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Discussion

The measurement and significance of green sheet properties for the properties of hardened fibre cement [Cooke AM. Cement and Concrete Composites 2005;27(6):604–610]

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The author should be complimented for not only acknowledging—at the top of page 605—but also elaborating—halfway the same page—*the fibre orientation effect on tensile strength*. Although a problem that should not be neglected in analyzing mechanical behaviour of dispersed fibre reinforced cementitious specimens under load, it is in a lot of cases, as in other contributions in this Special Issue of CCC on Natural Fibre Reinforced Cement Composites.

However, as with the rest of the world, the theory on mechanics of such materials dramatically changed during the past quarter of a century. A *sole* reference from the late seventies of the last century can be considered therefore a weak basis for approaching the present problem. The author is invited to check more modern publications focussing on the same problem, whether dealing with bulky material entities (3D problem) or sheets (2D problem). We will therefore discuss such methodological aspects, since we believe this to do right to the ambitions of the author. Moreover, the general trend of the paper is not seriously affected.

Fibres in bulky cementitious specimens or in construction elements generally have a 3D (spatial) distribution. In relatively thin elements, as discussed in the paper, an approximately 2D distribution is at issue. In modelling approaches, the fibre centres are conventionally assumed distributed "randomly". For design purposes, the fibre orientation is additionally assumed 3D isotropic uniformly random (IUR) 3D in bulky elements and 2D IUR in thin sheets. This can be visualized in the following way. In an imaginary way, all fibres included in the steel fibre reinforced element are collected while maintaining their orientations. Next, the fibres are joined with one of their ends in the origin, O. The other end of monosize fibres will cover uniformly random a sphere with radius l, the length of the fibre. This will be a circle when dealing with a sheet. In the case of curved fibres or multisize ones, the fibres are subdivided in short lineal monosize elements (sticks) that are treated as before. For the sheet, we take two lines through O with small opening angle $\Delta\alpha$. The number of fibres (or sticks) inside this small opening angle would be independent on the orientation of the set of lines for the 2D IUR fibre dispersion. One can operate similarly in the 3D case.

The fibres are mechanically activated in concrete by *cracking*. Hence, fibres in low volume fraction fibre reinforced cementitious composites only contribute *significantly* to stress transfer when bridging a crack, as also assumed in the paper. *The orientation distribution of these load-bearing fibres is not 2D IUR, however, even when they are so in bulk.* This is a common mistake in the literature. The crack simply misses fibres in its neighbourhood. This is obviously a function of the angle between crack and fibre. When we assume for simplicity reasons that the crack is perpendicular to the tensile loading direction, and that the fibre includes an angle θ with the loading direction, it can simply be seen that the *probability* of the fibre to intersect with the crack is proportional to $\cos\theta$. Hence, the orientation distribution of the reinforcing fibres is given by

$$f(\theta) = \cos\theta. \quad (1)$$

This is far from 2D "random" (IUR). What is more, the reinforcement is more effectively distributed than in the 2D random case, as implicitly underlying the

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analysis in the paper. On the other hand, this concerns only part of the locally available fibres, i.e., the effective number of fibres, N_{eff}, is given by

$$N_{\text{eff}} = N\frac{\int_0^{\pi/2}\cos\theta\,\mathrm{d}\theta}{\int_0^{\pi/2}\mathrm{d}\theta} = \frac{2}{\pi}N, \tag{2}$$

whereby N stands for the total amount of fibres. Hence, only (roughly) 2/3rd of the "available" fibres around the crack are effectively contributing.

The *efficiency* of these fibres to contribute to stress transfer perpendicular to the crack (in the article, in the machine direction) is therefore given by

$$\sigma_{\text{f}\perp} = a\tau_{\text{f}}V_{\text{f}}\cdot\frac{\int_0^{\pi/2}\cos^2\theta\,\mathrm{d}\theta}{\int_0^{\pi/2}\mathrm{d}\theta} = \frac{1}{2}a\tau_{\text{f}}V_{\text{f}}, \tag{3}$$

where $a\tau_{\text{f}}V_{\text{f}}$ is the so-called fibre factor that is also used in the paper. The orientation factor for stress transfer (i.e., 0.5) is available in the international literature [1,2]. This does not account for the length efficiency, as correctly mentioned in the paper.

