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A model for prediction of time from corrosion initiation to corrosion cracking

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Abstract

Prediction of time to corrosion cracking is a key element in evaluating the service life of corroded reinforced concrete (RC) structures. This paper presents a mathematical model that predicts the time from corrosion initiation to corrosion cracking. In the present model a relationship between the steel mass loss and the internal radial pressure caused by the expansion of corrosion products is developed. The concrete around a corroding steel reinforcing bar is modeled as a thick-walled cylinder with a wall thickness equal to the thinnest concrete cover. The concrete ring is assumed to crack when the tensile stresses in the circumferential direction at every part of the ring have reached the tensile strength of concrete. The internal radial pressure at cracking is then determined and related to the steel mass loss. Faraday's law is then utilized to predict the time from corrosion initiation to corrosion cracking. The model accounts for the time required for corrosion products to fill a porous zone before they start inducing expansive pressure on the concrete surrounding the steel reinforcing bar. The accuracy of the model is demonstrated by comparing the model's predictions with experimental data published in the literature.

Keywords: Concrete; Corrosion; Cracking; Expansion; Service life; Steel; Model

1. Introduction

The mechanism of steel reinforcing corrosion is generally accepted to be electrochemical in nature [1]. The alkaline environment of concrete results in formation of a passive film of iron oxides at the steel-to-concrete interface that protects the steel from corrosion. Passivation of steel can be destroyed by carbonation or by chloride attack. Once the passive film breaks down corrosion will start, in the presence of moisture and oxygen, resulting in a formation of expansive corrosion products (rust) that occupy several times the volume of the original steel consumed. The expansive corrosion products create tensile stresses on the concrete surrounding the corroding steel reinforcing bar. This can lead to cracking and spalling of concrete cover as a usual consequence of corrosion of steel in concrete.

Prediction of time to corrosion cracking is a key element in evaluating the service life of corroding reinforced concrete (RC) structures. This is because the appearance of the first corrosion crack is usually used to define the end of functional service life where rehabilitation of a corroding structural element is required [2,3]. The bond strength at the steel-to-concrete interface of a corroding RC element initially increases with the increase of corrosion up to cracking of concrete cover after which time the bond strength decreases as corrosion progresses [4]. This means prediction of time to corrosion cracking is also crucial to determine the relationship between bond strength and degree of corrosion.

1.1. Summary of previous studies

1.1.1. Models for service life prediction of corroded structures

A conceptual model for service life prediction of corroded RC structures developed by Tuutti [2] is shown in

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Nomenclature

a	internal radius of the cylinder
a_s	surface area of the steel reinforcing bar
b	exterior radius of the cylinder
A_b	original cross-sectional area of the steel reinforcing bar
C	clear concrete cover (mm)
D	diameter of steel reinforcing bar (mm)
E_c	elastic modulus of concrete
E_{ef}	effective elastic modulus of concrete
f'_c	compressive strength of concrete
f_{ct}	tensile strength of concrete
F	Faraday's constant
i_{cor}	corrosion rate
i	current density
I	current (A)
k	hole flexibility
M	atomic mass of metal
P	internal radial pressure
P_{cor}	internal radial pressure caused by corrosion
P_{cr}	internal radial pressure that causes cracking of concrete
T	time
T_{cr}	time from corrosion initiation to corrosion cracking
T_{free}	time required for corrosion products to fill a porous zone around a steel reinforcing bar
T_0	time required for CO_2 or Cl^- ions to diffuse to the steel-to-concrete interface and activate corrosion
T_{stress}	time through which the stress builds-up
m_l	percentage steel mass loss
M_l	mass of steel lost in time T
M_{loss}	mass of steel per unit length consumed to produce M_r
M_r	mass of rust per unit length
M_{st}	original mass of steel per unit length before corrosion damage
z	ionic charge (2 for $Fe \rightarrow Fe^{2+} + 2e^-$)
δ	internal radial displacement
δ_c	displacement in concrete
δ_l	thickness of steel lost to form rust
δ_0	thickness of porous zone
δ_r	thickness of rust
γ	ratio of molecular mass of steel to molecular mass of rust
ϕ_{cr}	concrete creep coefficient (2.35)
ν	Poisson's ratio (0.18 for concrete)
ρ_s	mass density of steel
ρ_r	mass density of rust
ψ	factor depends on D, C and δ_0

Fig. 1. According to this model, there are two distinct periods in deterioration caused by corrosion. The first is the initiation period, T_0, which represents the time required for CO_2 or Cl^- ions to diffuse to the steel-to-concrete interface and activate corrosion. The second is the propagation period, T_{cr}, which represents the time between corrosion initiation and corrosion cracking.

