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Influence of high strain rates on stress–strain relationship, strength and elastic modulus of concrete

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Abstract

Earlier papers showed a method for strength evaluation at high strain rate from the results of static tests, and a method for evaluating modulus of elasticity at static loading from tests based on stress-wave propagation. This paper presents new results on the influence of high strain rates on the stress–strain relationship, strength and modulus of elasticity using the thermofluctuation theory, principle of accumulation and development damages as well as nonlinear behavior of concrete under loading. Obtained equations explain and unify the influence of high rates on the mechanical properties of concrete for different kinds of uniaxial stress. Comparative study showed again substantial agreement with existing experimental results and general equations by the Comité Euro-International du Béton (CEB) Model Code. The developed approach diminishes the labor expenses for tests of concrete under high rates of loading.

Keywords: Stress–strain relationship; Strength; Elastic moduli; Nonlinear deformation

1. Introduction

As well known, strength of concrete depends on the rate of loading, and the values of strength increase as the stress or strain rate increases [1–11]. For describing this phenomenon many investigators used different models based on the linear and nonlinear fracture mechanics [12,13]. But the delayed failure, in which a specimen with a crack is loaded by a constant stress that is less than the critical load, ‘‘is not described by the classical concept of the mechanics of brittle fracture’’ [14]. Thus obtained experimental and theoretical data proved that the fracture of concrete does not occur as a critical event. It is rather the result of a process of nucleation, accumulation, and development of damage elements, i.e. time depended or kinetic phenomenon [15,16]. In accordance with the kinetic nature of solid strength, at the atomic–molecular level this process is controlled by events of breaking of interatomic bonds by thermal fluctuations [15,16]. It leads, in particular, to the dependence between the rate of fracture and temperature of concrete [17–19]. At the macroscopic level the fracture of stressed concrete proceeds by a process that develops in the body from the moment of load application, while the fracture itself is the final act in this process. However, once a structure is damaged under loading the elastic properties cannot be constant. Indeed, the nonlinear stress–strain behavior of concrete in direct compression, tension and flexure is well established from the experimental results at different rates of loading [1,2,11,20,21].

Therefore the main goal of this paper is to unify the influence of the rate of loading on the stress–strain relationship, the strength and modulus of elasticity, using the conceptual statements of the kinetic nature of solid strength (thermofluctuation theory) and nonlinear behavior of concrete.

2. Background

The approach to the solution of this problem is based on the simple and natural consideration that damage accumulates while a stressed body is prepared for fracture. If the

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applied loading represents an arbitrary sequence of stress σ in the time t, i.e. $\sigma = \sigma(t)$, then the principle of summation of damage leads to Bailey's condition [16,22], which is the criterion for fracture of a solid:

$$\int_0^{t_r} \frac{dt}{\tau[\sigma(t), T]} = 1, \tag{1}$$

where t_r is time until fracture, T – absolute temperature. Value of expression (1) consists that it summarizes the "parts" of lifetime at loadings, i.e. is relative fractions, for various values of σ, of the lifetime-under-loading. It corresponds to summation of the damage got from various values of loading.

Based on Eq. (1) and the kinetic nature of fracture of concrete, it was shown that under conditions of varying the loading rate in a range of 5–6 decimals orders the value of the stress (σ_r) at the time of fracture is a linear function of $\ln(\dot{\sigma})$ – rate of loading [23]:

$$\sigma_r = \frac{kT}{\gamma} \ln\left(\frac{\dot{\sigma}}{\dot{\sigma}_0}\right), \tag{2}$$

where γ – structurally sensitive coefficient; k – Boltzmann constant; $kT = 404 \times 10^{-23}$ J (2.44 kJ/mol) at the usual temperature of test ($T = 293$ K), $\log \dot{\sigma}_0 = -8.5$ ($\dot{\sigma}_0$ in MPa/s).

For large rate of loading σ_r is the dynamic strength, for small rate of loading σ_r is the quasi-static strength (σ_s). And

$$\gamma = (kT/\sigma_r)\ln(\dot{\sigma}/\dot{\sigma}_0). \tag{3}$$

It is known also that the structurally sensitive coefficient γ of solids can be estimated as [24]:

$$\gamma = U_0(\varepsilon_c/E_0), \tag{3a}$$

where U_0 is the activation energy of destruction; ε_c is the strain of breaking the interatomic bonds; E_0 – tangent modulus of elasticity.

The expressions ((2) and (3a)) were derived on the assumption that the coefficient γ constant in time of loading. In reality this coefficient can change in the course of the loading, since γ is related to the structure of concrete, and the fracture process entails the development of different defects (micro- and macro-crevices) in the loaded specimen [25]. Thus, in addition to Eq. (3) another means of estimating the value of γ for concrete can be obtain by using its relationship with the nonlinear parameter "b" in the parabolic equation of state:

$$\sigma = E_0\varepsilon - b\varepsilon^2, \quad \text{where } b = E_0^2/4\sigma_s = E_0/2\varepsilon_c$$
$$\text{at quasi-static loading.} \tag{4}$$

In this case the structurally sensitive coefficient γ equal [26]:

$$\gamma = W_a b/E_0 \sim (1/2)W_a(dE/d\sigma) \tag{5}$$

which describes its dependence on the change of the modulus of elasticity with stress, $dE/d\sigma$. $W_a \sim U_0/E_0 \sim 10^{-2}$ nm^3 is the volume of atom, $U_0 = 124$ kJ/mol for concrete.

Simultaneously Eq. (3) allows rearranging Eq. (5) for quasi-static loading ($\dot{\sigma}_0 = \dot{\sigma}_s$) in the form

$$b = \gamma(E_0/W_a) = \left(\frac{E_0}{W_a}\right)(kT/\sigma_r)\ln\left(\frac{\dot{\sigma}}{\dot{\sigma}_s}\right). \tag{6}$$

$\dot{\sigma}_s$ is the stress rate at quasi-static loading, and $\dot{\sigma} > \dot{\sigma}_s$.

Use of the thermofluctuation theory and nonlinear equation (4) allowed obtaining also the dynamic–static strength ratio σ_d/σ_s [27,28]:

$$\sigma_d/\sigma_s = 1 + (kT/U_0)(E_0/\sigma_s dE/d\sigma)\ln \dot{\varepsilon}_d/\dot{\varepsilon}_s, \tag{7}$$

where σ_d, σ_s, $\dot{\varepsilon}_d$, $\dot{\varepsilon}_s$ – strength and strain rate at dynamic and quasi-static loading, respectively. For a parabola ($E_0/\sigma_s dE/d\sigma) \sim 2$.

But, as was mentioned above, the fracture of solids is a process of nucleation, accumulation, and development of damage elements, i.e. time depended phenomenon. For concrete as an inhomogeneous precracked material we can consider both processes (accumulation, and development) at any level of loading. Obviously, up to the certain values of stress rate (time of loading) the processes connected with accumulation of defects prevail in the comparison with the processes connected with development of defects. As a consequence the internal stress of concrete decreases. At the certain values of stress (strain) rate of loading processes connected with development of defects start to prevail. The internal stress has no time to relax (inertia effects), and continues to increase with increasing of loading rate. Accordingly, it seems reasonable to assume that the nonlinear parameter "b" in Eq. (4) consists of two parts. The first part reflects such phenomenon (e.g. internal cracking) which decreases ($b < 0$) the stress in concrete, while the second leads to its increase ($b > 0$) at different strain and rate of loading due to the inertial resistance of internal cracking [13]. Therefore, a generalized approach, Eq. (4) may be considered as the relationship between the dynamic and quasi-static stress at different stress or strain rate:

$$\sigma_d = E_0\varepsilon - (E_0^2/4\sigma_s)\varepsilon^2 + \varepsilon^2\left(\frac{E_0}{W_a}\right)(kT/\sigma_s)\ln\left(\frac{\dot{\sigma}_d}{\dot{\sigma}_s}\right), \tag{8}$$

$$\sigma_d = E_0\varepsilon - (E_0/2\varepsilon_c)\varepsilon^2 + (2kT/W_a)\varepsilon^2/\varepsilon_c \ln \dot{\varepsilon}_d/\dot{\varepsilon}_s \tag{8a}$$

with $kT/W_a = 404 \times 10^{-23}$ J/10^{-2} nm$^3 = 404$ MPa at the temperature of test $T = 293$ K.

