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Factors affecting the external prestressing stress in externally strengthened prestressed concrete beams

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Abstract

The wide use of external prestressing system to strengthen reinforced and prestressed concrete members requires the full understanding of the behaviour of the strengthened members. At ultimate the stress in the external prestressing tendons need to be known in order to calculate the ultimate strength of the strengthened member. Several factors that can influence the increase in the ultimate stress in steel external prestressing tendons have been studied and well understood while the effect of these factors on tendons made from fibre reinforced plastics needs more research.

This research was carried out to study the effect of several factors on the increase in the ultimate stress in external Parafil ropes as well as external steel tendons. These factors were related to the external prestressing system, internal prestressed and ordinary bonded steel, beam geometry and material properties. Also, the accuracy of equations proposed by the Eurocode (EC2), ACI318 and BS8110 to calculate the ultimate stress in external steel and FRP prestressing tendons was examined.

The experimental and the analytical results showed that the studied factors have the same effect on both steel (up to yield) and Parafil ropes though this effect is greater in case of steel tendons. Also, factors such as tendon profile (straight or deviated), high strength of the concrete, effective tendon depth, number of deviators should be taken into consideration when calculating the ultimate stress in the external tendons.

Keywords: External tendons; Non-ferrous materials; Prestressed concrete; Tendon stresses; Deflection

1. Introduction

Over the last decade, the use of fibre reinforced plastics (FRP) in construction and strengthening has increased due to its attractive properties; high strength, high resistance to corrosion and light weight that speeds the construction process and reduces maintenance costs. FRP can be used as reinforcement in reinforced concrete or as prestressing tendons in internally or externally prestressed concrete members. Different types of FRP tendons can be used such as; carbon fibre reinforced plastics (CFRP), aramid fibre reinforced plastics (AFRP), glass fibre reinforced plastics (GFRP) and Parafil rope type G.

Analysis of external prestressed beams is different from that of both ordinary bonded prestressed beams and internal unbonded prestressed beams due to the lack of bond between tendons and concrete and due to the reduction in the effective depth of the tendons during loading (second-order effect). Also, using FRP instead of steel as external tendons adds another difficulty in the analysis of this system, as the behaviour of beams strengthened using FRP tendons may differ than those strengthened using steel tendons especially at ultimate. This is because of the low elastic modulus and the linear

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Notation

A_e	area of external prestressing tendons
A_p	area of internal prestressing steel
A_{pef}	effective area bonded steel
A_s	area of non-prestressed tensile steel
$A_{s'}$	area of non-prestressed compressive steel
d_e	effective depth of external prestressing tendon
d_{ef}	effective depth of bonded steel
d_p	effective depth of of prestressing steel
d_s	effective depth of non-prestressed tensile steel
f_{cu}	cube strength of concrete
f_{pe}	effective external prestressing stress
f_{pu}	ultimate strength of external prestressing tendon
f_{py}	yield stress of prestressing steel
f_{ys}	yield stress of non-prestressed tensile steel
h	total height of cross-section
L	span length between supports
S_d	distance between deviators
Δf_p	tendon stress increase $= f_{ps} - f_{pe}$

stress–strain relationship up to failure of FRP (Fig. 1) that can change the nature of the failure and affect the increase in the tendon stress.

To simplify the analysis of the unbonded prestressed beams many equations have been proposed by codes of practice to calculate the stress in the internal unbonded prestressing steel tendons at ultimate. These equations are also used to calculate the ultimate stress in external steel prestressing tendons. However, most of these equations are based on limited parameters and do not cover several factors that may have a significant effect if used with FRP tendons.

This paper aims to:

1. Study the effect of several factors, related to external prestressing systems, beam geometry, material properties and internal bonded non-prestressed and prestressed steel on the increase in the stress in external Parafil ropes at ultimate.
2. Compare the effect of these factors on the ultimate stress in the Parafil ropes with that on the ultimate stress of the external steel tendons (if applicable).
3. Check the accuracy of equations proposed by, ACI318-02 [1], BS8110-97 [2] and Eurocode2 (EC2) [3] when used to calculate the ultimate stress in external tendons (steel or FRP).

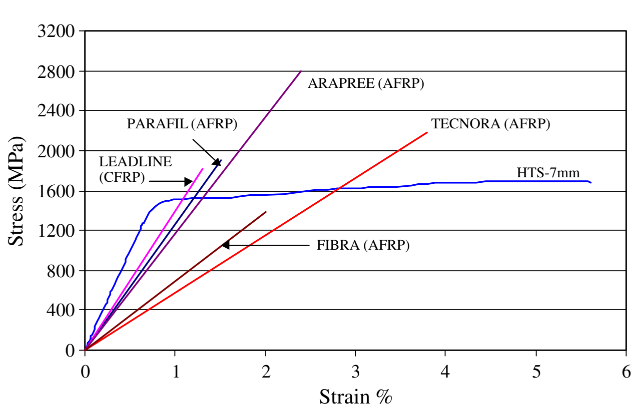

Fig. 1. Stress–strain curves of different types of FRP.

2. Review of previous work

2.1. Beams strengthened using steel tendons

Du and Tao [4] tested 22 unbonded partially prestressed beams to study the effects of varying amounts of non-prestressed reinforcement on the ultimate stress in unbonded prestressing tendons in partially prestressed concrete beams. Test results show that the stress in the unbonded tendons at ultimate is a function of the reinforcement percentages of both the unbonded tendons and the bonded non-prestressed reinforcement.

Yaginuma and Kitada [5] tested three series of unbonded partially prestressed concrete beams. All the beams were strengthened using straight tendons. The first series was externally strengthened without deviator, the second series was externally strengthened using deviators at a distance equal to half the beam depth from the concentrated load and the third series was internally strengthened. Two (span/depth) ratios were used 18 and 32. Yaginuma and Kitada [5] concluded that the increase in the prestressing stress decreased as the span/depth ratio increased. Also, the increase in the prestressing stress in the external tendons supported using deviators was similar to that in internal unbonded tendons while that in external tendons without deviators was significantly smaller.

Harajli [6] tested 16 beams, which were strengthened externally using steel tendons after the beam had been subjected to cyclic fatigue loading to induce fatigue deformation. The beams were reinforced concrete (RC), prestressed concrete (PC) and partially prestressed concrete (PPC) beams. Based on the test results, Harajli [6] concluded that external prestressing using a straight horizontal profile was relatively less effective in increasing the flexural resistance than a deviated profile because of the progressive reduction of the depth of the straight external tendon during loading.

Yaginuma [7] tested three unbonded prestressed beams, the first with external tendons, the second with internal and external tendons and the third with internal tendons. All beams prestressed by the same force and the total tendons area are almost the same. All beams were simply supported and tested under two points static loading. Yaginuma [7] found that the increase in the prestressing stress in the internally strengthened beam was the highest while that in the externally strengthened was the smallest. Also, the increase in the prestressing stress of the internal tendons was higher than that in the external tendons in the mixed type beam.