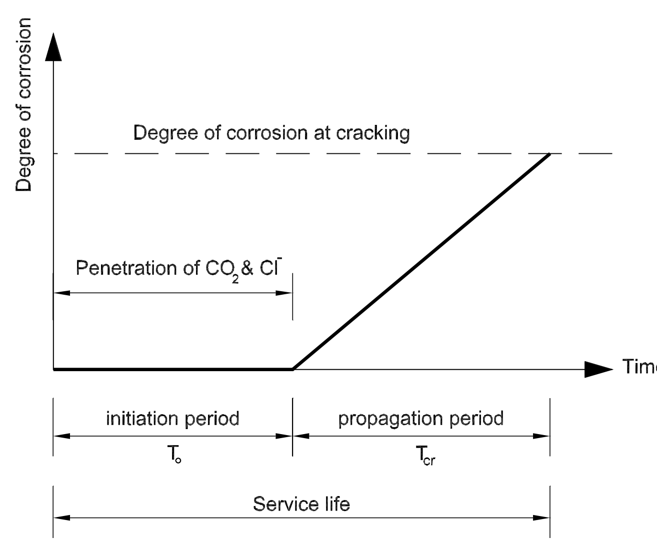

Fig. 1. Service life model of corroded structures [2].

Weyers [3] reported that not all corrosion products contribute to the expansive pressure on the concrete; some of them fill the voids and pores around the steel reinforcing bar and some migrates away from the steel-to-concrete interface through concrete pores. The researcher concluded that the conceptual model presented by Tuutti [2] underestimates the time to corrosion cracking compared with times obtained from field and laboratory observations. The researcher reported that there is a porous zone around the steel reinforcing bar caused by the transition from cement paste to steel, entrapped/entrained air voids, and capillary voids in the cement paste into which corrosion products diffuse. Fig. 2 shows a schematic diagram of the corrosion-cracking process as proposed by Weyers [3]. Fig. 3 shows a modified service life model in which the propagation period, T_{cr}, is divided into two different periods. The first one is the free expansion period, T_{free}, corresponding to the time required for corrosion products to fill the porous zone around the corroding steel reinforcing bar. The second period encompasses the time in which the stress builds-up, T_{stress}, as corrosion products, having filled the porous zone, exert an expansive pressure on the surrounding concrete. The model assumes that this pressure increases linearly as the volume of corrosion products increases until the internal tensile stresses exceed the tensile

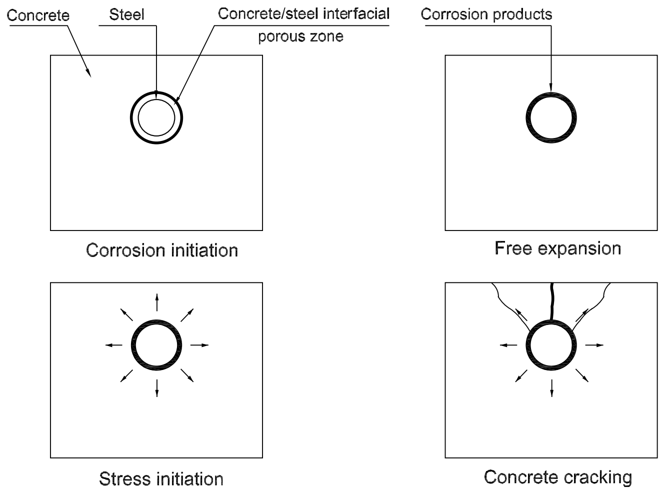

Fig. 2. Corrosion and cracking process [3].