The physical basis and obtained unified equations allow predicting the stress–strain dependence of concrete, its strength and elastic modulus at different strain (stress) rate.

3. Comparison with existing results

3.1. Stress–strain relationship

It is important to point out that the equation of state (4) coincides with the stress–strain relationship for uniaxial compression of concrete, Eq. (9), recommended by Comité Euro- International du Béton (CEB) Model Code [29]:

$$\sigma = -\frac{\frac{E_0}{E_c}\frac{\varepsilon}{\varepsilon_c} - \left(\frac{\varepsilon}{\varepsilon_c}\right)^2}{1 + \left(\frac{E_0}{E_c} - 2\right)\frac{\varepsilon}{\varepsilon_c}} f'_c \quad \text{for } \varepsilon < \varepsilon_{\lim}, \tag{9}$$

where f'_c – compressive strength (the peak compressive stress); $\varepsilon_c = -0.0022$; E_c – secant modulus from the origin to the peak compressive stress f'_c; $\varepsilon_{\lim}$ – has no significance other than limiting the applicability of Eq. (9).

Indeed, for a parabola, Eq. (4), $\varepsilon_c = 2f'_c/E_0$ and the secant modulus $E_c = f'_c/\varepsilon_c = 0.5E_0$ Substituting these relations into Eq. (9) yields the original equation of state (4).

For comparison purposes, for instance Eq. (8a), with experimental results it is necessary, according to principle of accumulation and development of defects, to make up an arbitrary set of increasing strains and strain rates. As an example of applying Eq. (8a), Table 1 and correspond-

Table 1
Stress–strain relationship for concrete in comparison at different strain rates by Eq. (8a)

Arbitrary strain	Stress (MPa) at arbitrary strain rates (s^{-1})								$\sigma = E\varepsilon$
	10^{-8}	10^{-7}	10^{-5}	$10^{-3.5}$	0.1	1	10	150	
0.00001	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34	0.34
0.00002	0.67	0.67	0.68	0.68	0.68	0.68	0.68	0.68	0.68
0.00004	1.34	1.34	1.34	1.34	1.35	1.35	1.35	1.35	1.36
0.00008	2.63	2.63	2.65	2.66	2.68	2.68	2.69	2.70	2.72
0.00010	3.26	3.27	3.29	3.30	3.33	3.34	3.35	3.36	3.4
0.00012	3.87	3.89	3.92	3.94	3.98	3.99	4.01	4.03	4.08
0.00014	4.48	4.50	4.54	4.57	4.62	4.64	4.66	4.69	4.76
0.00016	5.07	5.10	5.15	5.19	5.26	5.29	5.31	5.35	5.44
0.00018	5.65	5.69	5.76	5.81	5.89	5.93	5.96	6.00	6.12
0.00100	19.59	20.65	22.75	24.34	26.97	28.02	29.08	30.32	34
0.00120	20.05	21.57	24.61	26.88	30.68	32.20	33.71	35.50	40.8
0.00140	19.36	21.43	25.56	28.66	33.82	35.89	37.95	40.38	47.6
0.00160	17.51	20.21	25.61	29.66	36.40	39.10	41.80	44.97	54.4
0.00180	14.52	17.93	24.76	29.89	38,42	41.84	45.25	49.27	61.2
0.00200	10.37	14.58	23.02	29.34	39.88	44.10	48.31	53.27	68

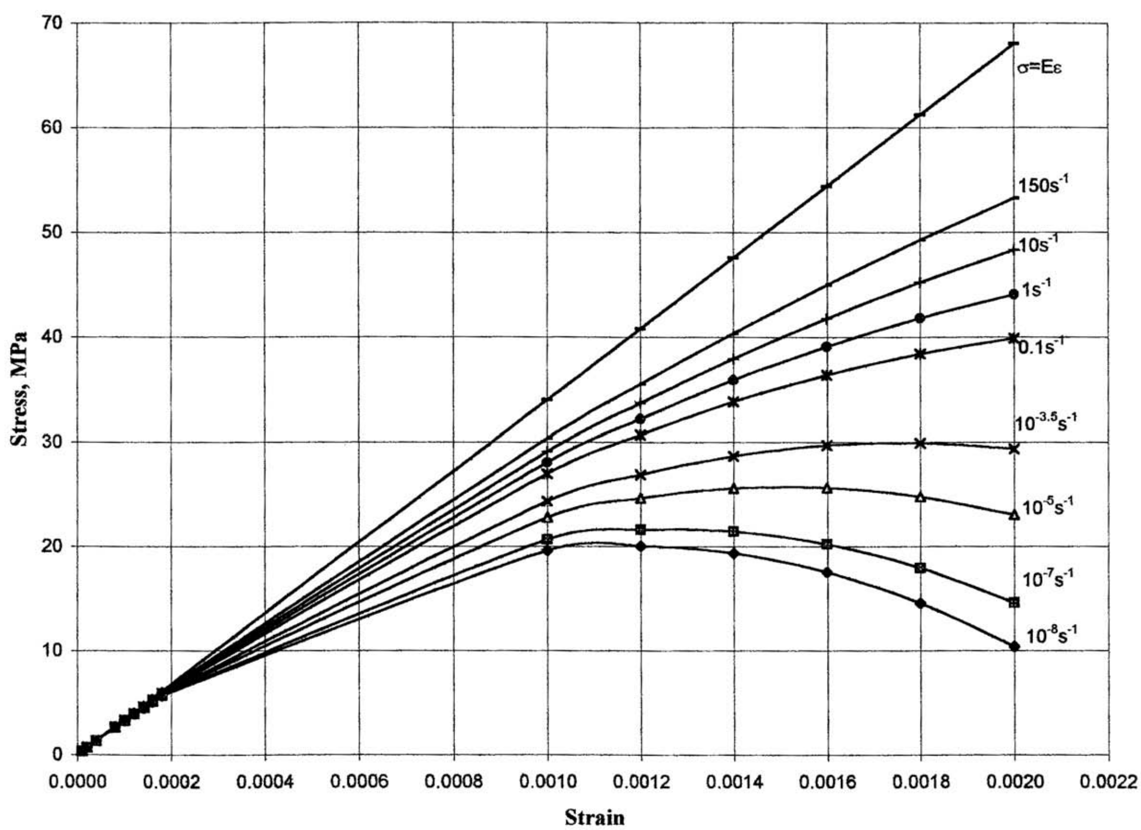

Fig. 1. General stress–strain curves in compression at wide range of strain rates.

ing Fig. 1 show the stress–strain dependences calculated for static and dynamic compressive loading using as a source the values under quasi-static loading ($\dot{\varepsilon}_s = 0.00034\ s^{-1}$) for concrete cylinders $f'_c = 30$ MPa, $E_0 = 34$ GPa [29], i.e. the critical strain $\varepsilon_c = 0.001765$. Presented data show the influence of a combination of strain in the range (10^{-5}–2×10^{-3}) and strain rate in the range (10^{-8}–150) s^{-1} on the values of stress. It can be seen that stress–strain curve remain linear up to higher values of stress. An ideal material, following Hook's law ($\sigma = E\varepsilon$), is not sensitive to the strain rate of loading. It is important to point out also, that the elastic modulus increases with an increase in strain rate, which is generally accepted, and the theoretical stress–strain curves of Fig. 1 are similar to experimental curves obtained by many investigators for concrete in compression at different strain rate in the range (10^{-5}–130) s^{-1} [1,2,5,11,20]. Besides, it can be seen (Table 1), that at strain rate $10^{-3.5}\ s^{-1}$ ($\sim 0.00032\ s^{-1}$) and strain 0.0018 the maximum static stress equals 29.89 MPa (last but one row, 5th column); then at strain rate 150 s^{-1} and the same strain (0.0018) the dynamic stress equals 49.27 MPa (last but one row, 9th column). Thus, the dynamic/static stress ratio equals 1.6.

Using a new measurement technique based on a new algorithm for a stress–strain diagram it was concluded that the nonlinear behavior of concrete under high strain rates does not become more brittle: the ultimate strain, i.e. the strain corresponding to compressive strength, increased of 1.4, although the strength increased of 1.7 [20].

Curbach and Eibl observed also the behavior of the ascending part of the stress–strain diagram for both compressively and tensile loaded concrete, and concluded that the nonlinear behavior of concrete under compressive loading is similar to that observed for tensile loading.

