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Models relating mixture composition to the density and strength of foam concrete using response surface methodology

E.K. Kunhanandan Nambiar, K. Ramamurthy *

Building Technology and Construction Management Division, Department of Civil Engineering, Indian Institute of Technology Madras, Chennai 600 036, India

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Abstract

There have been several investigations in the past on the influence of mixture composition on the properties of foam concrete. Conventionally strength is related to density alone and few models have been developed relating strength with density, porosity, gel–space ratio, etc. Very little work has been reported in the literature in predicting the properties of foam concrete from the knowledge of its mixture proportions. This paper discusses the development of empirical models for compressive strength and density of foam concrete through statistically designed experiments. The response surface plots helps in visually analysing the influence of factors on the responses. The relative influence of fly ash replacement on strength and density of foam concrete is studied by comparing it with mixes without fly ash and brought out that replacement of fine aggregate with fly ash will help in increase in the strength of foam concrete at lower densities allowing high strength to density ratio. Confirmatory tests have shown that the relation developed by statistical treatment of experimental results can act as a guideline in the mixture proportion of foam concrete.

Keywords: Foam concrete; Response surface methodology (RSM); Fly ash replacement level; Filler–cement ratio; Strength–density ratio

1. Introduction

Several of the earlier investigations on foam concrete give a detailed discussion on its composition, physical properties and uses [1–6]. The first comprehensive review on cellular concrete including foam concrete was presented by Valore [1] and a detailed treatment by Short and Kinniburgh [3] and Rudnai [4]. A rational proportioning method of preformed foam concrete was proposed by Mc Cormick [5] based on solid volume calculations. Richard [6] reviewed the thermal and mechanical characteristics of foam concrete. Hoff [7] reported a porosity-strength model for cellular concrete made with Portland cement, water and preformed foam. Durack and Weiqing [8] proposed a strength-gel space ratio relationship for foam concrete. Nehdi et al. [9] presented a nontraditional approach to the prediction of density and compressive strength of foam concrete mixtures based on the Artificial Neural Network (ANN) technology. For moist cured foam concrete, the relation between strength and volumetric composition, particularly water content and air voids, has been formulated by Tam et al. [10] using Feret's and Power's equation, for a small range of water–cement ratios (0.6–0.8) and sand–cement ratios (1.58–1.73). Kearsley and Wainwright [11–14] investigated the effect of replacing large volume of cement with fly ash and equations based on effective water–cement ratio and binder ratio have been developed to predict the strength of foam concrete made of cement paste of different densities at different ages. Most of these studies related strength to density, gel–space ratio or porosity. Limited work has been reported on predicting the properties of foam concrete from knowledge of its mixture proportion.

Further, studies on foam concrete using fly ash as partial/complete replacement for filler have proved that

* Corresponding author. Tel.: +91 44 22574265; fax: +91 44 22574252.
E-mail address: vivek@iitm.ac.in (K. Ramamurthy).

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Table 1
Physical and chemical properties of cement and fly ash used

Properties	Cement		Fly ash	
Physical				
Blaine fineness (m^2/kg)	391		252	
Specific gravity	3.13		2.09	
28-day compressive strength (MPa)	54.7		–	
	% by mass	IS 12269-1987 (17)	% by mass	ASTM C 618 (Class F) (18)
Chemical				
SiO_2	19.3	–	63.6	–
CaO	61	–	1.57	<10
Al_2O_3	5.687	–	28.19	–
Fe_2O_3	6.036	–	2.99	–
MgO	1.875	6	0.54	5 (max)
MnO	–	–	0.03	–
Na_2O	–	–	0.05	–
K_2O	–	–	0.003	–
SO_3	1.67	2.5	0.26	–
Loss on ignition	0.2963	4	0.85	–
Soluble residue	1.489	2	–	–
Al_2O_3/Fe_2O_3	0.94	0.66	–	–
$SiO_2 + Al_2O_3 + Fe_2O_3$	–	–	94.78	70 (min)

the use of fly ash results in higher strength to density ratio and facilitates its high volume utilization [15]. This study attempts to develop empirical models for predicting the density and compressive strength of foam concrete from the mixture composition details like filler–cement ratio, addition of fly ash as partial/complete replacement for sand and foam volume, through systematically designed experiments.

2. Experimental investigations

2.1. Materials and mixture composition

Foam was produced by aerating an organic based foaming agent (dilution ratio 1:5 by weight) using an indigenously fabricated foam generator to a density of 40 kg/m^3. 53 grade Ordinary Portland Cement [16], pulverized river sand finer than 300 μm (specific gravity = 2.52) and class F fly ash conforming to ASTM C 618-89 [17] were used. The properties of cement and fly ash used in this study are presented in Table 1.