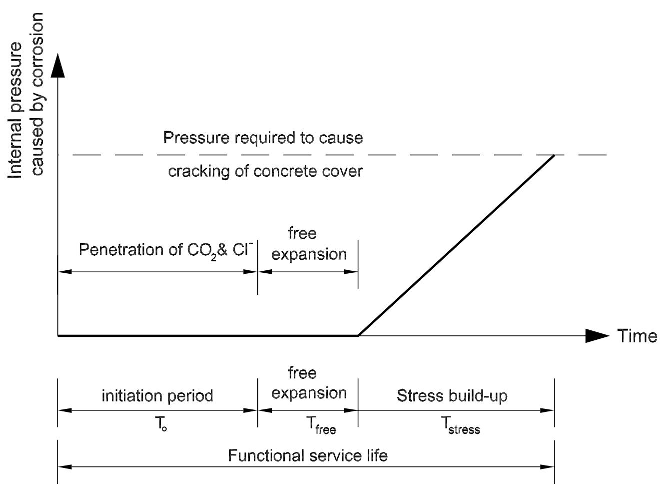

Fig. 3. Modified service life model.

strength of concrete at which time cracking of concrete cover occurs. It should be noted that this assumption is valid up to the point of cracking, after which time pressure distribution is not uniform. Cover cracking marks the end of functional service life of a corroded structure where structural rehabilitation is needed.

1.1.2. Models for time to corrosion cracking

The time required for corrosion initiation corresponds to the time required for CO_2 or Cl^- ions to diffuse to the steel-to-concrete interface and activate corrosion. Extensive research work has been devoted to develop models that predicts the time for corrosion initiation. Different models, validated in field studies, for the rate of carbonation progression and chloride ingress which predict the time for corrosion initiation can be found in the literature [5,6]. Some researchers intended to model the cracking behavior caused by corrosion using nonlinear fracture mechanics and/or finite element analysis [7–9]. In these models crack propagation was governed by energy considerations. It was concluded that stable crack growth can occur prior to reaching the surface of the concrete. This implied that additional expansion of corrosion products is necessary, beyond the expansion required for crack initiation. Although these models would better represent the cracking behavior, they might be too complicated to be used by practicing engineers. Few other researchers have attempted to develop simple mathematical models for prediction of time from corrosion initiation to corrosion cracking [10–12]. A summary of these models is presented hereafter.

Bazant [10] suggested a mathematical model to calculate the time between corrosion initiation and corrosion cracking of RC bridge decks. According to Bazant's model, the time from corrosion initiation to corrosion cracking is mainly dependent on corrosion rate, cover depth, spacing between steel reinforcing bars, diameter of the steel reinforcing bar, and properties of concrete such as tensile strength, modulus of elasticity, Poisson's ratio, and creep coefficient. Bazant's model assumes that all corrosion products create pressure on the surrounding concrete which would underestimate the time to corrosion cracking [11].

The work of Bazant [10] was extended by Liu and Weyers [11]. Liu and Weyers modeled the time from corrosion initiation to corrosion cracking based on the amount of corrosion products required to cause cracking of concrete cover. The model includes same parameters used in Bazant's model but it takes into account the time required for corrosion products to fill a porous zone around the steel reinforcing bar before creating an internal pressure on the surrounding concrete. In Liu–Weyers's model, the rate of steel mass loss caused by corrosion was assumed to decrease as time progresses. The rate of steel mass loss was assumed to be directly proportional to the square root of the product of the corrosion current and the time of corrosion exposure. For the same time of corrosion exposure, this assumption significantly underestimates the amount of steel weight loss compared with that obtained by using the well known Faraday's law [13]. Underestimating the rate of steel loss caused by corrosion would result in overestimating the time to corrosion cracking.

Morinaga [12] proposed an empirical equation (Eq. (1)) based on field and laboratory data to predict the time from corrosion initiation to corrosion cracking

$$T_{\mathrm{cr}} = \frac{0.602D\left(1+2\frac{C}{D}\right)^{0.85}}{i_{\mathrm{cor}}} \tag{1}$$

where T_{cr} is the time from corrosion initiation to corrosion cracking (days), D is the steel bar diameter (mm), C is the clear concrete cover (mm), and i_{cor} is the corrosion rate (10^{-4} g/cm^2/day). According to Morinaga's equation, the time from corrosion initiation to corrosion cracking is function of corrosion rate, concrete clear cover, and steel bar diameter. It is evident that the empirical equation proposed by Morinaga [12] does not account for the mechanical properties of concrete which would significantly affect the time to corrosion cracking. Accordingly, Morinaga's equation cannot be generalized to predict the time from corrosion initiation to corrosion cracking for a concrete having various mechanical properties.