The nonlinear stress–strain behavior of concrete for direct tensile loading observed by different investigators [3,4,9,10,13,17,18,21]. It has been shown [21] that the strength, strain and the secant modulus at peak stress, the limit of proportionality of stress and strain increase with stress rate, but the initial tangent modulus of elasticity is unaffected by stress rate. Similar results based on the thermofluctuation theory and nonlinear model (Eq. (8)) are shown in Fig. 2. This figure and detailed data in Table 2 contain comparison of the stress–strain dependence predicted by Eq. (8) with experimental results for direct tension [21]: strength 4.20 MPa, 5.70 MPa at the stress rate 2.5 MPa/s and 250 MPa/s., tensile strength $f_{ctm} =$ 3.12 MPa at stress rate of quasi-static loading 0.0225 MPa/s, and modulus of elasticity $E_0 = 30$ GPa. Taking into account that the strains at peak stress increase with rate of loading it can be seen that the strengths are 4.14 MPa, 5.77 MPa (Table 2, bold numbers), at 2.5 MPa/s and 250 MPa/s, respectively, i.e. are equal to the experimental data [21]. It proves, in addition, that the nonlinear equations (8 and 8a) can be used to predict mechanical characteristics of concrete at different kind of uniaxial stress.

3.2. Strength

Early investigations [27,28] showed similar approach of evaluation strength of concrete at high rate of loading from

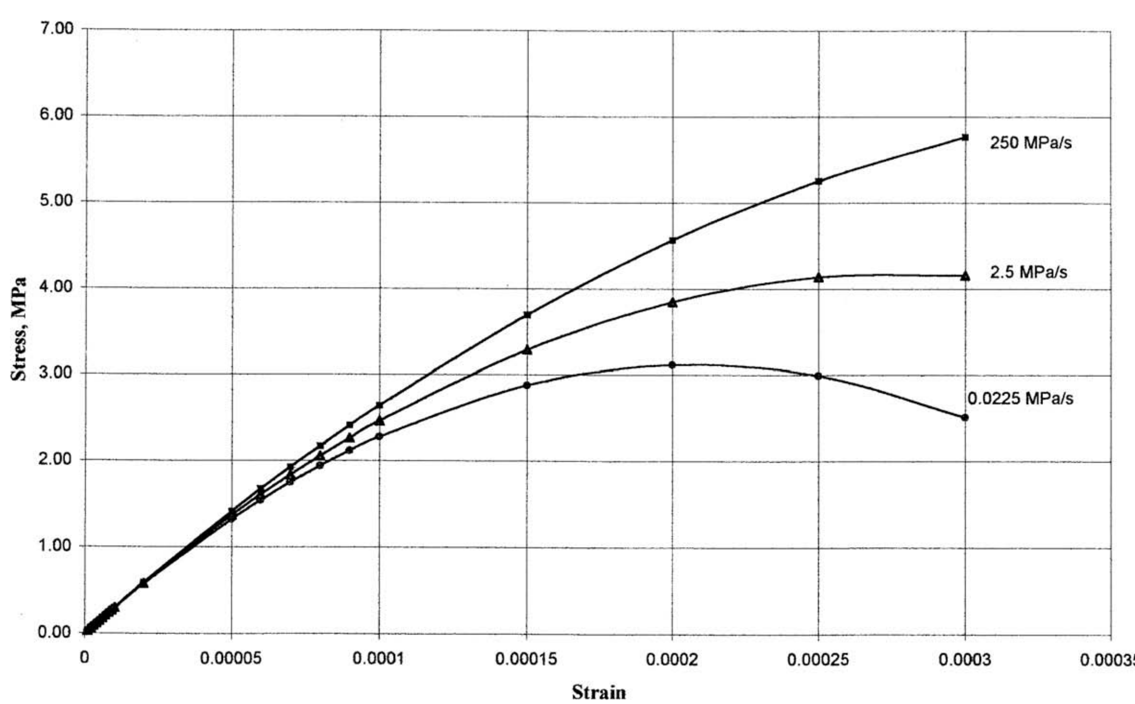

Fig. 2. Stress–strain curves in direct tension at different stress rates.

Table 2
Experimental data [21] and stress–strain relationship in direct tension at different stress rates by Eq. (8)

Strain	Arbitrary		Stress (MPa) at rates (MPa/s)		
	Strain rate (s^{-1})	Stress rate (MPa/s)	0.0225	2.5	250
0.000001	0.00000075	0.0225	0.03	0.03	0.03
0.000002	0.0000015	0.045	0.06	0.06	0.06
0.000003	0.000002	0.06	0.09	0.09	0.09
0.000004	0.0000025	0.075	0.12	0.12	0.12
0.000005	0.000005	0.15	0.15	0.15	0.15
0.000008	0.000065	1.95	0.24	0.24	0.24
0.000009	0.00007	2.1	0.26	0.27	0.27
0.000010	0.000075	2.25	0.29	0.29	0.30
0.000020	0.0001	3	0.57	0.58	0.59
0.000050	0.002	60	1.32	1.37	1.41
0.000060	0.004	120	1.54	1.61	1.67
0.000070	0.005	150	1.75	1.84	1.92
0.000080	0.006	180	1.94	2.06	2.17
0.000090	0.007	210	2.12	2.26	2.41
0.000100	0.008	240	2.28	2.46	2.64
0.000150	0.00835	250.5	2.88	3.29	3.69
0.000200	0.00835	250.5	**3.12**	3.85	4.56
0.000250	0.00835	250.5	2.99	**4.14**	5.25
0.000300	0.00835	250.5	2.51	4.16	**5.77**

results of quasi-static tests. But previous studies did not have demonstrated how can influence on the dynamic strength a combination of the strain and rate of loading. Based on the thermofluctuation theory, nonlinear model, principle of accumulation and development of defects, such combination is very essential for predicting the stress–strain curves at compressive and direct tinsel loading, as it illustrated in the section above. Naturally, these curves offer the possibility to find the peak of stress, i.e. the strength, or dynamic–static strength ratio at different stressing or straining rate. From Eqs. (8) and (8a) follow the expressions for the dynamic–static strength ratios:

$$\sigma_d/\sigma_s = 1 + \varepsilon^2\left(\frac{E_0}{W_a}\right)(kT/\sigma_s)\ln\left(\frac{\dot{\sigma}_d}{\dot{\sigma}_s}\right)/(E_0\varepsilon - (E_0^2/4\sigma_s)\varepsilon^2), \tag{8b}$$

$$\sigma_d/\sigma_s = 1 + (2kT/W_a)\varepsilon^2/\varepsilon_c \ln\dot{\varepsilon}_d/\dot{\varepsilon}_s(E_0\varepsilon - (E_0/2\varepsilon_c)\varepsilon^2). \tag{8c}$$

Figs. 3–8 demonstrate the stress–strain curves, values of dynamic–static strength ratios for two concrete compressive strength 30 MPa and 70 MPa, corresponding moduli of elasticity 34 GPa and 43 GPa under compressive and tensile loading, respectively. Under compressive loading the dynamic–static strength ratios are about 3 and 1.4 at strain 0.0025 and strength 30 MPa, 70 MPa, respectively. Under tensile loading the dynamic–static strength ratios are about 4.3 and 1.4 at strain 0.0002 and strength 2 MPa, 5 MPa.