Table 2
Factors and factor levels adopted

Notation	Factor	Low level	High level
F/C	Filler–cement ratio	1	3
FA	Fly ash as replacement for sand (%)	20	100
FV	Foam volume (% of total volume of concrete)	10	50

The mix proportions were arrived at as per the procedure given in ASTM C 796-97 [18]. As the standard deals with only cement slurry, the procedure was modified to include cement–sand–fly ash components. As the foam is added to the wet mix, the consistency of the wet mix is very important to get the desired design density [15], which is expressed in terms of water–solids ratio required to produce this consistency, and its range is narrow for a given mix. Based on several trails, the percent flow (consistency), measured in a standard flow table [19] (without raising/dropping of the flow table as it may affect the foam bubbles entrained in the mix) was arrived at as 45 ± 5%. Earlier studies by the authors have shown that within this range, the water–solids ratio does not affect compressive strength. As the water–solids requirement for obtaining this flow values varied for mixes with and without fly ash replacement (0.3–0.4 for mixes without fly ash and 0.35–0.6 for mixes with fly ash at different replacement levels), it was decided to split experimental programme into two mixes, namely cement–sand mixes (M_1) and cement–fly ash–sand mixes (M_2).

2.2. Experimental design and validation

With an objective of developing relationships between the parameters and response (empirical models for compressive strength and density), a statistical methods of experimental design based on response surface methodology (RSM) is adopted. For providing equal precision of estimates in all directions, a central composite design with rotatability or equal precision is selected [20]. The factors and factor levels are shown in Table 2. Mixture M_1 has

Table 3
Observed and predicted responses for confirmation of models

Mix type	Mixture composition			Compressive strength, MPa						Dry density, kg/m^3	
	F/C	FA, %	FV, %	7-Day		28-Day		90-Day			
				Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed
M_1	1	–	20	9.0	7.8	11.6	10.7	13.2	12.3	1282	1318
	2	–	35	2.41	2.14	2.67	2.30	2.91	3.01	998	1011
	3	–	50	0.25	1.20	0.52	1.23	0.84	1.54	802	787
M_2	1	70	10	13.8	12.3	18.9	17.8	24.7	23.0	1194	1224
	2	65	30	3.66	2.93	5.08	4.84	9.23	8.03	858	854
	3	50	40	1.03	1.87	1.10	1.96	3.83	3.01	664	650

