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Effectiveness of the use of steel fibres on the torsional behaviour of flanged concrete beams

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The behaviour under torsion of reinforced concrete beams with steel fibres as mass reinforcement is experimentally investigated. Short hooked-ended steel fibres with aspect ratio $l_f/d_f = 37.5$ are used. Test results of 35 beams with rectangular, L-shaped and T-shaped cross-sections tested in pure torsion are presented and discussed. Various configurations of conventional and fibre steel reinforcement are examined. The experimental program includes (i) plain concrete beams (control specimens), (ii) specimens with longitudinal reinforcing bars and (iii) specimens with bars and stirrups. All cases are examined with 0%, 1% and 3% steel fibre volume fractions. The use of steel fibres as the only shear torsional reinforcement is also reported herein, in an attempt to examine the effectiveness of fibres as a potential replacement of stirrups. Test results indicated that fibrous concrete beams exhibited improved overall torsional performance with respect to the corresponding non-fibrous control beams. The addition of steel fibres was essential to the tested beams without or with inadequate conventional steel reinforcement. Fibres prevented the sudden brittle failure of both rectangular and non-rectangular beams and proved to be under some circumstances adequate to provide for enhanced torsional moment capacities, even in the case of full replacement of stirrups with steel fibres.

1. Introduction

The addition of steel fibres in concrete mix has long been recognised as a non-conventional mass reinforcement that enhances the mechanical properties of concrete and provides for crack propagation control [1,2]. This ability is attributed to the tensile stress transfer capability of the steel fibres across crack surfaces, known as crack-bridging, and also to the fact that such fibres provide significant resistance to shear across developing cracks. Cracking of steel fibre concrete requires debonding and pull-out of the randomly distributed steel fibres in the concrete. Therefore, steel fibre concrete demonstrates a pseudo-ductile tensile response and enhanced energy dissipations capacities, relative to the brittle behaviour of plain concrete [3]. This has inspired investigations of the possibility of partially replacing conventional steel reinforcement – mainly stirrups – by steel fibres, especially in cases where design criteria recommend high steel ratio, which leads to very small stirrup spacing [4–6].

Concerning the phenomenon of torsion, early experimental efforts pointed out that the behaviour of an element in pure torsion is fully characterized by the behaviour of the material in direct tension [7,8]. Thus, in order to enhance the torsional response of concrete members, improvement of the poor performance of concrete in tension by incorporating steel fibres has been proposed and extensively studied in the last decades [9–23]. Most of the conducted research on the torsional behaviour of fibrous concrete elements presents experimental results of rectangular beams containing steel fibres without conventional reinforcement [9–12] and rectangular beams with steel fibres, bars and stirrups [13–18]. There is also an experimental investigation of fibrous concrete beams with circular cross-section under torsion [19]. All these experimental studies have shown that the use of steel fibres improves the cracking characteristics and the overall torsional behaviour of concrete beams with rectangular and circular cross-sections.

Analytical research on the torsional response of fibrous concrete elements has also been reported in the literature. An efficient theoretical model for the prediction of the full torsional behaviour of steel fibre concrete members without reinforcing bars has been presented by Karayannis [20,21]. Further, the well-known softened truss model theory [7] has been properly modified to include the influence of steel fibres and has successfully been applied in fibrous concrete rectangular beams with steel bars and stirrups [15,17,18]. Moreover, simplified semi-empirical expressions and formulas for code modifications to calculate the ultimate torsional capacity of fibrous concrete beams with rectangular cross-section have also been proposed [9–14,22].

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It is obvious that despite the plethora of the carried out research on this field of study and the fact that in real reinforced concrete structures most of the beams have L- or T-shaped cross-section (flanged beams), the published work on non-rectangular fibrous concrete beams under torsion is quite limited and mainly conducted by the authors [20,21,23]. Furthermore, the use of steel fibres as the only shear reinforcement instead of the use of common stirrups in torsional beams has not been thoroughly investigated.

In this paper the influence of the steel fibres on the torsional behaviour of L- and T-shaped beams is experimentally investigated. Test results of 35 beams in pure torsion with rectangular, L- and T-shaped cross-sections, with various reinforcement configurations are reported. The experimental program includes three groups of specimens; (i) without conventional steel reinforcement, (ii) with longitudinal reinforcing bars and (iii) with bars and stirrups (conventional steel reinforcement). The examined specimens also contain different volume fractions of short steel fibres: 0%, 1% and 3%.