Eqs. (7) and (8c) agree with Eqs. (10) and (11) recommended by CEB Model Code [29]:

$$\sigma_d/\sigma_s = (\dot{\varepsilon}_d/\dot{\varepsilon}_s)^{1.026\alpha_s} \quad \text{for } \dot{\varepsilon} \leqslant 30\ \text{s}^{-1}, \tag{10}$$

$$\sigma_d/\sigma_s = \eta_s(\dot{\varepsilon}_d/\dot{\varepsilon}_s)^{1/3} \quad \text{for } \dot{\varepsilon} \geqslant 30\ \text{s}^{-1}, \tag{11}$$

$$\alpha_s = \frac{1}{5 + 9\sigma_s/10\ \text{MPa}}, \tag{10a}$$

$$\log\eta_s = 6.156\alpha_s - 2. \tag{11a}$$

Indeed, as was mentioned above, $kT = 2.44$ kJ/mol, activation energy $U_0 = 124$ kJ/mol, $(E_0/\sigma_s dE/d\sigma) \sim 2$. According to CEB [29] one can use static strain rate as $\dot{\varepsilon}_s = 0.00003\ \text{s}^{-1}$, and, for example, the dynamic strain rate

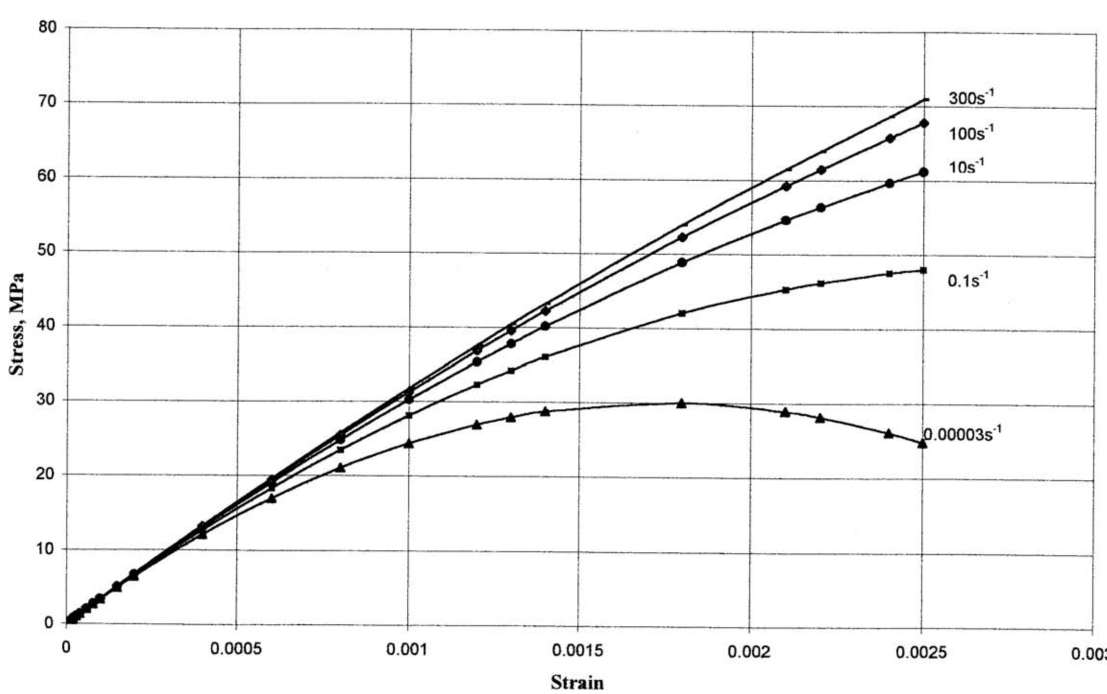

Fig. 3. Stress–strain curves in compression ($f'_c = 30$ MPa) for strain rates given [29].

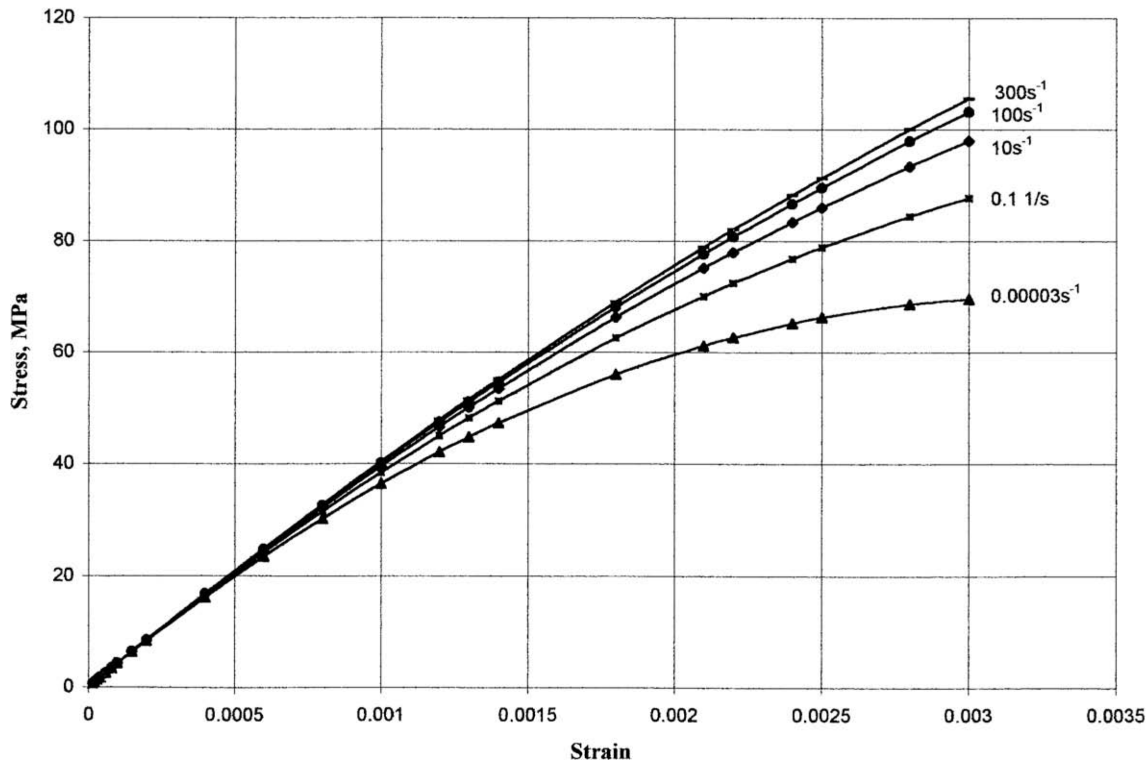

Fig. 4. Stress–strain curves in compression ($f'_c = 70$ MPa) for strain rates given [29].

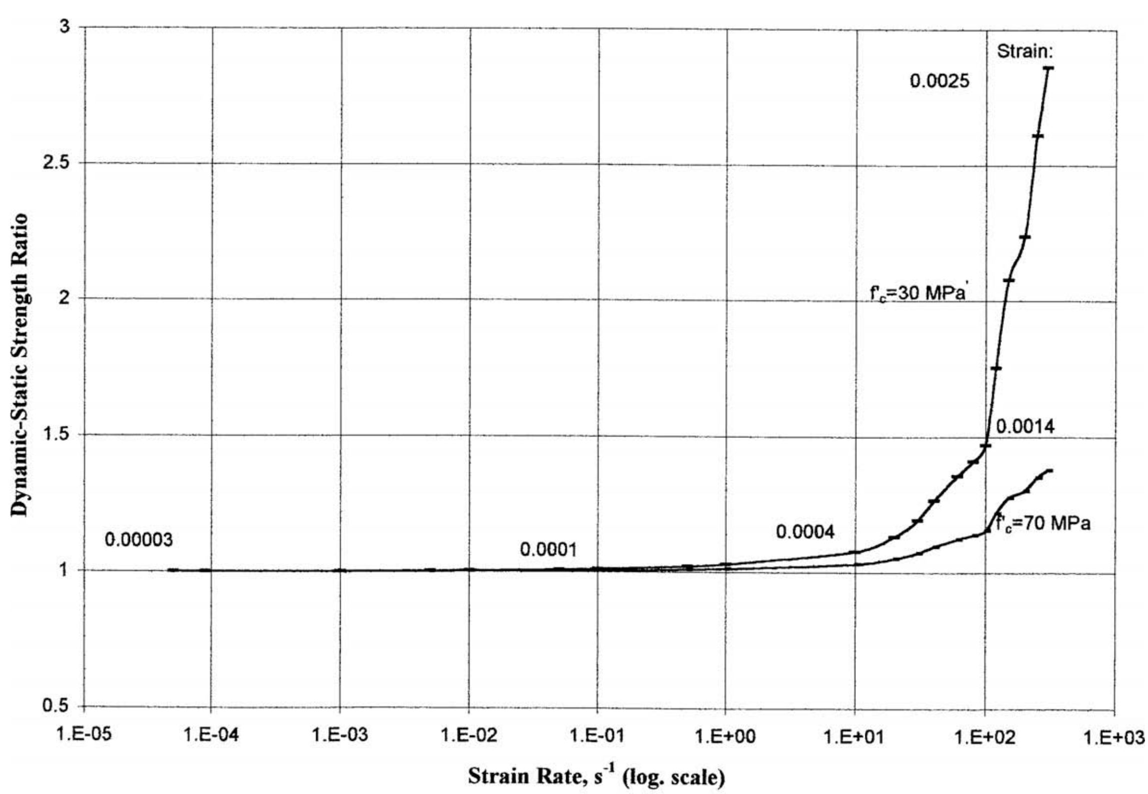

Fig. 5. Dynamic–static strength ratios for two concrete in compression vs. strain rates.