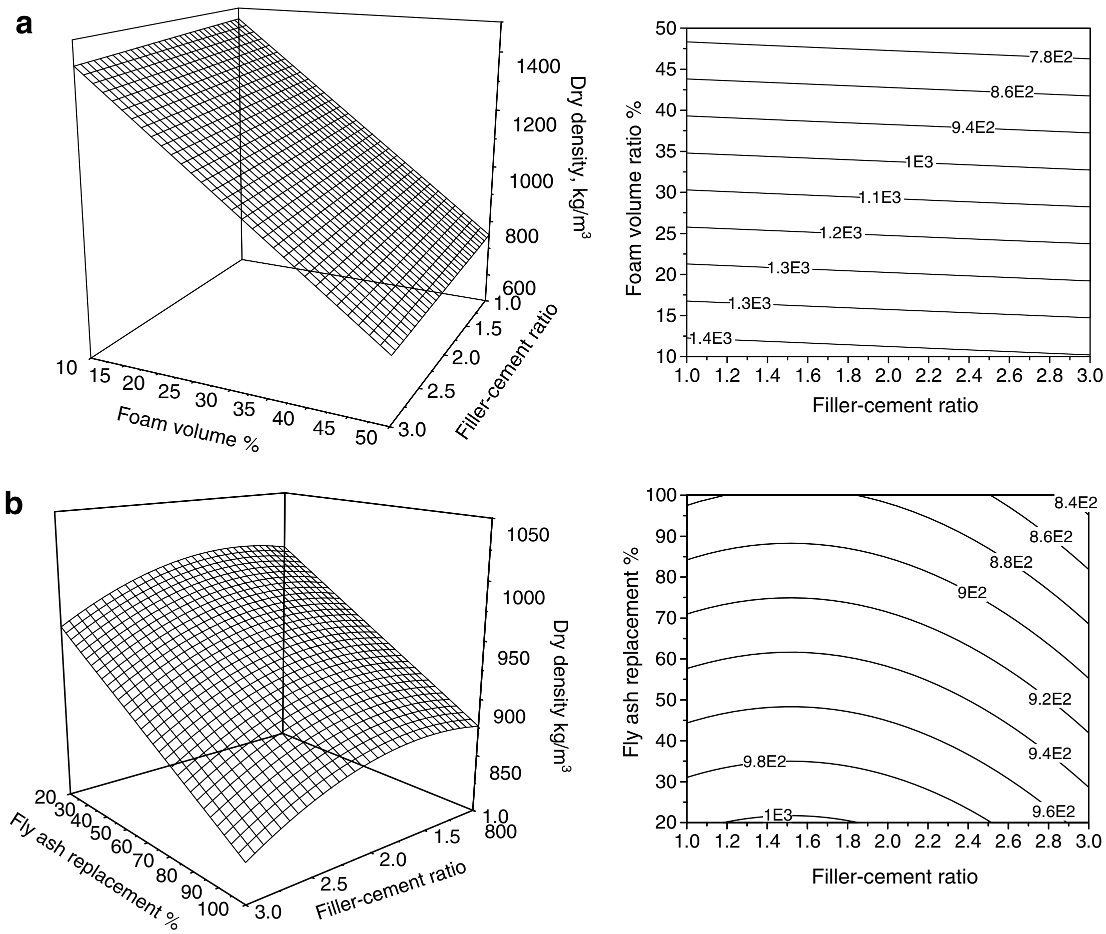

Fig. 1. Response surface and contour plot for dry density for (a) Mix M_1; (b) Mix M_2 (FV = 30%).

two variables (F/C and FA) and M_2 has three (F/C, FA and FV). The experimental design was done using Statistical Analysis Software (SAS) [21], resulting in 13 and 20 experimental runs for mixes M_1 and M_2, respectively.

The fresh density of foam concrete was measured by filling a standard container. The compressive strength was determined at the age of 7, 28, 90 days using six cube specimens of size 50 mm after curing by immersing in water in a curing tank. A sample size of six was arrived at as per ASTM E 122-72 [22]. The dry density was determined in accordance with the section on Oven-Dry Weight of Test method ASTM C 495-91a [23].

The results were analysed using SAS to determine the quadratic response surface adopting the following second-order model (to take care of curvature in the relationship) for predicting fresh density and compressive strength of foam concrete:

$$y = b_0 + \sum_{i=1}^{k} b_i x_i + \sum_{i=1}^{k} b_{ii} x_i^2 + \sum_{i<j} \sum b_{ij} x_i x_j$$

where, $x_i\,(i = 1, 2, \ldots, k)$ are quantitative variables and b_0, b_i are the least square estimates of the regression coefficients. The models for density and strength are given below.

For cement–sand foam concrete mix (Mix M_1):

$$\text{Dry density} = 1655.92 - 18.283 * (\text{F/C}) - 17.753 * (\text{FV})$$

$$\text{7-Day strength} = 25.328 - 5.903 * (\text{F/C}) - 0.737 * (\text{FV}) + 0.00474 * (\text{FV})^2 + 0.120 * (\text{F/C}) * (\text{FV})$$

$$\text{28-Day strength} = 32.342 - 6.098 * (\text{F/C}) - 1.004 * (\text{FV}) + 0.00768 * (\text{FV})^2 + 0.118 * (\text{F/C}) * (\text{FV})$$

$$\text{90-Day strength} = 38.212 - 7.327 * (\text{F/C}) - 1.225 * (\text{FV}) + 0.00929 * (\text{FV})^2 + 0.155 * (\text{F/C}) * (\text{FV})$$

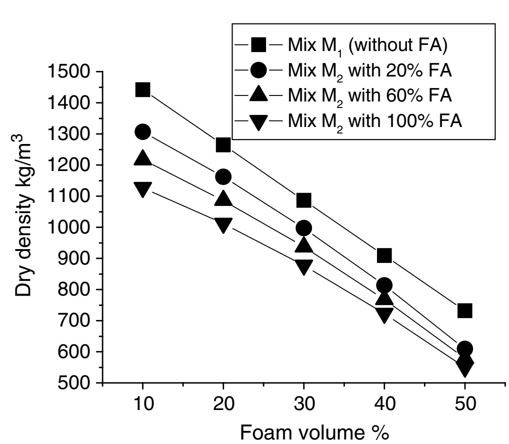

Fig. 2. Effect of fly ash replacement on density of foam concrete.