2. Research significance

This paper presents a simple mathematical model that can be used by practicing engineers and researchers to reasonably predict the time from corrosion initiation to corrosion cracking. Cover cracking marks the end of functional service life of a corroding RC structure where rehabilitation is required. The primary deficiencies of previous models are considered while developing the present model. The assumptions used in model formulations along with derivations for the model's main components are presented. The accuracy of the model is demonstrated by comparing the model's predictions to experimental data published in the literature.

3. Corrosion and expansion in concrete

Four basic assumptions were used in the present model to determine the internal radial pressure caused by the expansion of corrosion products:

- The corrosion products are formed uniformly around the steel reinforcing bar which results in a uniform expansive stresses around the steel bar. This assumption is typically used and widely accepted while modeling the volume expansion caused by corrosion [10–12].
- There is a porous zone around the steel reinforcing bar which the corrosion products must first fill before they start to induce internal pressure on the surrounding concrete. Weyers [3] reported that not all corrosion products contribute to the expansive pressure on the concrete; some of them fill the voids around the corroding steel reinforcing bar. Al Khalaf and Page [14] confirmed the presence of a porous layer of cement paste at the steel-to-concrete interface. Park and Paulay [15] reported that there is a soft and spongy porous layer of concrete forms around the steel reinforcing bars caused by bleeding of water under coarse aggregate particles and under the steel reinforcement.
- The volume expansion caused by corrosion creates strain only in concrete (i.e. strain in steel is neglected). This assumption is reasonable because the Young's modulus of steel is about one order of magnitude higher than that of concrete, and hence steel deformation would be small enough to be neglected compared with concrete deformation. The mechanical properties of concrete adjacent to the steel reinforcing bar are assumed to be the same as those of the bulk concrete. These assumptions were typically used by other researchers to predict internal pressure caused by corrosion [10,11,16,17].
- The concrete around the steel reinforcing bar is modeled as a thick-walled cylinder with a wall thickness equal to the thinnest concrete cover. The concrete ring is assumed to crack when the tensile stresses in the circumferential direction at every part of the ring have reached the tensile strength of concrete [4,10,11,18,19].

3.1. Internal radial pressure caused by corrosion

A uniform layer of corrosion products would create uniform expansive stresses at the interface surface between steel and concrete that results in a uniform radial displacement at the surface of the rust layer. In this section the relationship between the percentage steel mass loss and the corresponding radial pressure on concrete is determined. Considering the concrete around the bar to be a thick-walled concrete cylinder of a homogeneous material, the radial pressure, P_{cor}, required to produce a concrete displacement, δ_{c}, necessary to accommodate the increased volume as steel changes to rust is given by

$$\delta_{\mathrm{c}} = kP_{\mathrm{cor}} \tag{2}$$

where k is the hole flexibility constant that relates the radial displacement to the internal pressure acting on the thick-walled concrete cylinder. Since there is a porous zone at the steel-to-concrete interface, the corrosion products must first fill this zone before their expansion starts to create pressure on the surrounding concrete. This porous zone will be included in the derivation of the hole flexibility, k, from Eq. (3) that can be found in standard text books on elasticity (e.g. [20])

$$\delta = \frac{a}{E_{\mathrm{ef}}}\left[\frac{a^2+b^2}{b^2-a^2}+v\right]P \tag{3}$$

where δ is the internal radial displacement, P is the corresponding internal radial pressure, E_{ef} is the effective elastic modulus of a thick-walled cylinder, a is the internal radius of the cylinder, b is the exterior radius of the cylinder, and v is the Poisson's ratio of the cylinder. Assuming $D + 2\delta_0 = D'$ as D is the diameter of the steel reinforcing bar and δ_0 is the thickness of the porous zone and considering $a = D'/2$ and $b = C + D'/2$ where C is the wall thickness of the cylinder, Eq. (3) can be rewritten as follows:

$$\delta = \frac{D'}{2E_{\mathrm{ef}}}\left[\frac{D'^2}{2C(C+D')}+1+v\right]P \tag{4}$$

and then considering, $\psi = D'^2/2C(C+D')$, Eq. (3) will take the form:

$$\delta = \frac{D'}{2E_{\mathrm{ef}}}[\psi+1+v]P \tag{5}$$

Hence, the hole flexibility, k, is given by

$$k = \frac{(1+v+\psi)D'}{2E_{\mathrm{ef}}} = \frac{(1+v+\psi)(D+2\delta_0)}{2E_{\mathrm{ef}}} \tag{6}$$