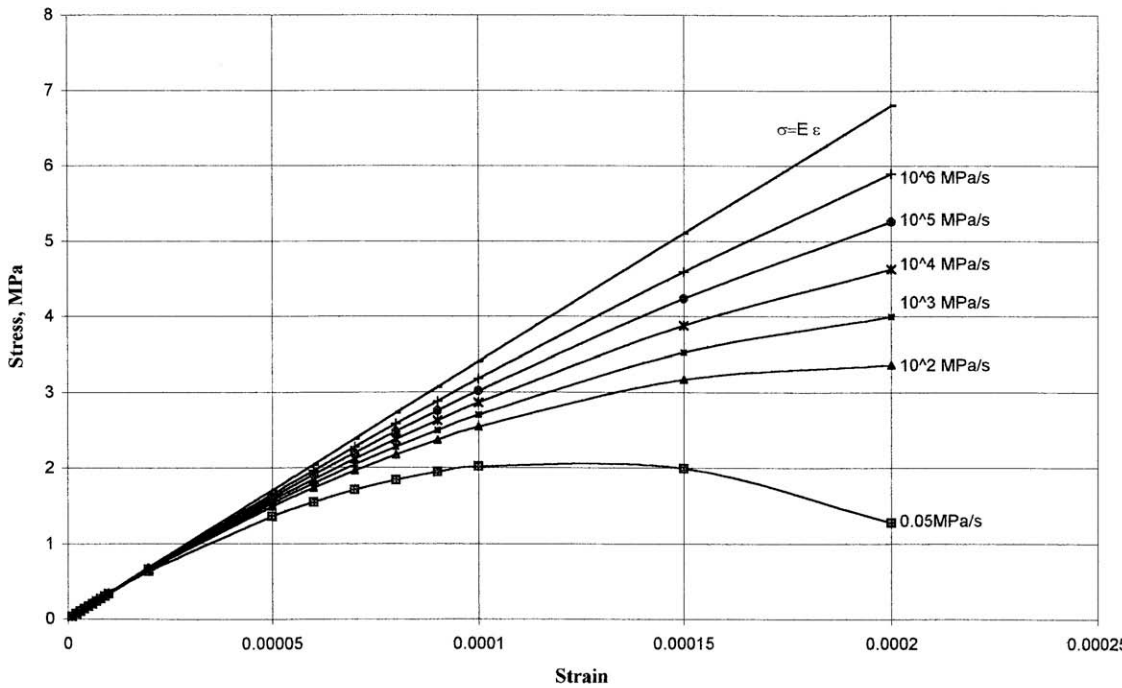

Fig. 6. Stress–strain curves in direct tension ($f_{ctm} = 2$ MPa) at wide range of stress rates.

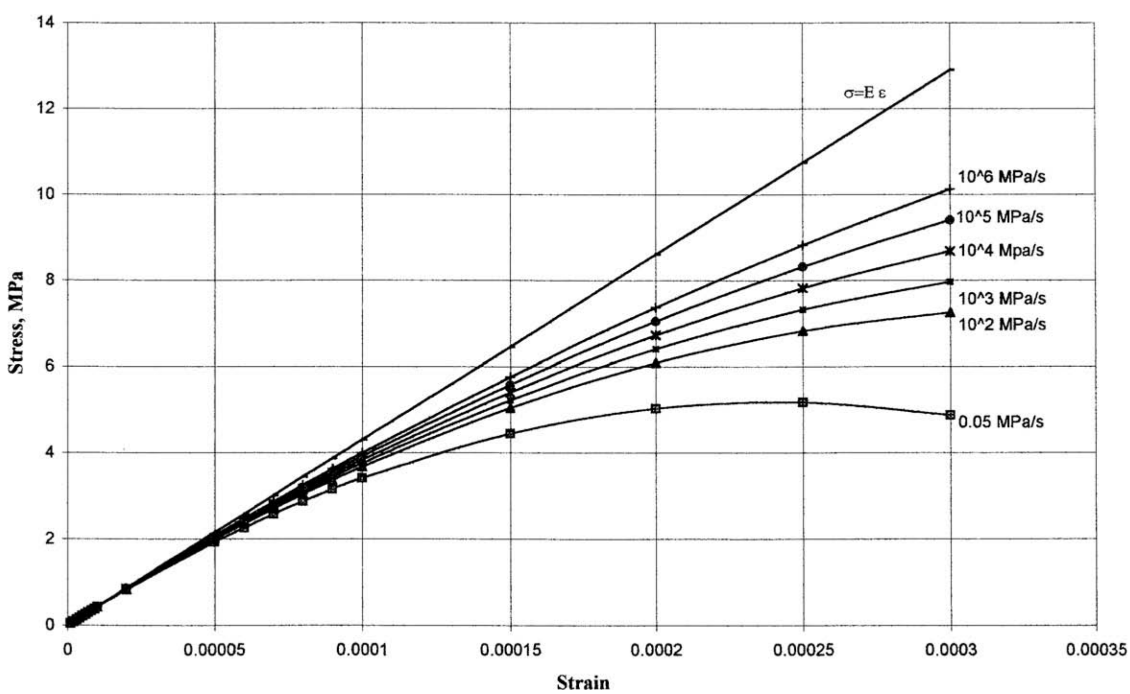

Fig. 7. Stress–strain curves in direct tension (f_{ctm}=5 MPa) at wide range of stress rates.

is $\dot{\varepsilon}_d = 0.01$ s^{-1}. Substituting all of these data into Eq. (7), we will obtain the dynamic/static strength ratio $\sigma_d/\sigma_s = 1 + (2.44/124) * 2 * \ln(0.01/0.00003) = 1.229$. Analogous calculation at the dynamic strain rate $\dot{\varepsilon}_d = 1$ s^{-1} leads to the dynamic/static strength ratio $\sigma_d/\sigma_s = 1.41$. Now let assume that the static strength of concrete $\sigma_s = 30$ MPa, so $\alpha_s = 1/(1 + 9 * 30/10) = 0.036$ by Eq. (10a). Then from Eq. (10) follows that the dynamic/static strength ratio at the strain rate $\dot{\varepsilon}_d = 0.01$ s^{-1} is $\sigma_d/\sigma_s = (0.01/0.00003)^{0.036} = 1.23$, and at dynamic strain rate $\dot{\varepsilon}_d = 1$ s^{-1} the ratio $\sigma_d/\sigma_s = 1.45$.

It can be seen that the difference between the corresponding magnitudes (1.229, 1.233) and (1.41, 1.45) of dynamic/static strength ratios σ_d/σ_s by Eqs. (7) and (10) does not exceed a few percents for the strain rate less than 30 s^{-1} (Fig. 9).