For cement–sand–fly ash foam concrete mix (Mix M_2):

$$\text{Dry density} = 1438.536 + 68.465*(F/C) - 2.629*(FA) - 12.333*(FV) - 22.563*(F/C)^2 - 0.098*(FV)^2 + 0.038*(FA)*(FV)$$

$$\text{7-Day strength} = 23.770 - 3.669*(F/C) - 0.778*(FV) + 0.00628*(FV)^2 + 0.0.082*(F/C)*(FV)$$

$$\text{28-Day strength} = 31.179 - 6.279*(F/C) + 0.117*(FA) - 1.053*(FV) - 0.00119*(FA)^2 + 0.00799*(FV)^2 + 0.138*(F/C)*(FV)$$

$$\text{90-Day strength} = 32.002 - 5.348*(F/C) + 0.193*(FA) - 1.028*(FV) - 0.00140*(FA)^2 + 0.00715*(FV)^2 + 0.095*(F/C)*(FV)$$

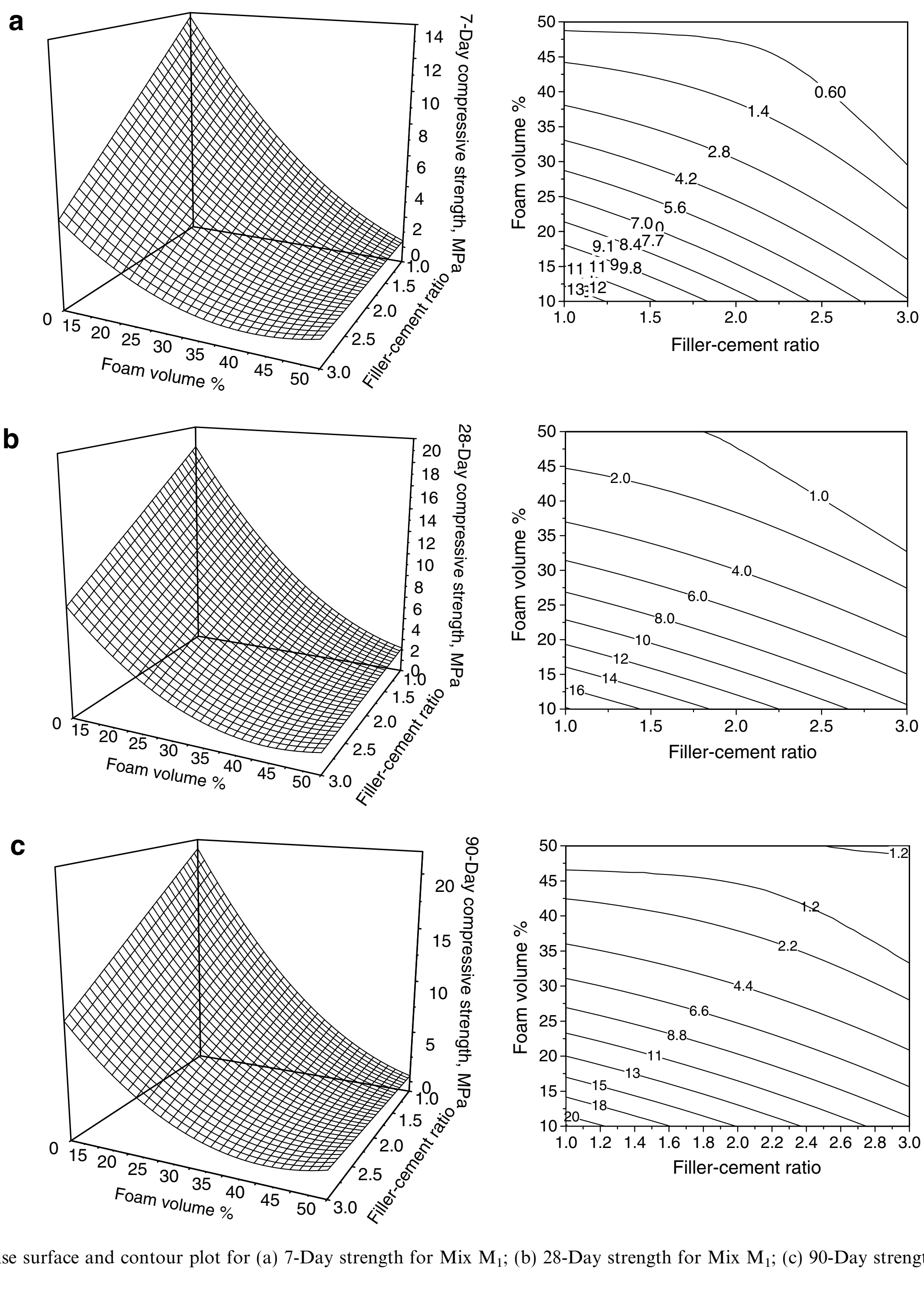

Fig. 3. Response surface and contour plot for (a) 7-Day strength for Mix M_1; (b) 28-Day strength for Mix M_1; (c) 90-Day strength for Mix M_1.