Furthermore, a short review of the proposed analytical models for the prediction of the torsional strength of steel fibre reinforced concrete beams is including in this study. The ultimate torsional moment of the tested fibrous beams without conventional reinforcement has been calculated using the analytical expressions of Craig et al. [9], Mansur & Paramasivam [10] and Wafa et al. [7] and the numerical model by Karayannis [21]. The entire torsional behaviour of the tested rectangular fibrous beams with longitudinal bars and stirrups has been predicted using the combined methodology proposed by the authors [17] and the one by Gunneswara Rao and Rama Seshu [18]. The purpose of this analytical investigation is to present and to compare the predictions of the proposed analytical approaches in order to estimate their effectiveness and accuracy on the experimental results of the tested beams of this study.

The present study contributes to the limited existing literature on torsional tests of fibrous concrete beams with non-rectangular cross-section. Rectangular and comparable flanged beams are jointly tested in order to acquire comparative results. Furthermore, the use of steel fibres as the only shear reinforcement in torsion is also presented herein, in an attempt to examine the effectiveness of steel fibres as a potential replacement for stirrups.

2. Experimental program

The experimental program includes 35 rectangular and flanged beams tested under pure torsion and sorted in three groups based on the conventional steel reinforcement. Tested beams were constructed by plain concrete (control specimens) and steel fibre concrete with 1% and 3% volume fractions of the used fibres.

2.1. Specimen characteristics

Rectangular, L-shaped and T-shaped sections comprise the examined cross-sectional schemes of the tested beams. Geometrical characteristics and dimensions of the specimens' cross-sections are shown in Fig. 1. All beams have the same height and web width; $h/b_w = 200/100$ mm. The ratio of the flange width, b_f, to the web width is equal to $b_f/b_w = 1$ (rectangular specimens), 1.5 (L-shaped beams), 2 (L- and T-shaped beams) and 3 (T-shaped beams).

Tested beams are sorted in three groups named as P, L and R, based on their conventional steel reinforcement. The first group, group P, includes 17 specimens; 3 with rectangular cross-section and 14 L-shaped and T-shaped beams without any conventional steel torsional reinforcement (without bars and stirrups). Second group, group L, includes 9 specimens; 3 rectangular and 6 T-shaped beams without stirrups and only longitudinal bars were added as conventional steel reinforcement. Third group, group R, includes 9 specimens; 3 rectangular and 6 T-shaped beams with bars and stirrups.

The longitudinal reinforcement of the beams of groups L and R was the same; four deformed bars of diameter 10 mm (4∅10) at

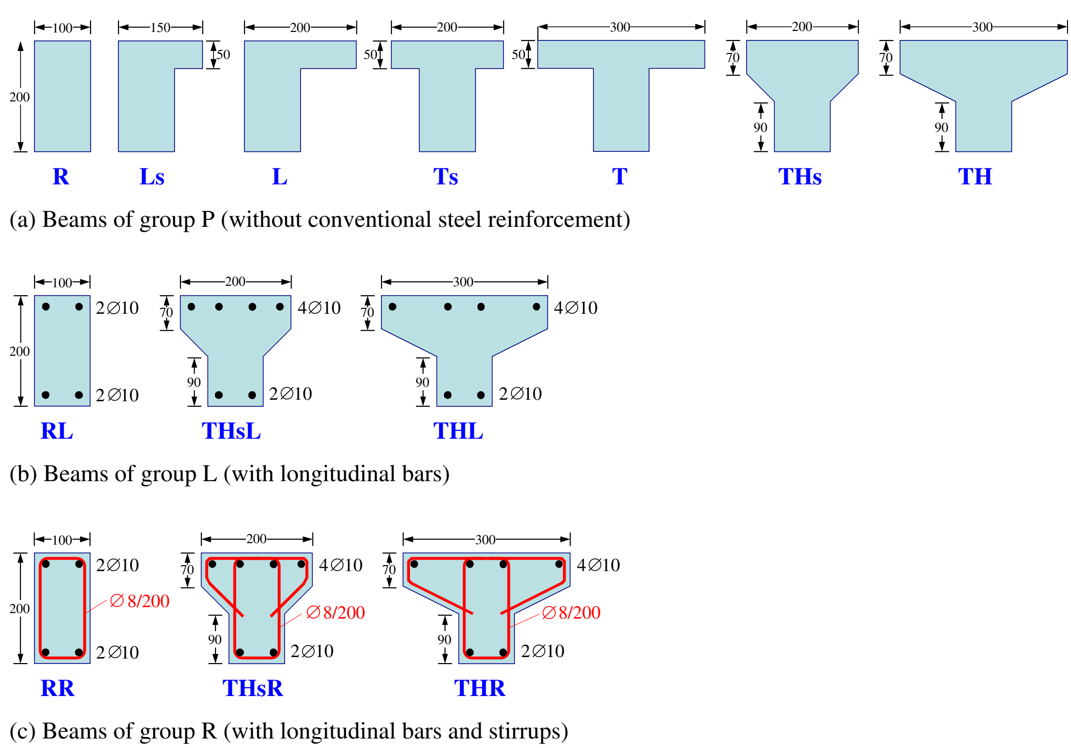

Fig. 1. Cross-sectional dimensions and conventional steel reinforcement of the tested beams.