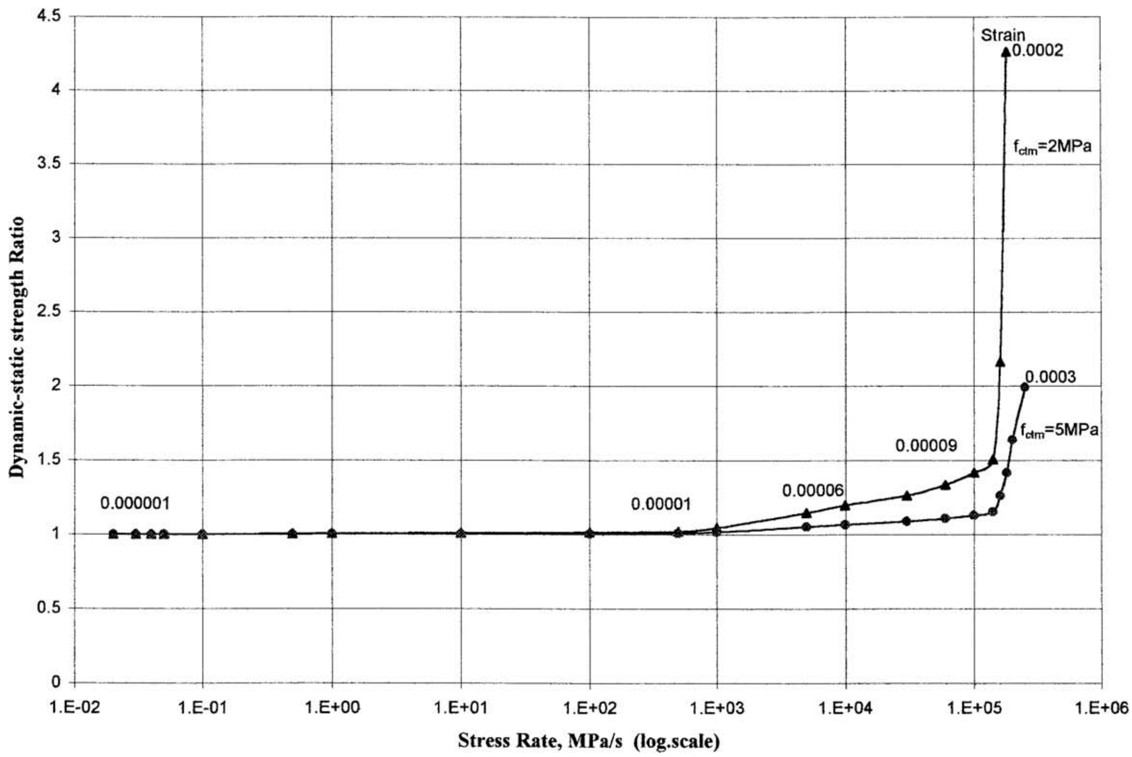

Fig. 8. Dynamic–static strength ratios for two concrete in direct tension vs. stress rates.

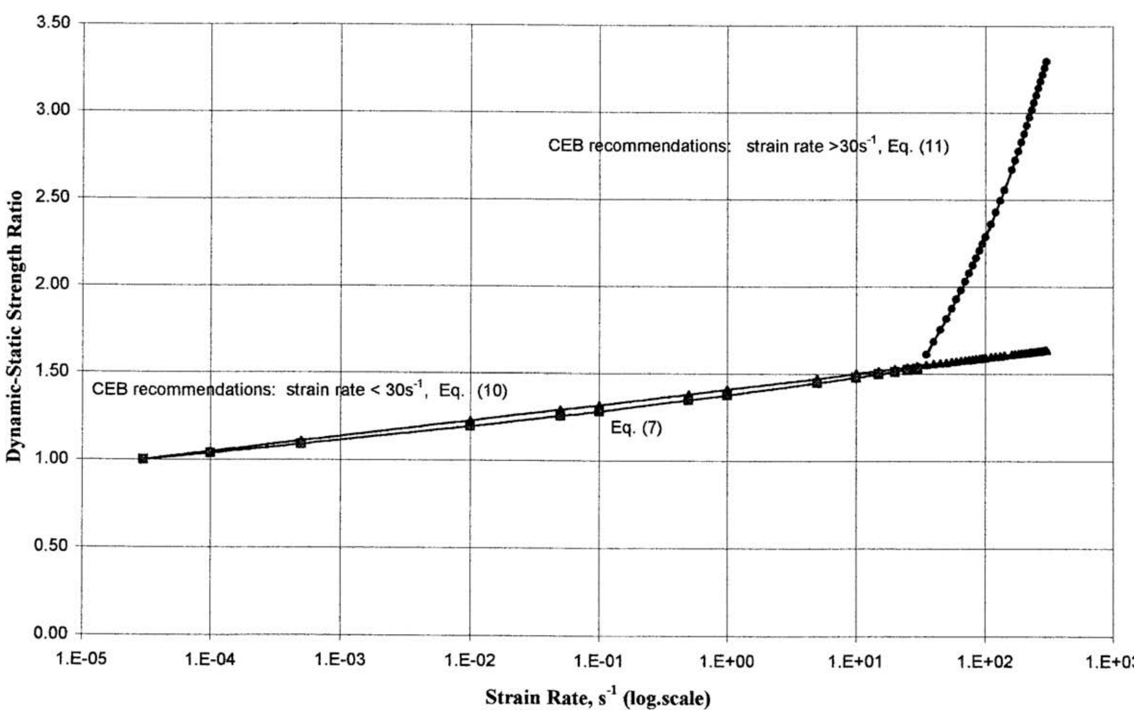

Fig. 9. Comparison the dynamic–static strength ratio by [29] with the predicted by Eq. (7).

This fact enables to establish physical meaning of the coefficient α_s. Indeed, because $0 < \sigma_d/\sigma_s < 2$ for the strain rate $<30\ s^{-1}$ the first degree Maclaurin polynomial for $\ln(\sigma_d/\sigma_s)$ is $(\sigma_d/\sigma_s - 1)$. Therefore, from Eq. (10) follows:

$$\ln(\sigma_d/\sigma_s) = \sigma_d/\sigma_s - 1 = 1.026\alpha_s \ln \dot{\varepsilon}_d/\dot{\varepsilon}_s \quad \text{or}$$
$$\sigma_d/\sigma_s = 1 + 1.026\alpha_s \ln \dot{\varepsilon}_d/\dot{\varepsilon}_s.$$

Comparison with corresponding Eq. (7) shows that

$$\alpha_s = (kT/U_0)(E_0/\sigma_s \mathrm{d}E/\mathrm{d}\sigma)/1.026,$$

i.e. α_s depends on the temperature of concrete, its activation energy of destruction, ($kT/U_0 = 0.02$), modulus of elasticity, strength, nonlinearity of deformation $E_0/\sigma_s \mathrm{d}E/\mathrm{d}\sigma \sim 2$, and in the order of magnitude coincides with the experimental values of α_s in Eq. (10a) for different strength of concrete.

Simultaneously Fig. 9 shows that the discrepancy between assumed Eqs. (7) and (11) for the strain rate more than 30 s^{-1} is very high. However, according to [29] and a number of results of different investigators [2,4,11,20], the critical compressive strains increase at high strain rates. For reasonable estimation of the range of compressive strain at high strain rates let assume that the critical axial strain $\varepsilon_c = 0.0022$ at the static strain rate, i.e. $\dot{\varepsilon}_s = 3 \times 10^{-5}\ \mathrm{s}^{-1}$, and the ultimate dynamic strain rate equal 300 s^{-1}. Then the effect of high strain rate on the strain at maximum stress can be find from corresponding expression [29] as $\varepsilon = \varepsilon_c(\dot{\varepsilon}_d/\dot{\varepsilon}_s)^{0.02} = 0.0022 * (300/3 \times 10^{-5})^{0.02} = 0.003$. After that Eq. (8c)

Table 3
Dynamic–static strength ratio by CEB Model Code, Eqs. (10 and 11) and predicted by Eq. (8c)