Aravinthan et al. [8] conducted a parametric study to investigate the effect of the following factors on the ultimate stress of the external tendon: span/external tendon depth ratio (L/d_{ps}), loading span/span ratio (L_p/L), ratio of area of bonded prestressed steel/total tendons area ($A_{ps,int}/A_{ps,tot}$), distance between deviators/span ratio (S_d/L), prestressing steel ratio (ρ_p) and reinforcing steel ratio (ρ_s). Also, Aravinthan et al. [9] tested four externally prestressed beams with different (L/d_{ps}) ratio. Based on the parametric and the experimental results Aravinthan et al. stated that (L/d_{ps}) ratio is one of the most important factors influencing the ultimate tendon stress and as this ratio increases, the ultimate tendon stress is significantly reduced.

Tan and Ng [10] studied the effect of deviators and tendon configuration (tendon area, force and eccentricity) on the behaviour of reinforced concrete beams strengthened in flexure. Tan and Ng [10] tested six identical reinforced beams after they had been externally strengthened using steel tendons. Of these, three beams had identical straight external strands with a different number of deviators along each of their spans. The other three beams each had a deviator at mid span, and their tendon configurations were varied by either increasing tendon eccentricity (and correspondingly decreasing the prestressing force), increasing the tendon area or draping the strands. The beams were loaded to failure under third-point loading. Test results indicated that using a deviator at the section of maximum deflection led to satisfactory service load behaviour and a higher load-carrying capacity, while the use of smaller effective prestressing force led to larger stress in both the internal reinforcement and external tendons. Also, an increase in the eccentricity of straight tendons with a correspondingly smaller prestressing force led to larger internal steel stresses, while draped tendons resulted in greater tendon stress increase. The use of a larger tendon area gave similar service load behaviour but a higher ultimate strength and lower ductility.

Harajli et al. [11] used a nonlinear analysis model, based on the incremental deformation method, to predict the entire response of concrete members originally designed with or strengthened by external prestressing. Based on the analytical results, Harajli et al. [11] stated that the second-order effect is the main factor that distinguishes the behaviour of external tendons from an internal unbonded tendon system and tendons without deviators mobilized lower stress increase for all types of loading. Also, tendon stress in beam loaded by a uniform load was higher than that in beam loaded by a single concentrated load.

Aravinthan et al. [12] tested four beams; two simple span beams (one monolithic (D-1) and one segmental (D-1a)) and two continuous beams (one monolithic ((A-1) and one segmental (A-1a)) to study the effect of a large eccentricity of the external prestressing tendons on the behaviour of the prestressed beams. Beam D-1a was internally strengthened using unbonded steel tendons while the rest were internally strengthened using bonded steel tendons. During the tests, yielding of external tendons was observed in the four beams and the ultimate tendon stress for the segmental beams was slightly less than that for monolithic beams. Aravinthan et al. [12] concluded that the tendon stress increased proportionally to the mid span deflection until yielding of tendon and that the ultimate flexural strength is influenced by the ultimate stress of external tendons as well as internal tendons.

2.2. Beams strengthened using FRP tendons

Jerrett and Ahmad [13] tested four prestressed beams under two concentrated loads. Each beam was prestressed internally with one or two 7-wire steel strands and strengthened externally using two CFRP (Leadline) tendons. Test results showed that the stress increase in CFRP tendons for the beams with one steel strand was significantly higher than that for the beams with two steel strands.

To study the behaviour of bridges strengthened using CFRP tendons, Grace and Abdel-Sayed [14] tested four bridge models strengthened using internal bonded and externally unbonded draped CFRP tendons under static, repeated, and ultimate loads. Two bridges were right angle bridges whereas the others were skew bridges. During testing, none of the externally draped tendons ruptured. At ultimate, the prestressing force in the external tendons increased to about twice the initial values. Grace and Abdel-Sayed [14] recommended that designers should combine internally bonded tendons with externally draped tendons to ensure better ductility and to force the structure to fail by crushing of concrete rather than by rupture of the internal tendons.

Ghallab and Beeby [15] tested three prestressed beams, externally strengthened after different levels of loading had been applied to examine the benefit of external prestressing using Parafil rope Type G and to evaluate the effect of the external prestressing on both the service load and ultimate load behaviour. Ghallab and Beeby [15] concluded that the degree of pre-loading and cracking reached before external prestressing has no effect on the increase in the external prestressing force in the uncracked and working stages if the internal prestressing steel has not reached yield. However, a greater working load prior to applying the external prestress results in a slightly lower increase in the external prestressing force at ultimate.

As can be seen from the previous review a few researches had been conducted to study the factors affecting the increase in the prestressing stress in the FRP tendons. Also, the effect of these factors, in case of steel or FRP tendons, on the code provisions is unclear. Following the effect of some of these factors on the ultimate stress of Parafil ropes (as an example of FRP tendons) is studied and compared with their effect on the ultimate stress of steel tendons (if applicable). Finally, the impact of these factors on the code provisions is studied.

3. Experimental work

Nine prestressed beams were tested up to failure after being externally strengthened using Parafil ropes type G. Each beam was strengthened using two ropes located symmetrically on both sides and deviated using one or two steel deviators as shown in Fig. 2. During prestressing and testing, the external prestressing force was measured using two load cells at the end of the ropes. All beams were tested under static loads; two concentrated loads at the third span or one load at the mid span. Also, seven beams tested previously by Ghallab [16] are used as companion beams to the current test specimens (PC1–PC7 in Table 1a). Those beams had the same geometry and were strengthened in the same way as the current test specimens. Table 1a shows the details of all the strengthened beams. To compare the effect of the studied factors on the ultimate stress of the external steel tendons, experimental results from several beams externally strengthened using steel tendons were collected from the literature. Details of these beams are shown in Table 1b. Table 2 shows the factors studied and the beam number used in the study.