The change in the diameter of the steel reinforcing bar caused by corrosion is shown in Fig. 4. The prediction of the internal radial pressure caused by corrosion requires

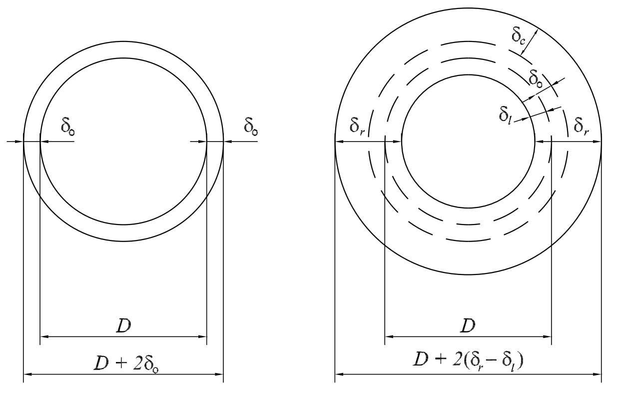

Fig. 4. Change in reinforcing bar diameter caused by corrosion.

a determination of thickness of rust, δ_r, thickness of steel lost to form rust around the steel reinforcing bar, δ_l, and thickness of the porous zone, δ_0. Let, M_r, denotes the mass of rust per unit length of one bar and, M_{loss}, denotes the mass of steel per unit length consumed to produce, M_r. As shown in Fig. 4, the formation of the rust layer results in increasing the bar diameter from $[D]$ to $[D + 2(\delta_r - \delta_l)]$. Equating the increase in volume per unit length, calculated from the difference between volume of rust produced and volume of steel consumed per unit length to the change in area expressed in terms of diameter change gives:

$$\underbrace{\frac{M_r}{\rho_r}}_{\text{Rust produced}} - \underbrace{\frac{M_{loss}}{\rho_s}}_{\text{Steel consumed}} = \frac{\pi}{4}\left[[D + 2(\delta_r - \delta_l)]^2 - D^2\right] \tag{7}$$

where ρ_r is the mass density of rust, ρ_s is the mass density of the steel, and D is the original diameter of the steel reinforcing bar. The thickness of rust, δ_r, can be expressed as follows:

$$\delta_r = \delta_l + \delta_0 + \delta_c \tag{8}$$

where δ_c is displacement in concrete that creates a radial pressure P_{cor}. Considering Eq. (8), Eq. (7) can be rewritten as follows:

$$\frac{M_r}{\rho_r} - \frac{M_{loss}}{\rho_s} = \frac{\pi}{4}\left[4D(\delta_0 + \delta_c) + 4(\delta_0 + \delta_c)^2\right] \tag{9}$$

The term, $4(\delta_0 + \delta_c)^2$, can be neglected in the calculations as $(\delta_0 + \delta_c)^2 \ll D$, and hence Eq. (9) will take the form:

$$\frac{M_r}{\rho_r} - \frac{M_{loss}}{\rho_s} = \pi D(\delta_0 + \delta_c) \tag{10}$$

Several researchers [7,10,11] have expressed the relationship between M_r and M_{loss} as follows:

$$M_{loss} = \gamma M_r \tag{11}$$

where γ is the ratio of molecular mass of steel to molecular mass of rust which ranges from 0.523 to 0.622. Combining Eqs. (2), (6), (10) and (11), with $\gamma = 0.622$, the radial pressure caused by corrosion, P_{cor}, is given by

$$P_{cor} = \frac{2M_{loss}E_{ef}[(1/0.622\rho_r) - (1/\rho_s)]}{\pi D(1 + \nu + \psi)(D + 2\delta_0)} - \frac{2\delta_0 E_{ef}}{(1 + \nu + \psi)(D + 2\delta_0)} \tag{12}$$

Negative values of P_{cor} in Eq. (12) represent the period in which rust fills the voids around the bar, are set equal to zero.