Strain rate (s^{-1})	Strain rate ratio	Strain	Eq. (8c) $\varepsilon_c = 0.001765$	CEB Eq. (10)	CEB Eq. (11)	Time (s)	
0.00003	1	0.00001	1.00	1.00		0.333333	Loading duration (s) 11.89
0.0001	3	0.00095	1.02	1.04		9.5	Average strain rate (s^{-1}) **0.000142132**
0.0005	17	0.00096	1.05	1.09		1.92	
0.01	333	0.00098	1.11	1.20		0.098	
0.05	1667	0.00110	1.16	1.26		0.022	
0.1	3333	0.00120	1.20	1.28		0.012	
0.5	16,667	0.00140	1.30	1.35		0.0028	
1	33,333	0.00145	1.35	1.38		0.00145	
5	166,667	0.00146	1.40	1.45		0.000292	
10	333,333	0.00147	1.43	1.48		0.000147	
15	500,000	0.00148	1.45	1.50		9.87E−05	
20	666,667	0.00149	1.47	1.51		7.45E−05	
25	833,333	0.00150	1.48	1.52		0.00006	
30	1,000,000	0.00155	1.51	1.53		5.17E−05	
35	1,166,667	0.00160	1.55		1.61	4.57E−05	Loading duration (s) 0.00065322
40	1,333,333	**0.00170**	1.62		1.68	4.25E−05	Average strain rate (s^{-1}) **1.48**
45	1,500,000	0.00180	1.70		1.75	0.00004	
50	1,666,667	0.00190	1.79		1.81	0.000038	
55	1,833,333	0.00200	1.90		1.87	3.64E−05	
60	2,000,000	0.00205	1.96		1.93	3.42E−05	
65	2,166,667	0.00208	1.99		1.98	0.000032	
70	2,333,333	0.00209	2.01		2.03	2.99E−05	
75	2,500,000	0.00210	2.03		2.08	0.000028	
80	2,666,667	0.00218	2.14		2.12	2.73E−05	
85	2,833,333	0.00219	2.15		2.16	2.58E−05	
90	3,000,000	0.00220	2.17		2.21	2.44E−05	
95	3,166,667	0.00225	2.25		2.25	2.37E−05	
100	3,333,333	0.00226	2.27		2.29	2.26E−05	
110	3,666,667	0.00228	2.31		2.36	2.07E−05	
120	4,000,000	0.00230	2.35		2.43	1.92E−05	
130	4,333,333	0.00240	2.54		2.49	1.85E−05	
140	4,666,667	0.00242	2.59		2.56	1.73E−05	
160	5,333,333	0.00244	2.65		2.67	1.53E−05	
170	5,666,667	0.00246	2.70		2.73	1.45E−05	
180	6,000,000	0.00248	2.75		2.78	1.38E−05	
190	6,333,333	0.00250	2.81		2.83	1.32E−05	
200	6,666,667	0.00252	2.86		2.88	1.26E−05	
210	7,000,000	0.00254	2.92		2.93	1.21E−05	
220	7,333,333	0.00256	2.98		2.97	1.16E−05	
230	7,666,667	0.00258	3.05		3.02	1.12E−05	
240	8,000,000	0.00260	3.11		3.06	1.08E−05	
250	8,333,333	0.00262	3.18		3.10	1.05E−05	
260	8,666,667	0.00263	3.22		3.14	1.01E−05	
270	9,000,000	0.00264	3.26		3.18	9.78E−06	
280	9,333,333	0.00265	3.30		3.22	9.46E−06	
290	9,666,667	0.00266	3.34		3.26	9.17E−06	
300	10,000,000	0.00267	3.38		3.30	8.9E−06	

offers the opportunity to reduce the discrepancy, because, by contrast to Eq. (7), it takes into account an arbitrary combination in increasing order of pairs "instantaneous strain rate and strain".

As an example, Table 3 and Fig. 10 illustrate the use of such pairs for concrete, in particular: compressive strength $f'_c = 30$ MPa, modulus of elasticity $E_0 = 34$ GPa, maximum of strain corresponding to the strength $\varepsilon_c = 0.001765$. As you can see (Table 3) the average strain rate was 1.4×10^{-4} s^{-1} and 1.48 s^{-1} for the strain range less than 0.0017 and more than 0.0017, respectively. Now the relative error between predicted values, i.e. Eq. (8c), and recommended values, Eq. (10), Eq. (11), in most cases (~94%) does not exceed 4% in the wide ranges of strain (10^{-5}–0.00267) and strain rate (3×10^{-5}–300) s^{-1}.

Thus, in general accordance with experiments and CEB recommendations, presented data show:

(1) relative increases of strength more for weaker concrete,
(2) compressive strength is less sensitive to stress rate than tensile strength,
(3) well prediction of the values of dynamic–static strength ratio.

3.3. Modulus of elasticity

For concrete the modulus of elasticity is usually defined as the chord between two values of strain or as a slope of the secant, which is drawn from the origin to a particular value of strain.

Table 4 gives the experimental data by Watstein for two concretes of widely different compressive strength (17 MPa and 45 MPa) at the age 28 days [2]. Specimens were tested at rate of straining ranged from 10^{-6} to about 10 in. per in. per s, and corresponding durations ranged from 30 min to 0.0003 s. At quasi-static loading the values of the secant modulus, E_{ssec}, were drawn to a point of $\varepsilon_{sec} = 0.001$ of strain. The ratio of the dynamic secant modulus, E_{dsec}, to the secant modulus E_{ssec} can be found from Eq. (8a):

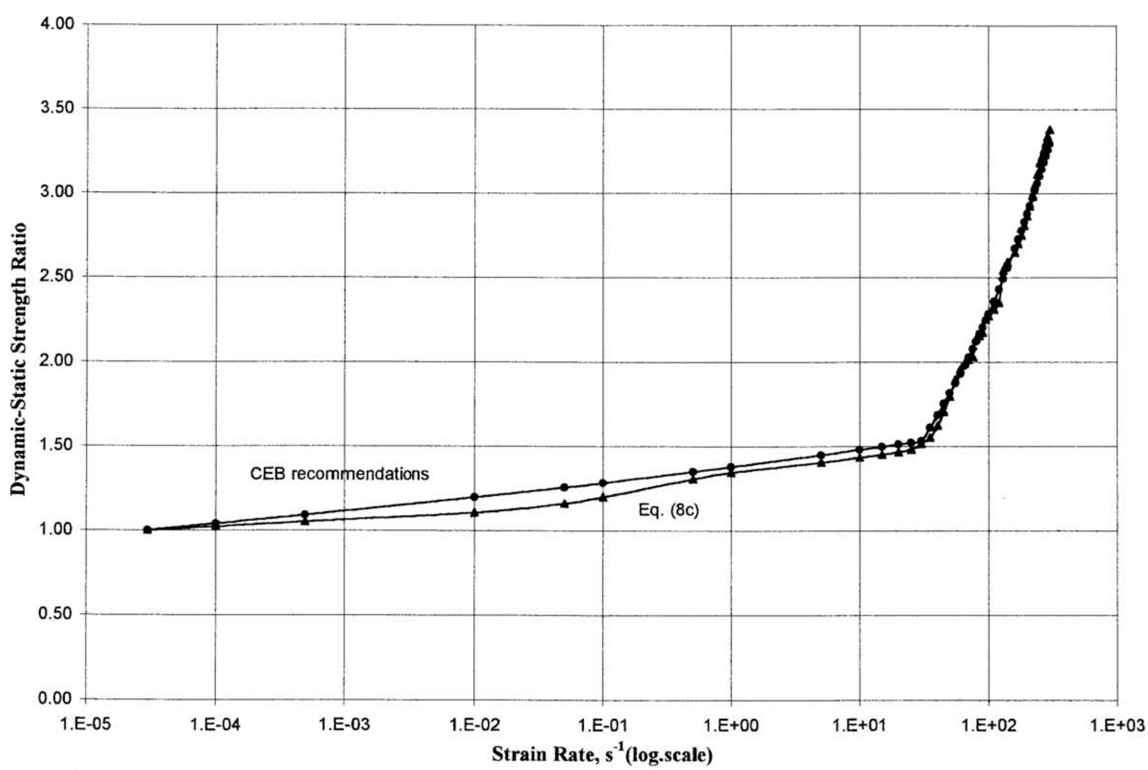

Fig. 10. Comparison the dynamic–static strength ratio by [29] with the predicted by Eq. (8c).

Table 4
Experimental [2] and predicted (Eq. (12)) ratios of modulus of elasticity

Concrete	E_{ssec} (MPa)	ε_c	Strain rate ratio	E_{dsec}/E_{ssec} Eq. (12)	E_{dsec}/E_{ssec} Experiment	Error (%)
W1	15,200	0.0023	3640	1.19	1.12	−6.2
W2	14,600	0.00205	531,000	1.36	1.32	−2.7
W3	15,800	0.00199	10,100,000	1.41	1.47	3.8
S1	26,200	0.00261	2960	1.09	1.07	−2.3
S2	25,400	0.00266	2,860,000	1.18	1.17	−0.7
S3	33,900	0.00219	6,690,000	1.17	1.33	12.0

$$\frac{E_{\mathrm{dsec}}}{E_{\mathrm{ssec}}} = 1 + \frac{2kT}{W_{\mathrm{a}}E_{\mathrm{ssec}}}\frac{\varepsilon_{\mathrm{sec}}}{\varepsilon_{\mathrm{c}}}\ln\left(\frac{\dot{\varepsilon}_{\mathrm{d}}}{\dot{\varepsilon}_{\mathrm{s}}}\right). \tag{12}$$

It can be seen the satisfactory consent between the experimental data (Table 4, columns 2–4, 6) and predicted values of $E_{\mathrm{dsec}}/E_{\mathrm{ssec}}$ (Table 4, last column).