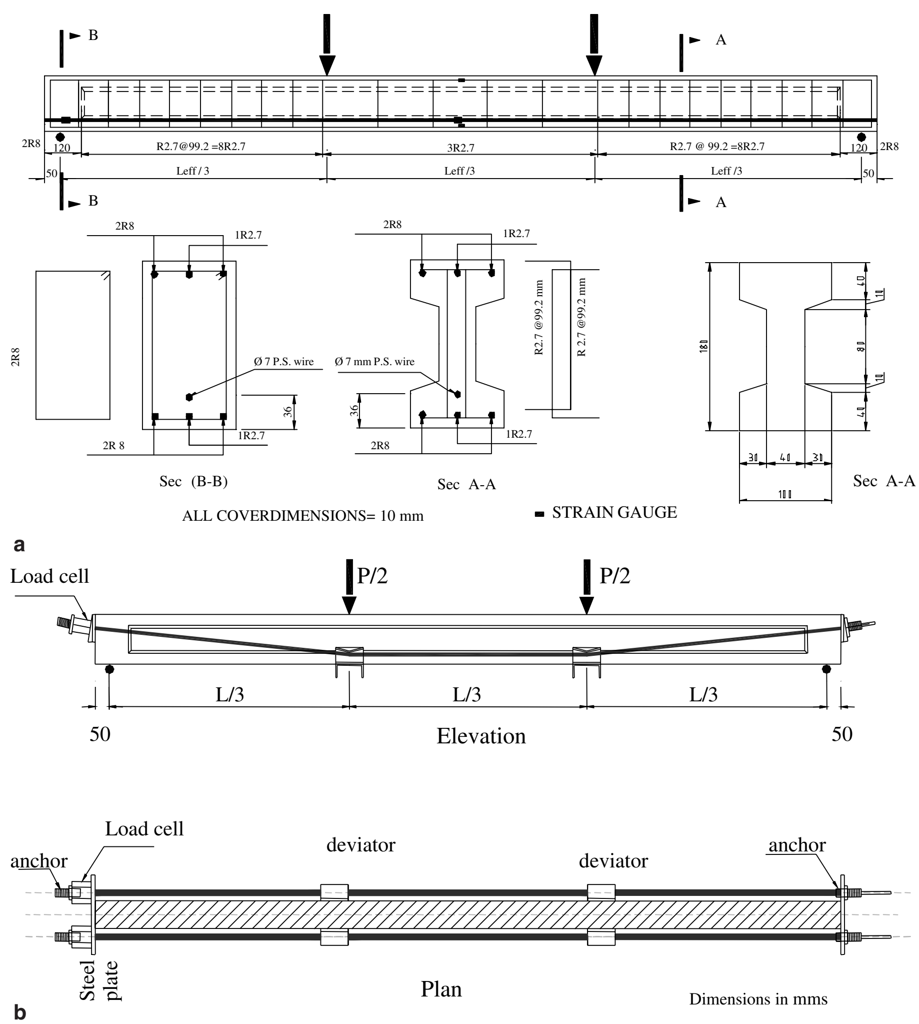

Fig. 2. (a) Dimensions and reinforcement details of beam test. (b) External prestressing tendons layout using two deviators at the third span.

Table 1

(a) Details of tested beams

Beam number	Details of tested beams f_{cu} (MPa)	(L/h)	As (mm^2)	Loading type, number and distance between loads	External prestressing			Tendon shape	Number and distance between deviators	Internal prestressing	
					Type and area (mm^2)	Effective stress (MPa)	(d_e/h)			Type and area (mm^2)	Effective stress (MPa)
PC1	53.3	14.4	2φ8	S-2-(L/3)	Parafil rope-61.1	785.3	0.79	Deviated	2-L/3	SW-38.5	1008.8
PC2	55.77	14.4	2φ8	S-2-(L/3)	Parafil rope-61.1	987.73	0.79	Deviated	2-L/3	SW-38.5	931.4
PC3	55.03	14.4	2φ8	S-2-(L/3)	Parafil rope-61.1	1182.8	0.79	Deviated	2-L/3	SW-38.5	896.73
PC4	47.7	14.4	2φ8	S-2-(L/3)	Parafil rope-61.1	982.2	0.79	Deviated	1–0	SW-38.5	954.3
PC5	52.2	14.4	2φ8	S-2-(L/3)	Parafil rope-61.1	990.5	0.89	Deviated	2-L/3	SW-38.5	919.3
PC6	45.7	14.4	2φ8	S-2-(L/3)	Parafil rope-61.1	998.9	1.07	Deviated	2-L/3	SW-38.5	892.5
PC7	43.3	14.4	2φ8	S-2-(L/3)	Parafil rope-61.1	990.5	0.79	Deviated	2-L/3	SW-38.5	915.2
PC8	79.27	14.4	2φ8	S-2-(L/3)	Parafil rope-61.1	984.6	0.79	Deviated	2-L/3	SW-38.5	1003
PC9	48.93	20	2φ8	S-2-(L/3)	Parafil rope-61.1	988	0.79	Deviated	2-L/3	SW-38.5	967
PC10	47.47	10	2φ8	S-2-(L/3)	Parafil rope-61.1	994	0.79	Deviated	2-L/3	SW-38.5	1004
PC11	48.57	14.4	–	S-2-(L/3)	Parafil rope-61.1	991.2	0.79	Deviated	2-L/3	SW-38.5	996.9
PC12	47	14.4	2φ8	S-2-(L/3)	Parafil rope-61.1	991.2	0.79	Deviated	2-L/3	–	–
PC13	63	14.4	2φ8	S-2-(L/3)	Parafil rope-61.1	987.23	0.79	Deviated	2-L/3	SW-38.5	359.2
PC14	49.63	14.4	2φ8	S-2-(L/3)	Parafil rope-61.1	998.85	0.79	Deviated	2-L/2	SW-38.5	968.8
PC15	47	14.4	2φ8	S-2-(L/3)	Parafil rope-61.1	984.5	0.79	Deviated	2-L/5.2	SW-38.5	956.6
PC16	43.9	14.4	2φ8	S-1-0	Parafil rope-61.1	814.24	0.79	Deviated	2-L/3	SW-38.5	961.3

(b) Details of beams collected from literatures

Beam number	Reference	Beam symbol. in reference	Details of tested beams			Loading type and distance between loads	External prestressing			Tendon shape	Number and distance between deviators	Internal prestressing	
			f^*_{cu} (MPa)	(L/h)	A_s (mm^2)		Type and area (mm^2)	f_{pe} (MPa)	(d_e/h)			A_{ps} (mm^2)	f_e (MPa)
PC17	Tan and Ng [10]	T-0	43.25*	10	2φ16	S-2-(L/3)	7-wire SS-109.65	1279	0.67	Straight	0	–	–
PC18		T-1	42.75*	10	2φ16	S-2-(L/3)	7-wire SS-109.65	1197	0.67	Straight	1–0	–	–
PC19		T-1A	38*	10	2φ16	S-2-(L/3)	7-wire SS-109.65	327	0.83	Straight	1–0	–	–
PC20		T-1D	40*	10	2φ16	S-2-(L/3)	7-wire SS-109.65	288	0.83	Deviated	1–0	–	–
PC21		T-2	35.9*	10	2φ16	S-2-(L/3)	7-wire SS-109.65	1182	0.67	Straight	2-L/3	–	–
PC22	Jerrett and Ahmad [13]	B-1	54.3	12.8	–	S-2-(L/7.3)	Carbon-101	1170	1.06	Deviated	2-L/7.3	197	1120
PC23		A-2b	60.3	12.8	–	S-2-(L/7.3)	Carbon-101	1210	1.06	Deviated	2-L/7.3	197	985
PC24		B-2	54.3	12.8	–	S-2-(L/7.3)	Carbon-101	1270	1.06	Deviated	2-L/7.3	197	1120
PC25	Khairallah and Harajli [17]	T2S	50*	9.84	3φ12	R-2-(L/3)	SW-39	935	0.67	Straight	0	–	–
PC26		T3S	47.4*	9.84	3φ14	R-2-(L/3)	SW-77	747	0.67	Straight	0	–	–
PC27		T4S	52.3*	9.84	3φ16	R-2-(L/3)	7-wire SS-75	994	0.67	Straight	0	–	–
PC28		T2D	54.4*	9.84	3φ12	R-2-(L/3)	SW-39	931	1.15	Deviated	1–0	–	–
PC29		T3D	48.8*	9.84	3φ14	R-2-(L/3)	SW-77	895	1.15	Deviated	1–0	–	–
PC30		T4D	48.4*	9.84	3φ16	R-2-(L/3)	7-wire SS-75	1001	1.15	Deviated	1–0	–	–
PC31	Aparicio et al. [18]	M2	40	12	704	S-2-(L/3)	SS-560	1014	0.92	Deviated	2-L/3	–	–
PC32		M3	40	12	704	S-2-(L/3)	SS-840	1101	0.92	Deviated	2-L/3	–	–
PC33		M4	40	12	704	S-2-(L/3)	SS-1120	1151.5	1.00	Deviated	2-L/3	–	–