3.2. Relationship between percentage steel mass loss and internal pressure caused by corrosion

To determine the relationship between the percentage steel mass loss, m_l, and the internal pressure caused by corrosion, the ratio of mass density of rust, ρ_r, to mass density of the original steel, ρ_s, is needed. The ratio suggested by other researchers [7,11] that is $\rho_r = 0.5\rho_s$ will be adopted in the current model. Substituting $\rho_r = 0.5\rho_s$ in Eq. (12), the internal radial pressure caused by corrosion can be expressed as follows:

$$P_{cor} = \frac{4.4(M_{loss}/\rho_s)E_{ef}}{\pi D(1 + \nu + \psi)(D + 2\delta_0)} - \frac{2\delta_0 E_{ef}}{(1 + \nu + \psi)(D + 2\delta_0)} \tag{13}$$

The percentage steel mass loss, (m_l), is given by

$$m_l = 100\left(\frac{M_{loss}}{M_{st}}\right) \tag{14}$$

where M_{loss} is the mass of steel per unit length consumed to form rust, and M_{st} is the original mass of steel per unit length before corrosion damage.

In order to express the internal radial pressure caused by corrosion as a function of the percentage steel mass loss, (m_l), the following relationship is used:

$$\left(\frac{M_{loss}}{\rho_s}\right) = \left(\frac{M_{loss}}{M_{st}}\right)\left(\frac{M_{st}}{\rho_s}\right) = \frac{(m_l)(A_b)}{100} = \frac{(m_l)(\pi D^2)}{400} \tag{15}$$

where A_b is the original cross-sectional area of the steel reinforcing bar. Combining Eqs. (13) and (15), the relationship between the percentage steel mass loss and the internal radial pressure caused by corrosion is given by

$$P_{cor} = \frac{m_l E_{ef} D}{90.9(1 + \nu + \psi)(D + 2\delta_0)} - \frac{2\delta_0 E_{ef}}{(1 + \nu + \psi)(D + 2\delta_0)} \tag{16}$$

Negative values of P_{cor} in Eq. (16) represent the period in which rust fills the voids around the bar, are set equal to zero.

4. Governing criterion for the pressure at cracking

A concrete ring model is shown in Fig. 5. As explained previously, the concrete around the steel reinforcing bar is modeled as a thick-walled cylinder with a wall thickness equal to the thinnest concrete cover. The concrete ring is

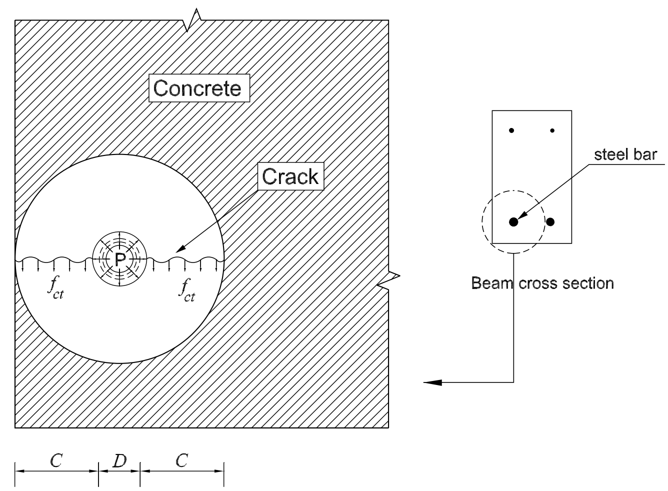

Fig. 5. Concrete ring model.

assumed to crack when the tensile stresses in the circumferential direction at every part of the ring have reached the tensile strength of the concrete. The radial pressure required to cause cracking of concrete cover, P_{cr}, is then given by

$$P_{\mathrm{cr}}[D+2(\delta_0+\delta_{\mathrm{c}})] = 2[C-(\delta_0+\delta_{\mathrm{c}})]f_{\mathrm{ct}} \tag{17}$$

where f_{ct} is the concrete tensile strength. Assuming, $[D+2(\delta_0+\delta_{\mathrm{c}})] = D$, and $[C-(\delta_0+\delta_{\mathrm{c}})] = C$ as, $2(\delta_0+\delta_{\mathrm{c}}) \ll D$, and $(\delta_0+\delta_{\mathrm{c}}) \ll C$, the radial pressure that causes cracking, P_{cr}, is given by

$$P_{\mathrm{cr}} = \frac{2Cf_{\mathrm{ct}}}{D} \tag{18}$$

The governing equation at cracking can then be determined by equating P_{cor} from Eq. (16) to P_{cr} from Eq. (18)

$$\frac{m_{\mathrm{l}}E_{\mathrm{ef}}D}{90.9(1+v+\psi)(D+2\delta_0)} - \frac{2\delta_0 E_{\mathrm{ef}}}{(1+v+\psi)(D+2\delta_0)} = \frac{2Cf_{\mathrm{ct}}}{D} \tag{19}$$