Table 5
Experimental [30] and predicted values, Eq. (14), of modulus of elasticity

t (s)	B (MPa)	C	Experiment E (MPa)	Eq. (14) E (MPa)	Error (%)
$\varepsilon_{\mathrm{sec}}/\varepsilon_{\mathrm{c}} = 0.25$					
100	26,700	0.018	26,700	26,700	0.00
10			27,185	27,650	−1.71
1			27,679	28,608	−3.36
0.1			28,181	29,575	−4.95
$\varepsilon_{\mathrm{sec}}/\varepsilon_{\mathrm{c}} = 0.5$					
100	25,800	0.024	25,800	25,800	0.00
10			26,427	26,729	−1.14
1			27,068	27,658	−2.18
0.1			27,726	28,588	−3.11
$\varepsilon_{\mathrm{sec}}/\varepsilon_{\mathrm{c}} = 0.75$					
100	24,300	0.03	24,300	24,300	0.00
10			25,040	25,694	−2.61
1			25,802	27,088	−4.98
0.1			26,588	28,481	−7.12
$\varepsilon_{\mathrm{sec}}/\varepsilon_{\mathrm{c}} = 1$					
100	21,700	0.033	21,700	21,700	0.00
10			22,428	23,558	−5.04
1			23,180	25,417	−9.65
0.1			23,958	27,275	−13.85

Hatano and Tsuisumi studied dynamic properties for concrete with the cement–sand–gravel ratio 1:3:5 and water–cement ratio of 0.5 [30]. They have established that at the reduction of time of test more noticeably is being increased the module of elasticity corresponding to the higher level of loading. The following dependence between modulus of elasticity, E_{sec}, ratio $\varepsilon_{\mathrm{sec}}/\varepsilon_{\mathrm{c}}$ and time of loading, t, for duration of test in the range 0.03–100 s was obtained:

$$E_{\mathrm{sec}} = B\exp(-C\log t). \tag{13}$$

On the other hand, taking into account that $\dot{\varepsilon}_{\mathrm{s}} = \varepsilon_{\mathrm{c}}/t_{\mathrm{s}}$ and $\dot{\varepsilon}_{\mathrm{d}} = \varepsilon_{\mathrm{d}}/t_{\mathrm{d}}$ (t_{s}, t_{d} – time at static and dynamic loading, $\varepsilon_{\mathrm{c}} \sim \varepsilon_{\mathrm{d}}$ – corresponding strain), from Eq. (12) follows:

$$E_{\mathrm{dsec}} = E_{\mathrm{sec}} + (2kT/W_{\mathrm{a}})(\varepsilon_{\mathrm{sec}}/\varepsilon_{\mathrm{c}})(2.3\log(t_{\mathrm{s}}/t_{\mathrm{d}})). \tag{14}$$

Table 5 contains details of the tests (columns 1–3: time of loading, ratio of strain, constants B, C), and comparison the experimental equation (13) with predicted modulus by Eq. (14). Although the theoretical values are slightly overestimate, it is possible to show the relationship between coefficient 'C', ratio of loading and temperature of concrete, as it was found out for coefficient α_{s} in the previous section.

According to CEB Model Code the effect of high strain rate on modulus of elasticity and on the strain at maximum stress in compression may be estimated from Eqs. (15) and (16), respectively [29]:

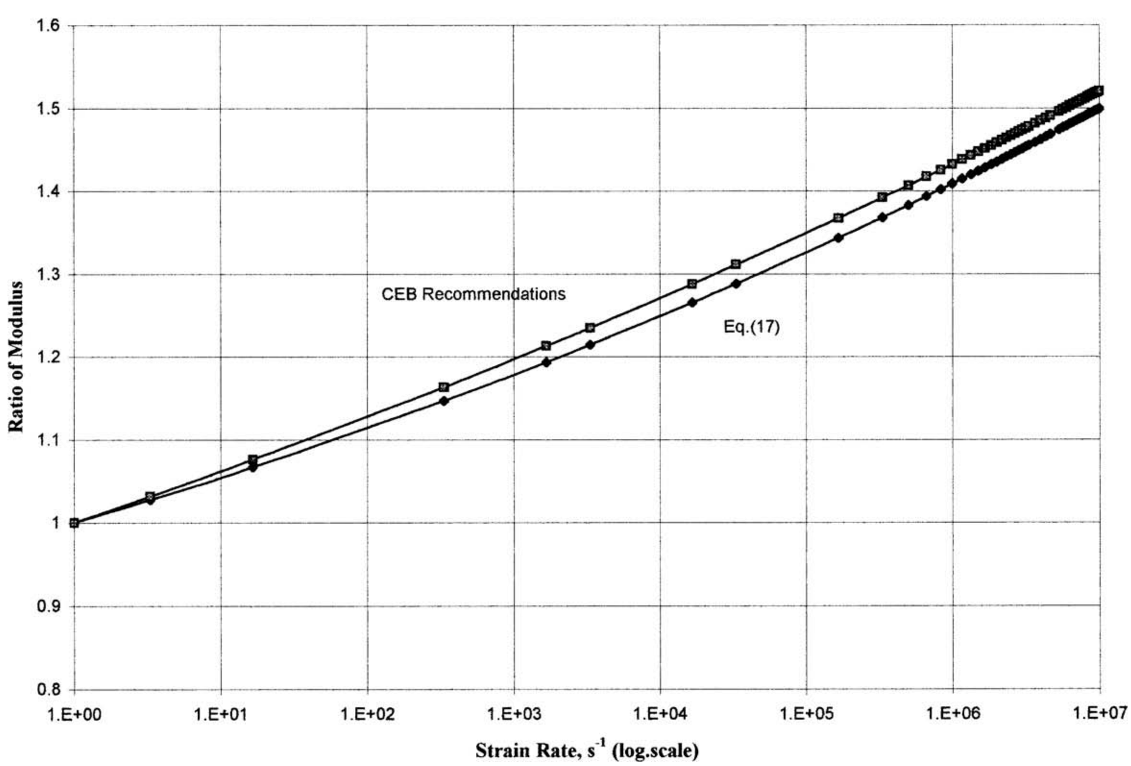

Fig. 11. Comparison the effect of strain rate on modulus of elasticity recommended by [29] with the predicted by Eq. (17).

$$E_{c,imp}/E_0 = (\dot{\varepsilon}_d/\dot{\varepsilon}_s)^{0.026}. \quad (15)$$

$E_{c,imp}$ is the impact modulus of elasticity, the average value of $E_0 = 36$ GPa, $\dot{\varepsilon}_s = 3 \times 10^{-5}$ s^{-1}:

$$\varepsilon_{imp}/\varepsilon_c = (\dot{\varepsilon}_d/\dot{\varepsilon}_s)^{0.02}. \quad (16)$$

ε_{imp} is the impact strain at maximum load.

Taking into account Eq. (16) for comparison purpose, we need to rewrite Eq. (8a) as:

$$\sigma_d = E_0\varepsilon - (E_0/2\varepsilon_c)\varepsilon^2 + (2kT/W_a)\varepsilon(\dot{\varepsilon}_d/\dot{\varepsilon}_s)^{0.02}\ln\dot{\varepsilon}_d/\dot{\varepsilon}_s$$

From this equation follows that the ratio of the impact modulus of elasticity to tangent modulus (at $\varepsilon = 0$) equal:

$$E_{c,imp}/E_0 = 1 + (2kT/W_aE_0)(\dot{\varepsilon}_d/\dot{\varepsilon}_s)^{0.02}\ln\dot{\varepsilon}_d/\dot{\varepsilon}_s. \quad (17)$$

It can be seen (Fig. 11) that Eq. (17) describes well the effect of high strain on modulus of elasticity.

As a whole presented data confirm again a quantitative relationship with the parameters of the thermofluctuation theory.

4. Conclusions

On the basis of the principle of summation of damages under loading of concrete an arbitrary sequence of instantaneous strain and strain rate is analyzed. The effect of such consideration is illustrated with the help of calculations of the stress–strain diagram, strength and elastic modulus for concrete for wide ranges of strain and strain rate. The obtained equations explain and unify the influence of the high rates on mechanical properties of concrete at different kind of uniaxial stress. Comparative study showed substantial agreement with existing experimental results and general equations by CEB Model Code.

Earlier papers demonstrated that nonlinear nondestructive methods could be used to evaluate the strength of concrete and elastic modulus at quasi-static loading [26,31,32]. On the other hand, no values of strength at high rate of loading can be obtained without reference of the values under quasi-static loading. Therefore presented data allow assuming the use of nondestructive methods will yield the possibility to predict mechanical properties of concrete also under high rate of loading.

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