(*continued on next page*)

Table 1 (*continued*)

Beam number	Reference	Beam symbol in reference	Details of tested beams			Loading type and distance between loads	External prestressing			Tendon shape	Number and distance between deviators	Internal prestressing	
			f^*_{cu} (MPa)	(L/h)	A_s (mm^2)		Type and area (mm^2)	f_{pe} (MPa)	(d_e/h)			A_{ps} (mm^2)	f_e (MPa)
PC34	Aravinthan and Mutsuyoshi [9]	A-1	35	16	3φ10	S-2-(L/5.8)	SC-277.4	955.3	0.77	Deviated	2-L/2.9	–	–
PC35		A-2	35	16	3φ10	S-2-(L/5.8)	SC-186	951.6	0.77	Deviated	2-L/2.9	–	–
PC36		B-1	35	16	3φ10	S-2-(L/5.8)	SC-142	831	1.15	Deviated	2-L/2.9	–	–
PC37		B-2	35	16	3φ10	S-2-(L/5.8)	SC-186	951.6	1.15	Deviated	2-L/2.9	–	–
PC38	Mutsuyoshi et al. [19]	No. 1	40	16	3φ10	S-2-(L/5.8)	SC-277.4	1009	0.77	Deviated	2-L/2.9	–	–
PC39		No. 2	40	16	3φ10	S-2-(L/5.8)	SC-277.4	1009	0.77	Deviated	2-L/1.73	–	–
PC40	Guimaraes and Araujo [20]	VGA1	69.4	10	4φ6.3	S-2-(L/3)	SC-305.6	1009.5	0.83	Deviated	1–0	–	–
PC41		VGA2	67.3	10	4φ6.3	S-2-(L/3)	SC-305.6	980.4	0.83	Deviated	1–0	–	–

S = static load; R = repeated load; L = effective span length; SW = steel wire; $^*f_{cu} = f'_c/0.8$; SC = steel cable; SS = steel strand; SW = steel wire.

4. Presentation and discussion of the experimental results

During loading, the increase in the external prestressing stress was small till cracking beyond which it started to increase rapidly as the load increased and reached its maximum value at the ultimate. The relation between the external force and the applied moment shows a relation similar to that between the deflection and the applied moment. In this study, the force in any rope never reached its nominal breaking load and no rope fractured during any test, while the literature mentioned yield or fracture of the external tendons before beam failure [12,13]. The increase in tendon stress at ultimate varied between 0.2 and 0.55 of the initial prestressing stress. In the following, the effect of each factor on the ultimate stress of Parafil rope is presented, discussed and compared with the effect of the same factor on the ultimate stress of steel tendons if applicable.

4.1. Ratio of effective prestressing stress to ultimate tendon strength (f_{pe}/f_{pu})

Fig. 3 shows the relation between the ratio of the effective prestressing stress; at the applying loading, to the ultimate tendon strength and the ratio of the increase in the external prestressing stress to effective stress for beams strengthened using different types of tendons. Tendons in each group have the same ultimate strength. As can be seen, increasing the effective prestressing stress tends to reduce the increase in the external prestressing force at ultimate. This was observed for beams strengthened using both steel and FRP tendons. This can be attributed to the effect of the external prestressing force that tends to close the cracks, prevents them from extending and improves the stiffness. Thus beams subject to high prestressing stress have less ductile behaviour and a lower increase in the tendon stress. Also, it can be seen that the increases in stress in the steel and CFRP tendons are higher than that in Parafil ropes. This can be attributed to the low Young's modulus of the Parafil ropes.

4.2. Effective depth of the external prestressing force (d_e)

The increase in the tendon stress is relative to its distance from the neutral axis; the greater the distance the higher the stress, as can be seen from Fig. 4a. Within the depth ($d_e \leqslant h$), increasing the tendon depth slightly increased the tendon stress while increasing the tendon depth to more than the section depth has a significant effect on the ultimate stress in the tendon.

The effect of the effective depth of the external prestressing tendon can be taken as (L/d_e) ratio where L is the beam span. Fig. 4b shows the relation between the

Table 2
Factors and group number

No.	Factors	Beam no. (Parafil ropes)	Beam no. (steel tendons)
1	Ratio of initial prestressing stress/ ultimate tendon strength (p_i/p_u)	(PC1, PC2, PC3), (PC22,PC24)*	PC28, PC30
2	Depth of external prestressing tendon/ beam depth ratio (d_e/h)	(PC2, PC5, PC6)	(PC35, PC37)
3	Span/depth of external prestressing tendons ratio (L/d_e)	(PC10, PC6, PC5, PC2, PC9)	(PC35, PC37)
4	Number of deviators (N_d)	(PC2, PC4)	(PC17, PC18, PC21), (PC38, PC39)
5	Distance between deviators/span ratio (S_d/L)	(PC4, PC15, PC2, PC14)	(PC18, PC21), (PC38, PC39)
6	Concrete strength (f_{cu})	(PC7, PC2, PC8)	–
7	Span/beam depth ratio (L/h)	(PC10, PC2, PC9)	–
8	Loading type	(PC16, PC1)	–
9	Internal prestressing stress	(PC12, PC13, PC2)	–
10	Reinforcing steel ratio	(PC11, PC2)	(PC25, PC26), (PC28, PC29)
11	Prestressing steel ratio	(PC12, PC2)	–

* CFRP.