In the cross direction the fibre efficiency for load transfer should be equal to the value in Eq. (3) for 2D random fibres in bulk of the sheet. Similarly operating, we find

$$\sigma_{\text{f}\parallel} = a\tau_{\text{f}}V_{\text{f}}\cdot\frac{\int_0^{\pi/2}\sin^2\theta\,\mathrm{d}\theta}{\int_0^{\pi/2}\mathrm{d}\theta} = \frac{1}{2}a\tau_{\text{f}}V_{\text{f}}. \tag{4}$$

Hence the ratio of the orthogonal efficiency factors, as defined in the paper, equals $\text{XR} = 1$ for the ideal case of 2D "random" fibres in bulk. In the author's experiments this ratio is considerable smaller (about 0.42: Table 1). Hence, the real fibre distribution is *more aligned with the machine direction*.

This effect can be modelled by assuming the alignment effect proportional to $\cos\theta$. XR would transform for this case into

$$\text{XR} = \frac{\int_0^{\pi/2}\sin^2\theta\cos\theta\,\mathrm{d}\theta}{\int_0^{\pi/2}\cos^3\theta\,\mathrm{d}\theta} = \frac{1}{2}. \tag{5}$$

Generally speaking, one can take higher order cosine terms to better match experimental data. For example, taking $\cos^2\theta$ would diminish the strength ratio XR to 0.33. This is well below the author's experimental value, however.

The average angle between the machine direction and the fibre reinforcement can readily be obtained under such conditions. 2D randomly oriented fibres would on average enclose an angle of 45° with the machine direction, of course. Hence, bulk situation can be characterized by

$$\bar{\theta}_{\text{2D}} = \frac{\int_0^{\pi/2}\theta\,\mathrm{d}\theta}{\int_0^{\pi/2}\mathrm{d}\theta} = \frac{\pi}{4}. \tag{6}$$

For the fibre portion that *reinforces the crack* we find

$$\bar{\theta}_{\text{2D,crack}} = \frac{\int_0^{\pi/2}\theta\cos\theta\,\mathrm{d}\theta}{\int_0^{\pi/2}\cos\theta\,\mathrm{d}\theta} = 0.72\frac{\pi}{4}. \tag{7}$$

As we have seen earlier, the result of the stochastic selection procedure by the crack is favourable in both orthogonal directions! Similarly operating for the $\cos\theta$-weighed fibre dispersion, we find an average value of the fibre orientation with respect to the machine direction of $0.60\pi/4$. Additionally, for the $\cos^2\theta$-weighed fibre dispersion this would be $0.51\pi/4$. Comparison of these outcomes learns the alignment imposed by the machine in the author's experiments to be quite obvious!

The author can find in the literature another *possible and practical* approach to partially ordered fibres, which is based on the *assumption* that the real fibre dispersion in bulk is a mixture of the 2D IUR fibres and a portion of fibres aligned in the machine direction (in the present case, this leads to a *partially linear fibre structure*) [3,4]. Although seemingly more complicated, the approach is even more straightforward. The two strength components would be

$$\sigma_{\text{f}\perp} = \tfrac{1}{2}aV_{\text{f2D}}\tau_{\text{f}} + aV_{\text{f1D}}\tau_{\text{f}}, \tag{8}$$

$$\sigma_{\text{f}\parallel} = \tfrac{1}{2}aV_{\text{f2D}}\tau_{\text{f}}, \tag{9}$$

where the subscripts 2D and 1D refer to the 2D "random" and the 1D aligned fibre portions, respectively, the subscripts $\perp$ and $\parallel$ indicate the orientation of the respective strength components with respect to the crack plane, and the coefficient 0.5 is derived in Eqs. (3) and (4). To adjust the system so that it meets the experimental data governed by $\text{XR} = 0.42$, it can easily be demonstrated that $V_{\text{f2D}}/V_{\text{f}} \approx 0.59$ and $V_{\text{f1D}}/V_{\text{f}} \approx 0.41$. This demonstrates again the degree of alignment to be quite significant! The average value of the angle between the machine direction and the fibres reinforcing the crack is

$$\bar{\theta}_{\text{pl,crack}} = 0.59\frac{\int_0^{\pi/2}\theta\cos\theta\,\mathrm{d}\theta}{\int_0^{\pi/2}\cos\theta\,\mathrm{d}\theta} = 0.43\frac{\pi}{4}, \tag{10}$$

a value deviating from that indicated by the earlier approach, since the *type of alignment* in the fibre reinforcement is modelled differently. Image analysis data would be required to come up with a more appropriate characterization of 2D fibre dispersion in bulk of the plate. Still, as suggested by the author, the XR value can be employed to reveal the machine-induced alignment effect, which is a result of practical significance.

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