the corners of the rectangular beams and six deformed bars of diameter 10 mm (6∅10), 4∅10 at the top and 2∅10 at the bottom of the T-shaped beams. The transverse reinforcement of the beams of group R was 8 mm diameter mild steel closed stirrups at a uniform spacing of 200 mm. Steel yield strength was 415 MPa for the bars and 344 MPa for the stirrups. Cross-sectional and conventional steel reinforcement details of the specimens are shown in Fig. 1 and summarized in Table 1.

Table 1 also presents the ratios of the steel reinforcements of the tested beams. It is noted that the ratio values for the bars and stirrups have been calculated based on the following expressions:

$$\rho_\ell = \frac{A_{sl}}{A_c} \tag{1a}$$

$$\rho_t = \frac{A_{st} \cdot p_t}{A_c \cdot s} \tag{1b}$$

where: A_c: gross area of the concrete cross-section; A_{sl}: total area of steel longitudinal bars; A_{st}: area of one single-legged steel stirrup; p_t: perimeter of the stirrup; s: spacing of stirrups.

Beams from plain concrete (without fibres) were used as control specimens (13 specimens). The steel fibre concrete beams (22 specimens) contained short hooked-ended steel fibres with length to diameter ratio (aspect ratio) equal to l_f/d_f = 30 mm/0.80 mm = 37.5 (DRAMIX ZC 30/.80). Two different steel fibre volume fractions, V_f, were used; a moderate one equal to 1% or 78.5 kg per 1 m^3 concrete and a high one equal to 3% or 235.5 kg per 1 m^3 concrete (see also Table 1).

The cement used in this experimental work was a locally manufactured general purpose ordinary Portland type cement (type 35IIa, Greek type pozzolan cement containing 10% fly ash). Sand with a high fineness modulus and coarse aggregates with a maximum size of 9.5 mm were used. The concrete mixture was made using cement, sand and crushed stone aggregates in a proportion 1:2.8:1.2, respectively, and water to cement ratio equal to 0.43.

The maximum size of the used coarse aggregates (9.5 mm), the length (l_f = 30 mm) and the aspect ratio (l_f /d_f = 37.5) of the used steel fibres were chosen in accordance with the RILEM TC 162-TDF recommendations [24], based on the minimum dimension of the tested beams. It is known that the size and the quantity of aggregates greater than 5 mm, the volume fraction and the aspect ratio of the added steel fibres strongly affect the workability of concrete and the homogenous distribution of the fibres in the concrete mix. Fibrous concrete mixtures with high volume fraction of steel fibres (V_f = 3%) have workability difficulties. However, the use of short fibres with low aspect ratio, the use of pozzolan cement containing 10% fly ash and the following procedure for mixing and casting the fibrous concrete mixture significantly reduced this deficiency.

First, cement (with fly ash), sand and crushed stone aggregates were dry-mixed for about 3 min in a 0.05 m^3 laboratory mixer. Second, free of clumps steel fibres were dispersed by hand gradually and very slowly in small amounts to avoid fibre balling, while mixing was continued for approximately 5–10 min. Third, water (without plasticizer) was added gradually and mixed for another 7–10 min at least, in order the produced mixture to obtain uniform

Table 1
Reinforcement of the tested beams and concrete strength values.