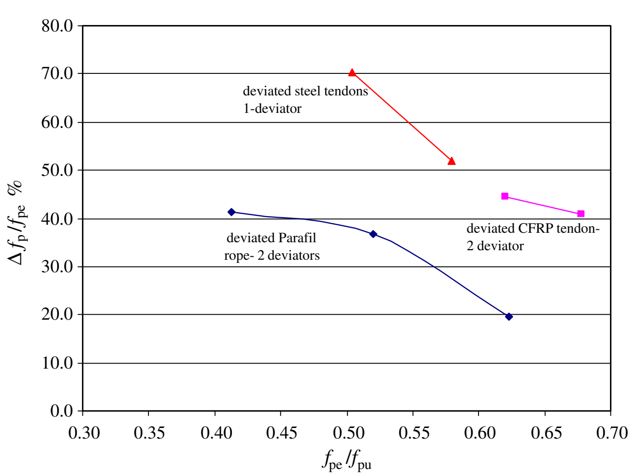

Fig. 3. Relation between the ratio of increase in effective external prestressing stress and the effective external prestressing stress ratio.

increase in the tendon stress and (L/d_e) ratio. This figure emphasizes the previous remark as reducing the (L/d_e) ratio resulted in increasing the ultimate stress (beam PC6 with $d_e > h$ has the highest increase in tendon stress). The low increase in the stress for beam PC10 (which has the lowest (L/d_e) ratio) can be attributed to its low ductility.

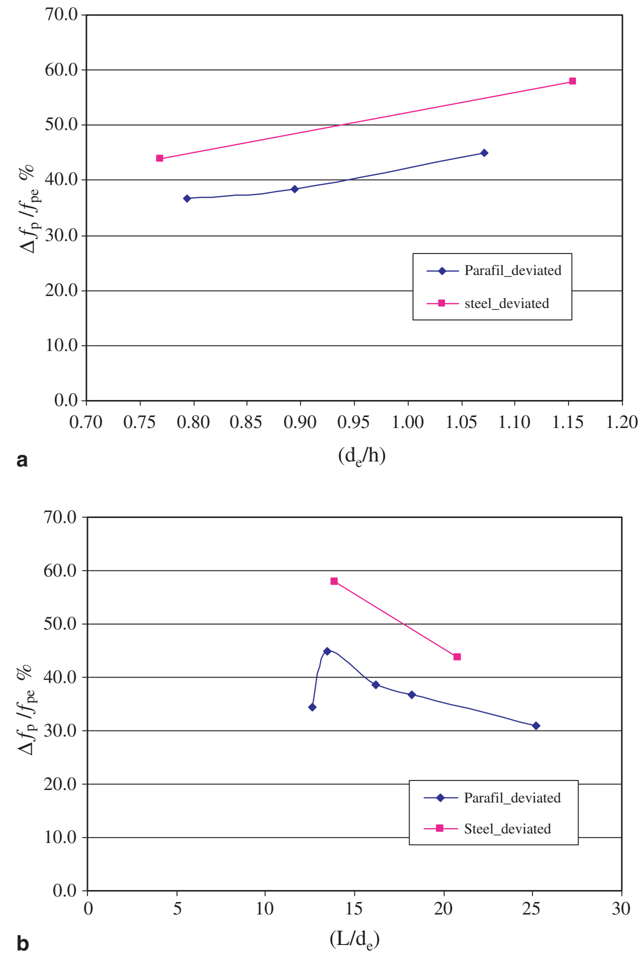

Fig. 4. (a) Relation between the ratio of increase in effective external prestressing stress and (eccentricity of the external prestressing force/ depth) ratio. (b) Relation between the ratio of increase in effective external prestressing stress and the span/effective external tendon depth.

4.3. Number of deviators

Fig. 5 shows the relation between the increase in the ultimate tendon stress and the number of deviators used for strengthening. All beams in each group had almost the same (f_{pe}/f_{pu}) as shown in Table 1a and b. Also, all beams were subjected to two concentrated load at the third span and strengthened using deviators located at the mid span (one deviator) and/or at the third span (two deviators or three deviators). The increase in the number of deviators significantly increased the ultimate stress in the deviated tendons and had less effect on the ultimate stress of the straight tendons.

The improvement in the ultimate stress of the deviated tendons when using more than one deviator can be attributed to the increase in tendon eccentricity along a wider distance (between deviators) thus increases the ultimate strength of the beam. In case of straight tendons, using more than one deviator kept the straight tendons in position, reduced the loss in tendon

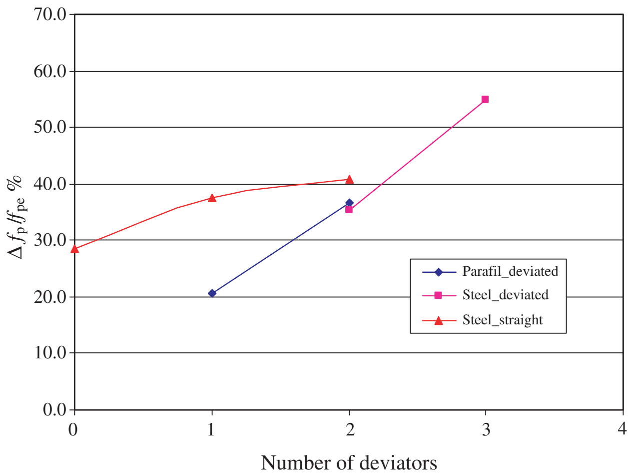

Fig. 5. Relation between the ratio of increase in effective external prestressing stress and the number of deviators.

eccentricity (second-order effect) during loading and improved the ultimate stress of the tendons.

4.4. Effect of the ratio of the distance between deviators to the span (S_d/L)

Fig. 6 shows the effect of the ratio of (S_d/L) on the increase in the stress in the external tendons at ultimate. All beams were strengthened using straight or deviated tendons and loaded by two concentrated loads. The distances between loads (L_p) varied between $L/3$ and $L/5.8$.

Generally, the optimum value of the increase in the external tendon stress at ultimate was reached when the distance between deviators (S_d) $\approx L/3$. For beams subjected to two concentrated loads at the third span and strengthened using tendons deviated at mid span, the critical section is under the concentrated load where the bending moment is high and the eccentricity of the external tendons is lower than at the mid-span. Hence, the failure occurred at a lower load at that section than when using two deviators at the third span. When the distance between the two deviators exceeds that between the two concentrated loads, the effect of the reduction in the tendon eccentricity at the mid span become significant and results in a rapid failure of the beam. This was also observed when the distance between the two concentrated loads was less than $L/3$.

The effect of the distance between deviators on the ultimate stress in the straight external tendons was less than that with the deviated tendons. For beam loaded by two concentrated loads at the third span there was a slight difference between the ultimate stress in the straight external prestressing tendons supported by two deviators and the ultimate stress in the straight external prestressing tendons supported by one deviator at mid span. This is because the variations in the eccentricity of the external tendon at the critical section as well as the tendon elongation when one deviator was used at mid span or two deviators at different distances (within the studied ratios) were small.