5. Time to corrosion cracking

The well known Faraday's law will be used to introduce the time in Eq. (19). Several researchers have used Faraday's law to estimate steel mass loss from monitored current density [21–24]. The ability of Faraday's law to predict the actual steel mass loss at different current density levels was confirmed in an earlier study carried by the authors [25].

$$M_{\mathrm{l}} = \frac{MIT}{zF} \tag{20}$$

where T is the time (s), M_{l} is the mass of steel lost in time $T(g)$ to form rust, I is the current (A), F is Faraday's constant (96,500 A s), z is the ionic charge (2 for $Fe \rightarrow Fe^{2+} + 2e^-$), and M is the atomic mass of the metal (56 g for Fe). Hence, the relationship between the time, T, and the mass of steel consumed to form rust, M_{l}, is given by

$$T = \frac{24125(M_{\mathrm{l}}/a_{\mathrm{s}})}{7i} \tag{21}$$

where i is the current density (A/cm^2), and a_{s} is the surface area of the steel reinforcing bar. The ratio $(M_{\mathrm{l}}/a_{\mathrm{s}})$ can be expressed as follows:

$$\frac{M_{\mathrm{l}}}{a_{\mathrm{s}}} = \frac{(m_{\mathrm{l}})(\pi D^2/4)\rho_{\mathrm{s}}}{100(\pi D)} = \frac{m_{\mathrm{l}}D\rho_{\mathrm{s}}}{400} \tag{22}$$

where m_{l} is the percentage steel mass loss, and ρ_{s} is the mass density of steel.

Combining Eqs. (21) and (22) with $\rho_{\mathrm{s}} = 7.85$ g/cm^3, and adjusting the units Eq. (21) can be rewritten as follows:

$$T = \frac{78.3 m_{\mathrm{l}} D}{i} \tag{23}$$

where T is the time (days), m_{l} is the percentage steel mass loss, D is the diameter of the steel reinforcing bar (mm), and i is the current density (μA/cm^2).

Combining Eqs. (19) and (23), the time from corrosion initiation to corrosion cracking, T_{cr}, is given by

$$T_{\mathrm{cr}} = \left[\frac{7117.5(D+2\delta_0)(1+v+\psi)}{iE_{\mathrm{ef}}}\right]\left[\frac{2Cf_{\mathrm{ct}}}{D} + \frac{2\delta_0 E_{\mathrm{ef}}}{(1+v+\psi)(D+2\delta_0)}\right] \tag{24}$$

where E_{ef} is the effective elastic modulus of concrete that is equal to $[E_{\mathrm{c}}/(1+\phi_{\mathrm{cr}})]$, E_{c} is the elastic modulus of concrete, ϕ_{cr} is the concrete creep coefficient (2.35 as per the CSA Standard A23.3-94 [26]), and v is the Poisson's ratio of concrete (0.18).

The thickness of the porous zone is typically in the range of 10–20 μm [27]. In the present model lower and upper bounds for the time from corrosion initiation to corrosion cracking will be calculated with δ_0 equal to 10 and 20 μm, respectively.

6. Examination of the model's accuracy

To asses the accuracy of the proposed model, experimental results published in the literature will be compared to the results predicted by the proposed model. Test parameters of the experimental data included various concrete covers, bar diameters, and current density levels. Eqs. (25) and (26) are used to determine the tensile strength and the Young's modulus of concrete, respectively, when they are not reported in the literature. Carasquillo et al. [28] proposed Eq. (25) to predict the concrete modulus of rupture for a concrete strength in the range of 20–83 MPa. Legeron and Paultre [29] verified Eq. (25) based on a statistical study on 395 data points from a large number of research programs. Eq. (26) is recommended by the CSA Standard [26].

$$f_{\mathrm{ct}} = 0.94\sqrt{f'_{\mathrm{c}}}\ \mathrm{MPa} \tag{25}$$

$$E_{\mathrm{c}} = 4500\sqrt{f'_{\mathrm{c}}}\ \mathrm{MPa} \tag{26}$$

where f'_{c} is the compressive strength of concrete.

