-and-tie concept with two different values of efficiency factor for concrete depending upon the amount of web reinforcement provided in the beam. CIRIA Guide 2 method is basically the one proposed by Kong et al. [11] with some restrictions and inclusion of material safety factors. Tan et al.'s equation is a modification of the CIRIA Guide 2 method to cover a wide range of shear-span/effective-depth ratio and concrete strength. Mau and Hsu's method considers an integrated composite action between concrete and steel along with the softening of concrete due to biaxial state of stress. Subedi's method is based on equilibrium of forces in one side of the assumed failure plane along with the use of splitting capacity of web concrete under biaxial stress condition.

As discussed earlier, the effect of increasing the amount of web reinforcement for beam B0 has been predicted by the above methods keeping all the physical and material properties the same, but varying the web reinforcement ratio in both horizontal and vertical directions. Different methods consider the effect of horizontal and vertical web reinforcements in different ways with varying relative effectiveness. Equal amounts of horizontal and vertical web reinforcements have been considered in the present exercise, which is just one case among infinitely possible combinations. As can be seen from Fig. 2, equations of Kong et al. and Tan et al. predict marginal increments in ultimate load with an increase in web reinforcement. ACI Code, CIRIA Guide 2, and Mau and Hsu's explicit method, all predict no improvement in ultimate load with the addition of web reinforcement in excess of 0.2%.

Though most conservative in predicting the strength of the reference beam B0, only Subedi's method predicts noticeable strength increase with increase in web reinforcement beyond some lower amount. For the present exercise, very little strength increase is expected for increasing the web reinforcement ratio up to 0.42% in each direction, after which the rate of strength increase is much higher up to a reinforcement ratio of 0.62%. Beyond that web reinforcement, the beam supposed to attain its bearing strength limit. However, if the bearing strength limit (which may vary with the confinement provided in the bearing regions) is not imposed, the strength increase with further increase in web reinforcement can be shown by the dotted line of Fig. 2. Therefore, Subedi's method looks promising in establishing the effectiveness of bonded FRP reinforcement. But unfortunately, the method predicts almost no strength increase with increase in only vertical web reinforcement (less than 1.0% strength increase with additional 1.0% vertical web reinforcement to beam B0), which contradicts the test result of beam B1 strengthened with vertical wraps. According to this method, higher effectiveness of web reinforcement can only be obtained when the capacities of both horizontal and vertical web reinforcements are higher than the corresponding components of splitting capacities obtained from web concrete and reinforcement. Some modifications in the 'criteria tests' for effectiveness of web reinforcement given by Subedi [15] may eliminate this disagreement on the performance of only vertical web reinforcement.

Therefore, from the above discussion it appears that none of the methods can readily be used for predicting the improved load carrying capacities of the strengthened deep beams by incorporating the effect of bonded FRP reinforcement into it as the strength increase due to conventional web reinforcements predicted by these methods are negligible.

There are also different opinions regarding the effect of web reinforcement in deep beams derived from experimental studies. By varying the amount of web reinforcement, de Paiva and Siess [19] observed negligible improvement in shear strength of deep beams. According to CIRIA Guide 2 [12], Leonhardt suggested that the shear capacity of deep beams cannot be improved by the addition of web reinforcement. On the other hand, some researchers [11,20–22] observed noticeable improvement (up to 30%) in shear strength of deep beams. Higher percentages of improvement have also been reported for deep beams with larger beam depths [23]. Therefore, there is a need to identify the criteria for the effectiveness of web reinforcement and develop proper shear strength prediction method for deep beams.

During the course of this experimental study, it was observed that failure generally occurred through the diagonal crack between the inner side of the support and the compression zone under the load. This crack had the maximum width among all the cracks throughout the loading history, except for the initial part. After strengthening, the maximum width of diagonal cracks was smaller compared to that in the control beam at a particular load level. Due to the restraint provided by FRP reinforcement, the critical diagonal crack propagated and grew more slowly with increasing load, thus requiring higher loads for the critical diagonal crack to penetrate into the upper part of the beam. This has led to higher ultimate loads.

Higher amount of bonded reinforcement restrained the growth of diagonal cracks better, thereby resulting in higher ultimate load of the beams. This becomes evident when the results of beams B4 and B5 are compared. In beam B5, the widths of the diagonal cracks were very small and the crack having the maximum width bypassed the strengthening grid system. Failure occurred through that bypassed crack as can be seen in Fig. 5.

The effectiveness of bonded FRP materials in restricting the growth of diagonal crack widths depends upon the reinforcement ratio, orientation of placement and on the bond characteristics, as observed and identified in this study. However, more research is needed to quantify the contributions of both internal and external web reinforcement in order to come up with a suitable prediction method for both the cases. This forms the subject matter of a study currently being conducted at the National University of Singapore.

4. Concluding remarks

This paper deals with shear strengthening of reinforced concrete deep beams by using an externally bonded FRP system in the beam web. Despite casting serious doubts on the extent of strengthening by adding reinforcement in the web by the available methods of analysis and design, test results of six identical beams, strengthened by different FRP systems, as presented and discussed in this paper clearly show that such a scheme is possible and practicable. Although FRP grids placed in normal orientation demonstrated to be the most effective system as far as the amount of material used in strengthening concerned, other systems were found to be almost equally effective. An enhancement of shear strength in the order of about 40%, as achieved in this study, represents more than what is usually needed in a practical situation.

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