4.5. Effect of concrete strength

Increase in concrete strength results in an increase in the prestressing force in the external tendons at ultimate, as can be seen from Fig. 7. Also, by comparing the effect of concrete strength with that of the other factors considered, concrete strength can be seen to be one of the main factors influencing the ultimate tendon stress.

4.6. Effect of span/depth ratio (L/h)

As can be seen from Fig. 8, variation in (L/h) ratio has a slight influence on the value of the increase in the tendon stress, with a tendency to reduce as (L/h) increases. The reduction in tendon stress at the low

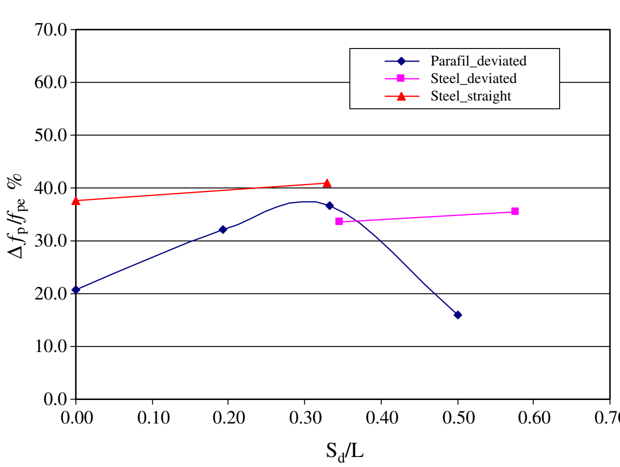

Fig. 6. Relation between the ratio of increase in effective external prestressing stress and (deviator distance/span distance) ratio.

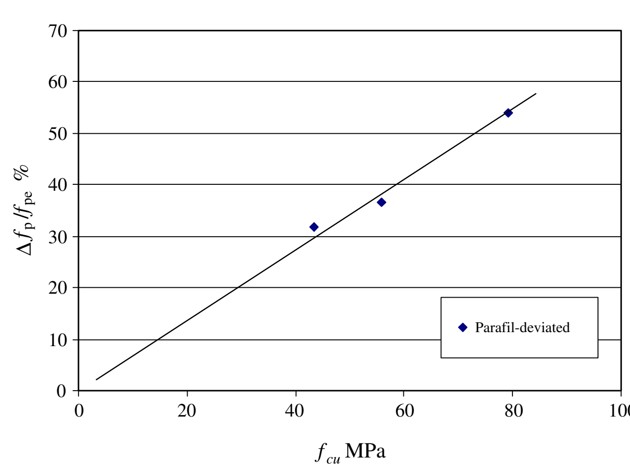

Fig. 7. Relation between the ratio of increase in effective external prestressing stress and the concrete strength.

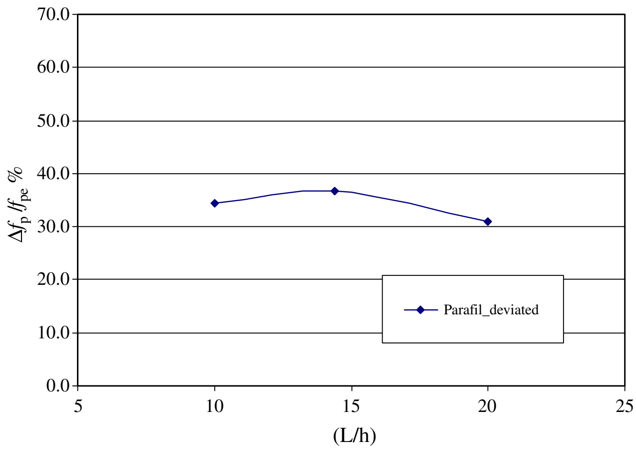

Fig. 8. Relation between the ratio of increase in effective external prestressing stress and (span/depth) ratio.

(L/h) ratio can be attributed to the reduction in ductility while the reduction in the tendon stress at the high L/h ratio can be related to the reduction in the tendon eccentricity (second-order effect). Therefore, the design equations proposed to calculate the tendon stress in the unbonded internal prestressed beams would be less accurate when used for externally prestressed beams with high L/h ratio.

4.7. Load type

Two load types were examined in this study; a concentrated load at the mid span and two concentrated loads at the third span (produces almost the same moment as the uniform load). The increase in the tendon stress in the beam subjected to loads at the third span is 32% greater than that in the beam with a single load at mid span. This is because, during loading, cracks started to appear on the beam surface and spread as the load increased. This continued up to the formation of the plastic hinge where the strain concentrated and stress increased up to failure. This resulted in the beam with the single load developing a smaller equivalent plastic hinge length at failure, and hence a smaller deflection, than the beam with two loads (or a uniform load). As the increase in the tendon stress depends on the deformation of the full member, the higher the deflection, the greater the increase in tendon stress.

4.8. Effect of internal bonded steel

Internal bonded steel refers to bonded prestressed and/or non-prestressed steel. The effective area and effective depth of bonded steel can be written in the form

$$A_{pef} = A_p + A_s \frac{f_{ys}}{f_{py}} \quad \text{and} \quad d_{ef} = \frac{d_s A_s \frac{f_{ys}}{f_{py}} + A_p d_p}{A_{pef}} \qquad (1)$$

where f_{py} is the yield stress of the prestressed steel and f_{ys} is the yield stress of the non-prestressed steel. Parameters such as the effective depth, the effective area and the internal prestressing stress may influence the ultimate stress of the external prestressing tendons. Fig. 9a shows the relation between the ratio of the increase in the ultimate external tendon stress and the ratio of internal bonded steel stress, (f_{pe}/f_{pu}), while Fig. 9b shows the relation between the ratio of increase in the ultimate external tendon stress and the ratio of the effective area of internal steel (A_{pef}/bd_{ef}).

The variation in the initial internal stress had only a slight effect on the ultimate stress of the external prestressing tendons as increasing the initial prestressing steel stress by about 52% resulted in increasing the ultimate external prestressing stress by only 2%.