Table 1
Parameters of the experiments used for demonstration of model's accuracy

Reference	Bar diameter (mm)	Cover thickness (mm)	Current density ($\mu A/cm^2$)	Tensile strength (MPa)	Young's modulus (MPa)
Andrade et al. [21]	16	20	100	3.55	22[c]
Mangat and Elgarf [23]	10	20	800	6.30[a]	30[a]
Cabrera and Ghoddoussi [30]	12	69	244[b]	6.97[a]	33[a]
Alonso et al. [31]	16	20	100	3.85	22[c]
	16	50	100	3.85	22[c]
	16	70	100	3.85	22[c]
	16	70	10	3.85	22[c]
El Maaddawy et al. [32]	16	33	150	4.9	28[a]

[a] Calculated value based on Eq. (25) and/or Eq. (26).
[b] Current density was estimated using reported bar diameter, time to corrosion cracking, and steel mass loss at cracking based on Faraday's law.
[c] Calculated value based on Eq. (26) assuming concrete compressive strength of 25 MPa.

In all experimental studies used to demonstrate the model's accuracy, chloride ions were added to the mixing water during construction of test specimens in order to provoke corrosion, and hence no time was required for chloride ingress. Accordingly, all observed times to corrosion cracking are times from corrosion initiation to corrosion cracking. All observed times were compared to the lower and upper bounds predicted by the proposed model. Test parameters of the experiments used as input data in the present model are given in Table 1 while all observed and predicted times from corrosion initiation to corrosion cracking are given in Table 2. From Table 2 it can be seen that most of the observed times from corrosion initiation to corrosion cracking fall within the range predicted by the proposed model. The data reported by Alonso et al. [31] confirmed that the time from corrosion initiation to corrosion cracking increases with the increase in concrete cover or with the decrease in current density level which agreed with the model's predictions. However, some of the results observed by Alonso et al. [31] were slightly higher than the upper limit predicted by the model (9–11% higher). This might be ascribed to a migration of some of corrosion products away from the steel-to-concrete interface through concrete pores which reduced the internal radial pressure thereby increasing the time from corrosion initiation to corrosion cracking. Another reason that might have contributed to this slight increase in the observed time from corrosion initiation to corrosion cracking is that crack initiation normally does not lead to simultaneous crack propagation to the surface. The need for an additional displacement to drive the crack would lengthen the actual time required for the appearance of crack on concrete surface. Considering the complexity of the corrosion process, it is evident that the model can give reasonable prediction for the time from corrosion initiation to corrosion cracking.

Table 2
Comparison between predicted and experimental results

Reference	Observed time[a] (h)	Predicted time[a] (h)	Remarks on observed time
Andrade et al. [21]	96	84–119	Within limit
Mangat and Elgarf [23]	14.4	12–16	Within limit
Cabrera and Ghoddoussi [30]	108	96–110	Within limit
Alonso et al. [31]	113	89–123	Within limit
	208	156–191	9% Higher[b]
	264	203–237	11% Higher[b]
	2643	2028–2372	11% Higher[b]
El Maaddawy et al. [32]	95	78–101	Within limit

[a] The time is from corrosion initiation to corrosion cracking.
[b] Compared with predicted upper limit.

7. Conclusion

In this paper a mathematical model that can predict the time from corrosion initiation to corrosion cracking was developed. The model accounts for the time required for corrosion products to fill a porous zone around the steel reinforcing bar before they start inducing expansive pressure on the surrounding concrete. In the present model a relationship between the steel mass loss and the internal radial pressure caused by corrosion was developed. The concrete around the steel reinforcing bar was modeled as a thick-walled cylinder with a wall thickness equal to the thinnest concrete cover. The concrete ring was assumed to crack when the tensile stresses in the circumferential direction at every part of the ring had reached the tensile strength of the concrete. The internal radial pressure at cracking was then determined and related to the steel mass loss. Faraday's law was then utilized to predict the time from corrosion initiation to corrosion cracking. A comparison of model's predictions with experimental results published in the literature showed that the model can give reasonable prediction for the time from corrosion initiation to corrosion cracking.

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