The models contain those terms that are statically significant (i.e. those terms that had a *T*-statistics greater than that of chosen significance level, 0.05 in this study). The adequacy checks on strength and density models (*R*-squared value, *F*-value and *T*-statistics) satisfy statistical requirements (details presented in Appendix A). Adequacy of the models was also checked by examining the residuals for trends by looking at the diagnostic plots like normal probability plots and residual plots. To confirm the accuracy of the prediction relations, a few tests were carried out with some of the factor levels chosen from the experimental series. The predicted and observed responses for

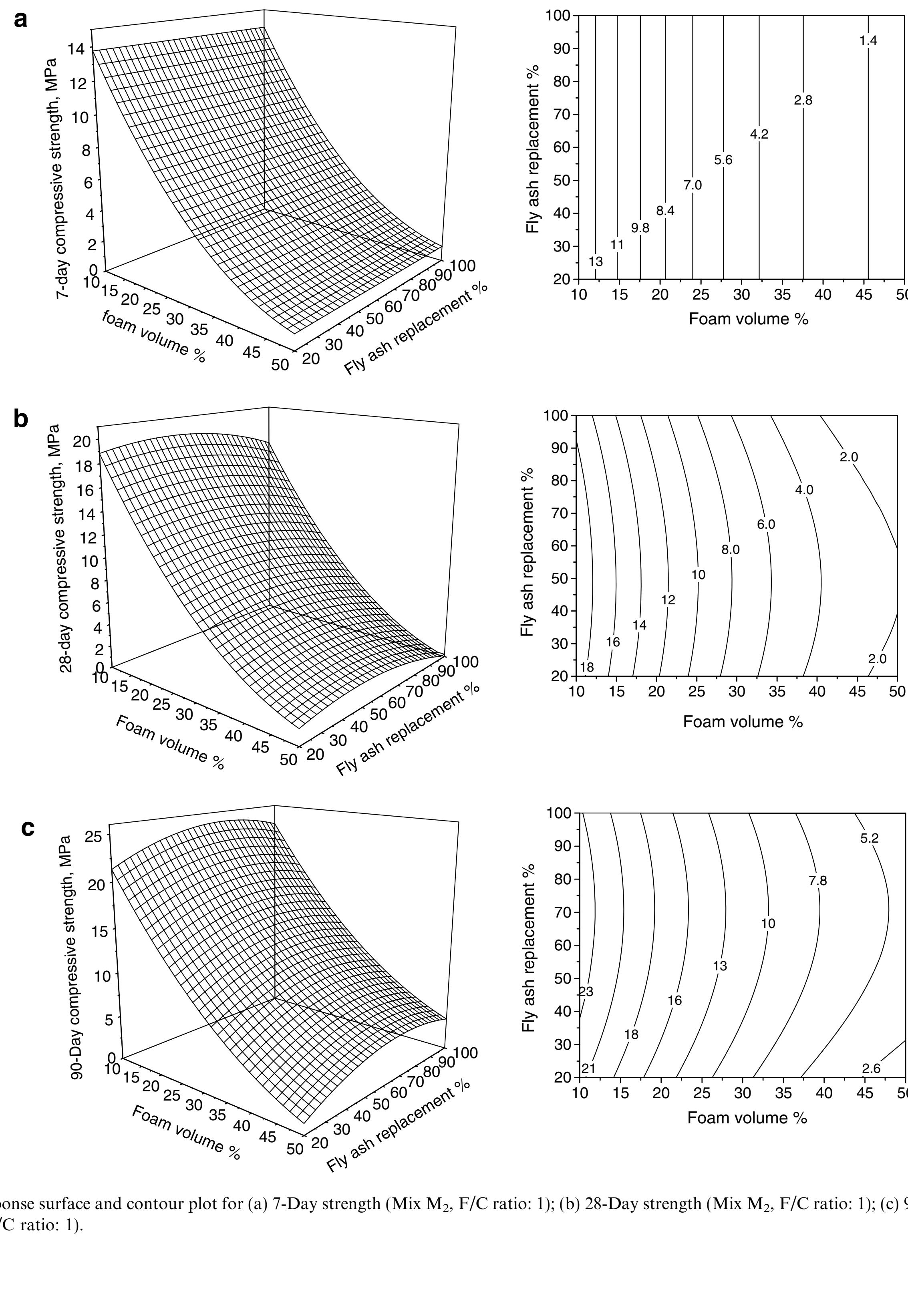

Fig. 4. Response surface and contour plot for (a) 7-Day strength (Mix M_2, F/C ratio: 1); (b) 28-Day strength (Mix M_2, F/C ratio: 1); (c) 90-Day strength (Mix M_2, F/C ratio: 1).

both compressive strength and density were found to match within satisfactory limits as given in Table 3. The behaviour of foam concrete for different mixture compositions are discussed in the next sections by plotting the response surface graphs developed from above equations.