During loading, the internal bonded steel assisted the external prestressing tendons in resisting the applied moment up to yielding of the internal steel, then the

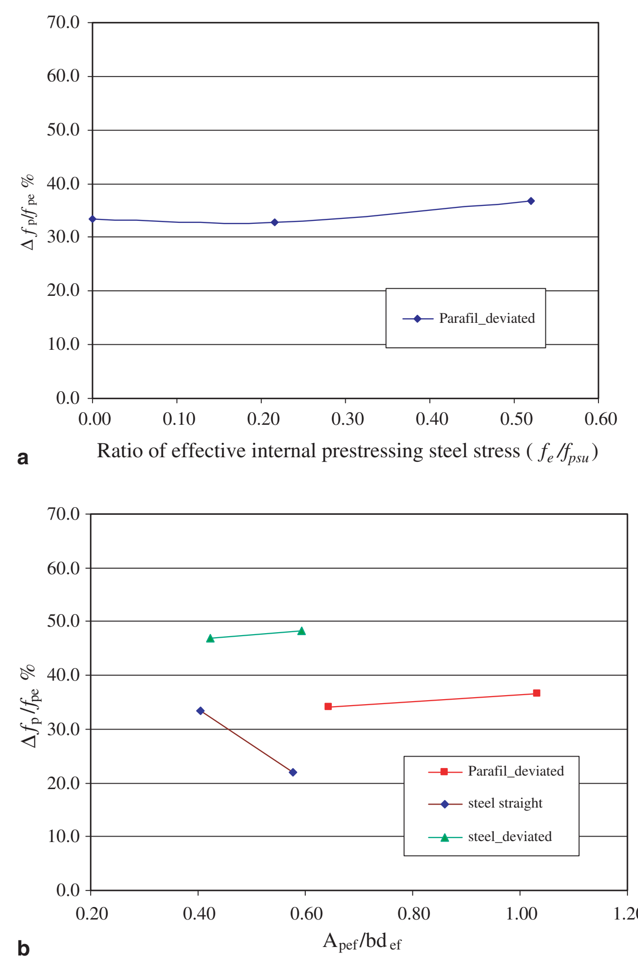

Fig. 9. (a) Relation between the ratio of the increase in external prestressing stress and the ratio of the effective internal prestressing steel stress. (b) Relation between the ratio of increase in effective external prestressing stress and the effective area of the internal bonded steel.

additional applied moment was fully resisted by the external tendons. Also, the presence of the bonded steel improves the ductility which results in an improvement in the ultimate stress in the external tendons as shown in Fig. 9a. However, increasing the internal bonded steel results in the beam behaving in a less ductile manner and the ultimate stress in the external tendons will be less.

In the case of the straight external tendons without any support form deviators, the presence of bonded steel has an inverse effect on the ultimate stress of the straight tendons (without deviators) due to the reduction in the eccentricity of the external prestressing tendons. This is accompanied by an improvement in the ductility of the beam.

4.8.1. Code equations

Most of the equations proposed by codes of practice to calculate the stress in the external tendons at ultimate can be written in the form:

$$f_{ps} = f_{pe} + \Delta f_p \quad (2)$$

These equations were originally proposed for unbonded tendons but have also been used for external tendons. For design purpose and for simplicity, several factors are neglected in these equations. The effect of some of these factors on the ultimate stress in external tendons was considered significant in the literature. Table 3 shows a comparison between factors included in the ACI-318 [1], BS8110 [2] and Eurocode 2 [3] (draft) equations.

To investigate the effect of the factors studied on the accuracy of the code equations; the experimental have been compared with the corresponding analytical results. Fig. 10a–d show part of this comparison.

4.8.2. Eurocode 2

The Eurocode is highly conservative and less accurate than both ACI318 and BS8110. It should therefore only be used as a guide in the early design stage.

4.8.3. ACI318

In general, the ACI code was more accurate than both the Eurocode and the BS code when calculating the ultimate stress in external steel tendons and less accurate when calculating the ultimate stress in Parafil ropes. This is because the equations proposed by the ACI are based on experimental results of beams with internally unbonded steel tendons.

From the comparison between the experimental results for Parfil ropes and steel tendons and the analytical results calculated by the ACI equation, the accuracy of ACI equation was found to decrease as

- (L/d_e) ratio decreased,
- concrete strength increased (especially for high strength concrete),
- (d_e/h) ratio increased (especially when $d_e > h$).

Also, the error between the actual and the calculated results was higher for the effect of number of deviators, (d_e/h) ratio, (L/d_e) ratio and (S_d/L) ratio in case of deviated steel tendons compared with deviated Parafil ropes.

From the comparison between the experimental and calculated results of deviated Parafil ropes, the following factors are found to have a slight influence on the accuracy of ACI equation:

- (L/h) ratio within the studied ratios.
- Loading type.
- Effective stress of the internal prestressing steel.
- Ratio of internal bonded steel area.

4.8.4. BS8110

Generally, the BS8110 equation underestimated the actual results and its accuracy was greater in case of Parafil rope than in case of steel tendons. This can be attributed to the lower Young's Modulus of Parafil rope compared to that of steel which results in a smaller increase in stress for the same strain and to the restriction applied to the calculated stress; $f_{ps} \leqslant 0.7 f_{pu}$.

The error between the actual and the analytical results is almost constant when considering the effect of the variation in the effective external prestressing stress, ratio of the external tendons area and (S_d/L) ratio. Also, the calculated ultimate stress is significantly affected by

Table 3
Factors included in codes equations

Code	Factors													
	External prestressing effect				Tendon profile			Internal bonded steel			Beam properties and loading type			
	f^1_{pe}	f^2_{pe}	A_{pe}	d_e	Shape	N_d	S_d/L	A_{ps}	f_{ps}	A_s	f_{cu}	L/h	L/d_e	p
BS8110	■	–	■	–	–	–	–	–	–	–	■	■	■	–
ACI-318	■	–	■	–	–	–	–	–	–	–	■	■*	–	–
EC-03	■	–	–	–	–	–	–	–	–	–	–	–	–	–

$f^1_{pe} = \Delta f_{pe}$ not influenced by f_{pe}, $f^1_{pe} \Delta f_{pe}$ influenced by f_{pe}, N_d = no. of deviators, S_d = distance between deviators, L = span length, P = loading pattern.

* L/h is not directly included in the equation.