3. Response surface for dry density

The response surface and contour plot for dry density of foam concrete mixes M_1 and M_2 are shown in Fig. 1(a) and (b), respectively. In cement–sand mixes (Mix M_1), the density decreased linearly with an increase in foam volume. The variation in density with filler–cement ratio was very less compared to that with foam volume. For mixes with fly ash replacement (Mix M_2), the density varies nonlinearly with filler–cement ratio and shows a peak value as indicated by the contour plot. For a given foam volume, replacement of fine aggregate with fly ash, due to its relatively lower specific gravity, reduces the density of foam concrete (Fig. 2). Alternately, to achieve a particular density of foam concrete, use of fly ash results in a reduction in foam volume.

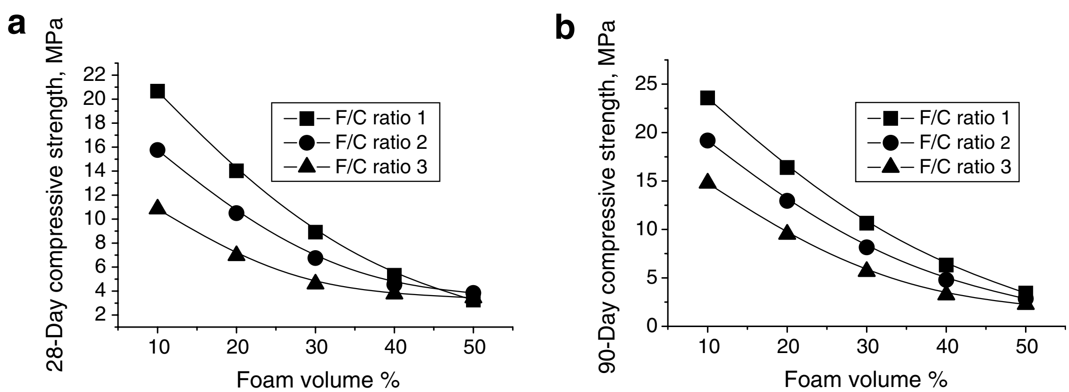

Fig. 5. Foam volume–filler cement ratio interaction for Mix M_2 at (a) 28-Day, (b) 90-Day.

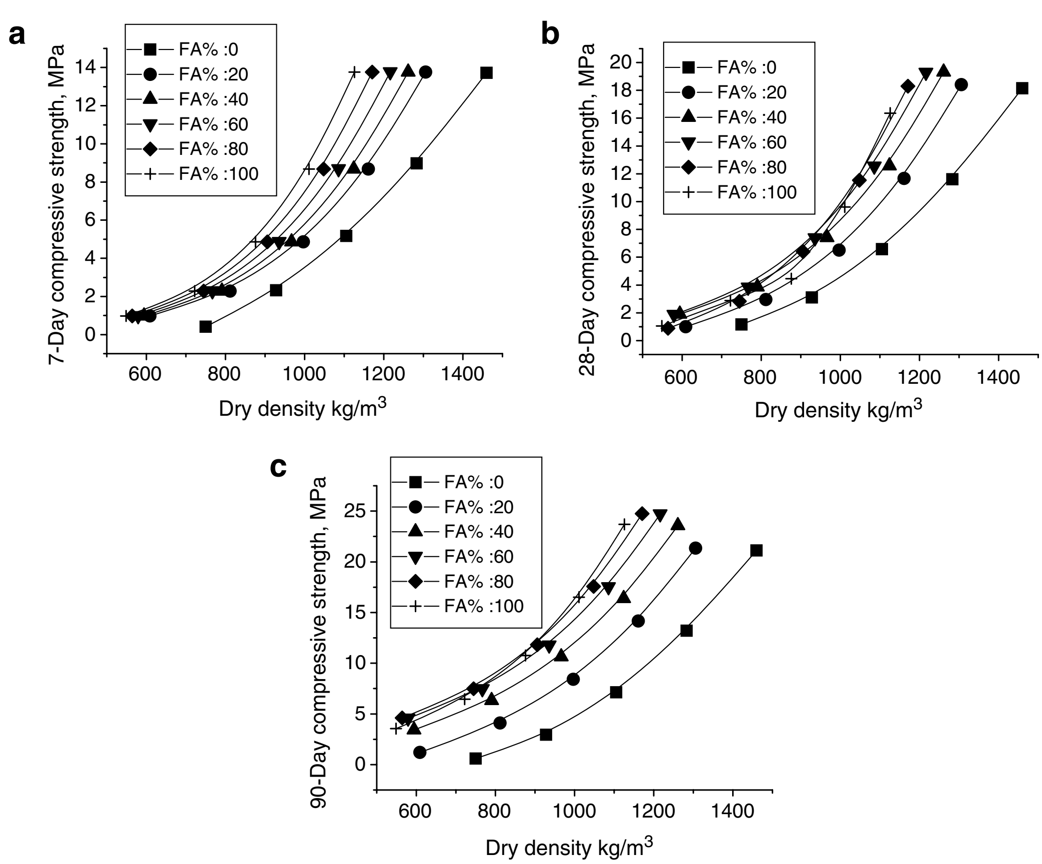

Fig. 6. Strength–density relationship of foam concrete mixes (filler–cement ratio 1).

4. Response surface for compressive strength

Fig. 3(a)–(c) shows the response surfaces and contour plots for compressive strength of cement–sand mix (M_1) at the age of 7, 28, 90 days. Compressive strength varies linearly with filler–cement ratio and nonlinearly with foam volume. For a given foam volume, an increase in filler–cement ratio reduces the compressive strength, while the effect of filler–cement ratio on compressive strength is predominant at lower foam content, indicating that at higher foam content the strength (which is very low) is mainly controlled by the entrained pores.

As Mix M_2 contains three factors, the filler–cement ratio is kept at fixed level (one) to draw the response surface and contour plots for compressive strength of cement–fly ash–sand mixture and are shown in Fig. 4(a)–(c). At 7 days, fly ash replacement percentage has no effect on the compressive strength of foam concrete. At 28 and 90 days, the strength increases with fly ash replacement percentage up to a certain level and decreases slightly after that, an optimum value of fly ash replacement are estimated to be 49% and 71%, respectively.

Comparing Figs. 3 and 5 it can be seen that, for foam concrete mixes with 10% foam volume and when filler cement ratio varied from one to three, there is about 12–55% and 6–15% of increase in strength for cement–fly ash–sand mix than the cement–sand mix at 28 and 90 days, respectively. The rate of strength development between 28 and 90 days is higher for cement–sand–fly ash mix. The contribution of fly ash to the compressive strength due to its pozzolanic reaction and its dependency on age is evident from this. Similar observations of strength enhancement by replacing fine aggregate by fly ash were reported in other studies on foam concrete [8,15,24].

Fig. 5(a) and (b) shows the effect of filler–cement ratio on compressive strength for varying foam volume content and for a typical fly ash replacement level of 40% and the behaviour is found to be similar to that of cement–sand mix. A given strength can be achieved at higher filler–cement ratio by reducing the foam volume, thus producing an economical mixture with lower cement content.

5. Strength–density relations

Using the above models, the strength and dry density for different levels of parameters have been determined first. A typical variation of strength with dry density for foam concrete with a filler–cement ratio 1 at different ages is plotted in Fig. 6. For both cement–sand and cement–fly ash–sand mixes, compressive strength increased with increasing dry densities, as expected, due to the lower pore volume. For a given density, as percentage fly ash replacement increases the compressive strength increases at all ages. Together with the pozzolanic reaction, the reduced foam volume also contributes to this strength enhancement. Alternatively for a given strength requirement, the density of concrete is lower with increase in fly ash content.

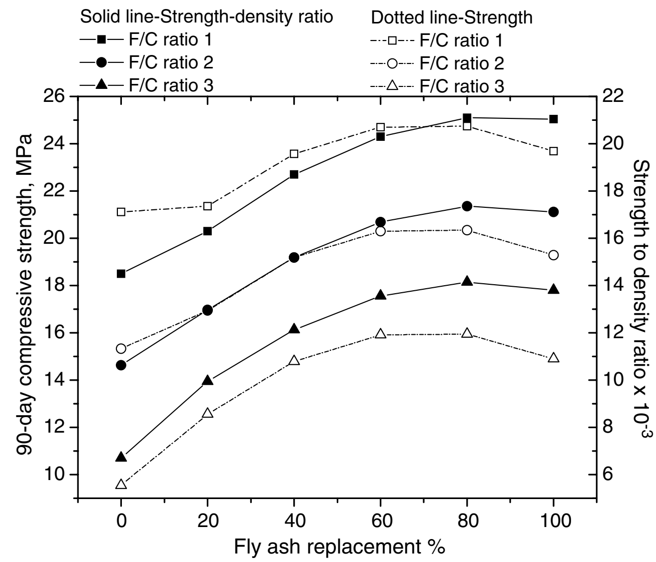

Fig. 7. Strength and strength to density variation for foam concrete (FV—10%).