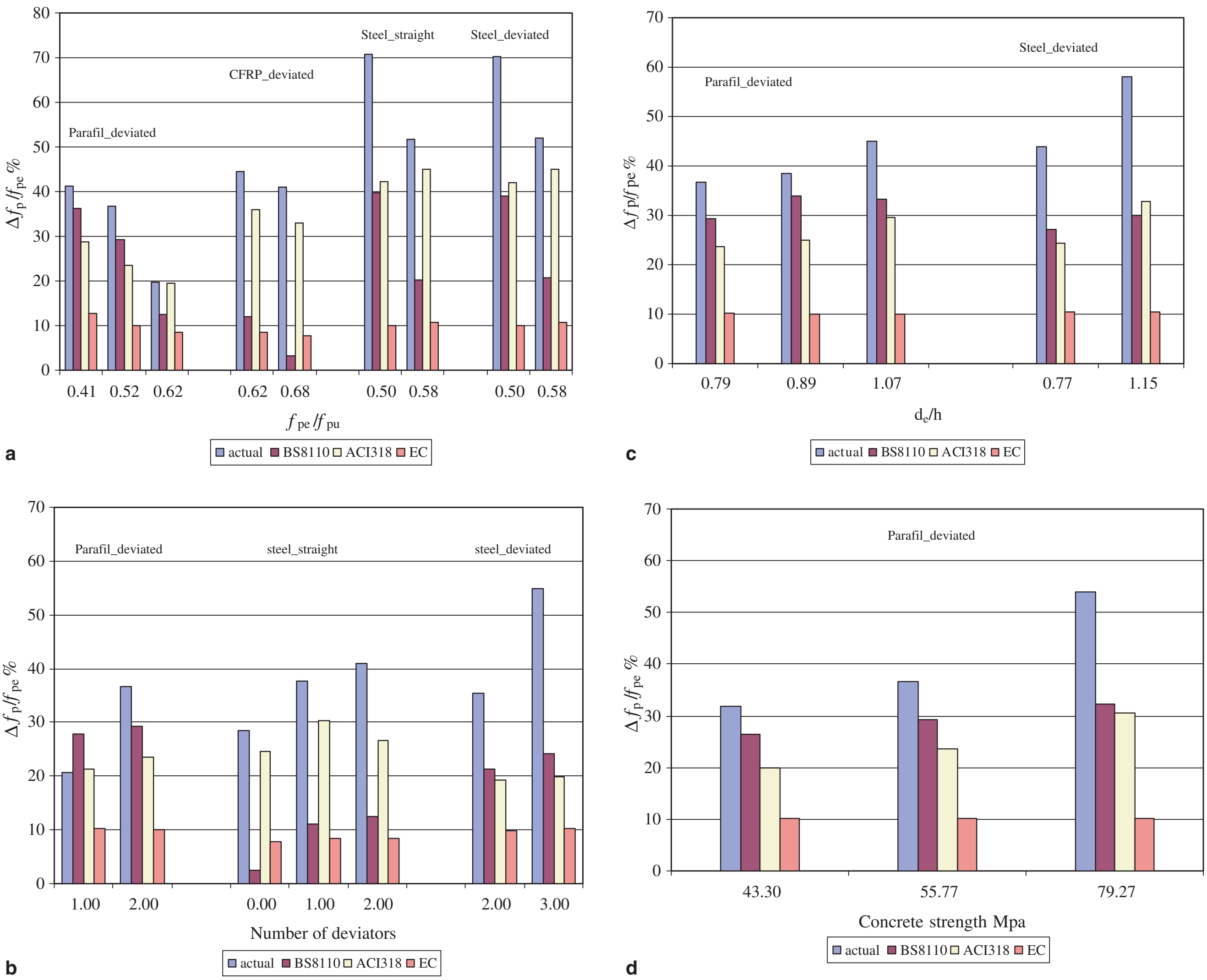

Fig. 10. (a) Relation between the ratio of increase in effective external prestressing stress and the effective external prestressing stress ratio. (b) Relation between the ratio of increase in effective external prestressing stress and the number of deviators. (c) Relation between the ratio of increase in effective external prestressing stress and (eccentricity of the external prestressing force/depth) ratio. (d) Relation between the ratio of increase in effective external prestressing stress and the concrete strength.

the number of deviators, effective depth of the external prestressing force, concrete strength and slightly affected by the variation in L/h ratio, internal prestressing stress, ratio of bonded steel area and loading pattern.

The accuracy of BS8110 equation can be improved by changing the restriction limit as follows:

$$f_{ps} = f_{pe} + \frac{7000}{L/d}\left(1 - 1.7\frac{f_{pu}A_{ps}}{f_{cu}bd}\right) \leqslant f_{pu}\ (\text{FRP}) \quad \text{OR} \quad f_{py}\ (\text{steel}) \tag{3}$$

Fig. 11 shows a comparison between the results calculated by the code equation and the modified equation. The modified equation can be used in design by applying a partial factor of safety factor (γ) to the restricted value; (f_{pu}/γ or f_{py}/γ).

5. Conclusions

Several factors influence the stress in external prestressing tendons under the ultimate load. The effect of some of these factors can be ignored while others should be taken into consideration.

This paper investigates the effect on the stress in external tendons at ultimate of the following factors related to the external prestressing system: initial prestressing stress, effective depth of the external tendons, location of deviators and the distance between deviators, area and prestress in internal bonded tendons, span/depth ratio, concrete strength and loading arrangement. The accuracy of prediction of the stresses using the equations proposed by the Eurocode, ACI318 and BS8110 was also investigated. Experimental results from

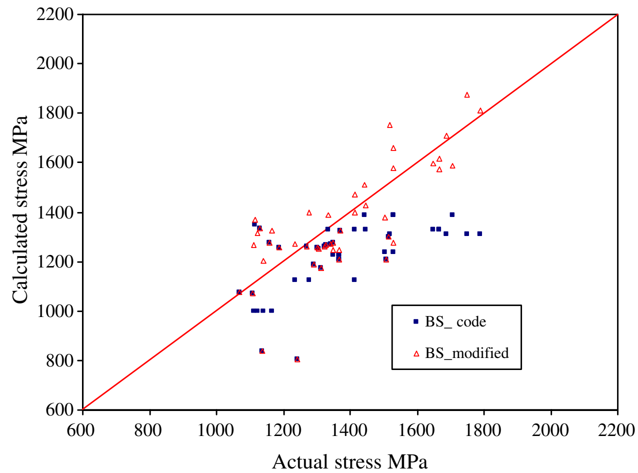

Fig. 11. Comparison between results obtained by BS equation and modified equation.

16 beams after externally prestressed using Parafil ropes as well as experimental results from beams using external steel tendons collected from literatures were used in this study.

These comparisons showed that:

1. Factors which influenced the stress in steel tendons had the same effect on the stresses in Parafil ropes. However, the magnitude of this effect is mainly as a result of the difference in the value of the Young's modulus.
2. The stress in internal tendons, the internal bonded steel ratio and the span/depth ratio had only a slight effect on the stress in the deviated external prestressing tendons. As expected, the effective depth of the external prestressing tendons (d_e) has a significant effect on the ultimate stress in the tendons if $d_e \geqslant h$.
3. Increasing the concrete strength increased the stress in the external tendons.
4. The ultimate stress in deviated external tendons is significantly influenced by the value of the prestress, the number of deviators, concrete strength and the ratio of the distance between deviators to the span.
5. The tendon profile (straight without supporting deviators or deviated), the presence of deviators and the Young's modulus of the external tendons should be considered in analysis of externally prestressed beams.
6. The equation proposed by the Eurocode underestimates the ultimate stress in the external tendons and shows the least accuracy.
7. The ACI equation shows a low accuracy when predicting the effect of the external prestressing stress, number of deviators, (distance between deviators/span) ratio, and effective depth of the external prestressing force. More studies are needed to include these factors and to modify the ACI equation to be suitable for high concrete strengths.
8. BS8110 generally gave good agreement with the actual results for Parafil rope while it gave a lower accuracy for steel tendons. A modified equation, which gives an improved accuracy, is proposed.

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