The enhancement of strength with fly ash is not pronounced at lower density range (higher % of foam volume) especially at lower ages. This is due to the fact that at lower

Table A.1a
Model statistics for cement–sand mix M_1

Responses	*R*-squared	Adjusted *R*-squared	*F*-value	Prob > *F*
7-Day strength	0.962	0.942	50.02	0.0001
28-Day strength	0.958	0.937	42.61	0.0001
90-Day strength	0.971	0.957	67.57	0.0001
Dry density	0.987	0.986	1933.8	0.0001

Table A.1b
Effect estimates of factors for cement–sand mix M_1

Responses	Estimates	Effect estimates of factors				
		F/C	FV	F/C * FV	$(F/C)^2$	$(FV)^2$
7-Day strength	*T*	−6.3406	−11.7082	3.2876	–	3.4608
	Prob > *T*	0.0002	0.0001	0.0111	–	0.0086
28-Day strength	*T*	−4.9163	−11.7611	2.2562	–	3.9186
	Prob > *T*	0.0012	0.0001	0.450	–	0.0044
90-Day strength	*T*	−5.4032	−14.3841	3.1005	–	4.9584
	Prob > *T*	0.0006	0.0001	0.0147	–	0.0011
Dry density	*T*	−3.198	-62.109	–	–	–
	Prob > *T*	0.0095	0.0001	–	–	–

Table A.2a
Model statistics for cement–sand–fly ash mix M_2

Responses	*R*-squared	Adjusted *R*-squared	*F*-value	Prob > *F*
7-Day strength	0.972	0.964	128.83	0.0001
28-Day strength	0.978	0.969	55.46	0.0001
90-Day strength	0.975	0.964	84.48	0.0001
Dry density	0.985	0.984	530.7	0.0001

Table A.2b
Effect estimates of factors for cement–sand–fly ash mix M_2

Responses	7-Day		28-Day		90-Day		Dry density	
Estimates	T	Prob > T	T	Prob > T	T	Prob > T	T	Prob > T
F/C	−5.275	0.0001	−7.587	0.0001	−6.256	0.0001	−3.765	0.0024
FA	–	–	−3.611	0.0031	2.9406	0.0114	−10.39	0.0001
FV	−20.76	0.0001	−20.98	0.0001	−20.50	0.0001	−55.07	0.0001
F/C * FA	–	–	–	–	–	–	–	–
F/C * FV	3.259	0.0053	4.4109	0.0007	2.1791	0.0483	−2.40	0.0320
FA * FV	–	–	–	–	–	–	–	–
$(F/C)^2$	–	–	–	–	–	–	–	–
$(FA)^2$	–	–	−4.103	0.0012	3.3604	0.0051	2.36	0.0342
$(FV)^2$	6.754	0.0001	6.9168	0.0001	4.4130	0.0007	−4.144	0.0011

density range, it is the foam volume which controls the strength of foam concrete, rather than the material properties. At 90 days, the contribution of fly ash at lower density is seemed to be better than other lower ages.

For a given strength and density, the percentage fly ash replacement can be appropriately selected from the strength–density relation as shown in Fig. 6. Knowing the filler–cement ratio and fly ash replacement level the foam volume requirement can be calculated using the equation developed for dry density. The water requirement for the foam concrete mixture with these parameters can be determined based on the consistency (flow value of 45 ± 5 %). Thus the prediction relation developed using statistical treatment of experimental results can serve as guidelines in the design of foam concrete mixes.

Fig. 7 shows the effect of fly ash replacement on the 90-day strength and strength–density ratio for foam concrete with 10% foam volume and for different filler–cement ratios. The strength–density ratio increases with finer filler like fly ash. It can be seen that even though there is a marginal reduction in strength after around 70% fly ash replacement level, while the strength to density ratio remains nearly same beyond this level. Hence a 100% replacement level will allow higher utilization of fly ash with same strength to density ratio, with a small reduction in strength.

6. Conclusions

Conclusions pertain to the experimental investigations conducted and are valid for the factors and factor level considered are summarised below.

The prediction relation developed using statistical treatment of experimental results can serve as guidelines in the design of foam concrete mixes. Fly ash when used as partial/complete replacement for fine aggregate; cause an increase in compressive strength. The replacement level for maximum strength depends on age of testing; it is 49% at 28 days and 71% at 90 days. Replacement of sand with fly ash in the mixture generally reduces the density, resulting in reduced foam volume requirement for a given density and thus indirectly contributes to strength enhancement over and above the pozzolanic and filler effects. The strength–density ratio is high for foam concrete mixes containing fly ash as compared to sand.

Appendix A

See Tables A.1(a)–A.2(b).

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