Beam code name	Cross-section	Bars (∅ in mm)	ρ_ℓ (%)	Stirrups (mm)	ρ_t (%)	Volume of steel fibres – V_f (%)	$f_{c.cube}$ (MPa)	$f_{ct,split}$ (MPa)
Group P: Beams without bars and stirrups (plain concrete and steel–fibre-reinforced-concrete beams)								
RP0	R	–	–	–	–	–	16.89	1.48
RP1	R	–	–	–	–	1	19.80	2.05
RP3	R	–	–	–	–	3	19.26	2.26
LsP0	Ls	–	–	–	–	–	19.24	1.93
LsP1	Ls	–	–	–	–	1	21.00	2.25
LP0	L	–	–	–	–	–	18.38	2.00
LP1	L	–	–	–	–	1	30.00	2.85
TsP0	Ts	–	–	–	–	–	20.70	2.00
TsP1	Ts	–	–	–	–	1	25.00	2.36
TP0	T	–	–	–	–	–	19.52	1.90
TP1	T	–	–	–	–	1	29.14	2.83
THsP0	THs	–	–	–	–	–	30.96	2.17
THsP1	THs	–	–	–	–	1	27.26	2.03
THsP3	THs	–	–	–	–	3	33.04	3.24
THP0	TH	–	–	–	–	–	39.56	2.97
THP1	TH	–	–	–	–	1	26.00	1.91
THP3	TH	–	–	–	–	3	29.51	4.36
Group L: Beams with longitudinal bars								
RL0	RL	4∅10	1.57	–	–	–	18.37	1.56
RL1	RL	4∅10	1.57	–	–	1	17.04	1.79
RL3	RL	4∅10	1.57	–	–	3	16.44	2.05
THsL0	THsL	6∅10	1.62	–	–	–	30.96	2.52
THsL1	THsL	6∅10	1.62	–	–	1	24.96	2.15
THsL1	THsL	6∅10	1.62	–	–	3	26.67	2.88
THL0	THL	6∅10	1.24	–	–	–	32.44	2.41
THL1	THL	6∅10	1.24	–	–	1	27.63	2.03
THL3	THL	6∅10	1.24	–	–	3	21.11	2.88
Group R: Beams with bars and stirrups								
RR0	RR	4∅10	1.57	∅8/200	0.63	–	20.12	1.46
RR1	RR	4∅10	1.57	∅8/200	0.63	1	18.96	1.86
RR3	RR	4∅10	1.57	∅8/200	0.63	3	16.89	2.57
THsR0	THsR	6∅10	1.62	∅8/200	0.71	–	36.15	2.26
THsR1	THsR	6∅10	1.62	∅8/200	0.71	1	32.29	2.05
THsR1	THsR	6∅10	1.62	∅8/200	0.71	3	26.59	1.86
THR0	THR	6∅10	1.24	∅8/200	0.66	–	33.19	2.10
THR1	THR	6∅10	1.24	∅8/200	0.66	1	24.07	1.70
THR3	THR	6∅10	1.24	∅8/200	0.66	3	25.48	3.14

material consistency, adequate workability and homogeneous fibre distribution. Right afterwards, the prepared fresh fibrous concrete mixture was carefully placed in the specimens' moulds using wide scoops and vibrated using frequency vibrating table. Sufficient time of vibration was provided to guarantee suitable consolidation and to prevent fibre protrusion from the finished surface. Extra time of about 2 min was required for further vibration in fibrous concrete mixtures with high volume fraction of steel fibres (V_f = 3%). During mixing and casting of the fresh mixture, no steel fibre gravitation was observed.

Supplementary compression and splitting tests in order to determine the cube compression strength of the plain concrete mixture and the tensile strength of the composite mixture (steel fibre concrete) were also included in the program and results are given in Table 1. These strength values are averages from three standard 150 mm cubes and three standard 150 × 300 mm cylinders, respectively.

2.2. Test setup

The experimental setup is a commonly used torsional test rig and it is shown in Fig. 2. The total length of the specimens was 1.60 m. Beams were supported on two roller supports 1.30 m apart. These supports ensured that the beam was free to twist and to elongate longitudinally in both end directions during the test. The load was applied through a diagonally placed steel spreader beam on the ends of two properly configured over-reinforced concrete arms at the end parts of the specimens, as shown in Fig. 2. The end parts of the specimens were also heavily reinforced with high volume of stirrups in order to bear without cracking the imposed torsional loading at the ends of the 500 mm long concrete arms. This way, the examined test region was the central part of the specimens. During the test procedure, torsional helical diagonal cracking and, finally, failure were localized within this test region. The heavily reinforced end parts and the over-reinforced concrete arms of the beams remained quite intact.

The load was imposed consistently in low rate and measured by a load cell with accuracy equal to 0.05 kN. The average angle of twist per unit length of the tested beams was estimated using the measurements of a set of linear variable displacement transducers with accuracy 0.02 mm, which was placed at the overhanging ends of the specimens to measure the deflections of the concrete arms as the beam twisted.

In order to acquire information about the failure modes of the tested beams, the strains of the steel reinforcement were measured by electrical resistance strain gauges.

The beams were tested in monotonically increasing torque moment until the ultimate torsional strength and subsequently in increasing twist until the total failure of the specimen or the maximum twist capacity of the test apparatus.

3. Test results and discussions

3.1. Torsional strength values and behavioural curves

The torsional behaviour of all the tested beams of groups P, L and R is presented in Figs. 3–5, respectively, in terms of experimental curves for torsional moment, T, versus angle of twist per unit length, ϑ. In these figures, the measured torsional response of the steel fibre concrete beams is compared with the response of the corresponding control specimens (beams without fibres). It is noted that all the torsional curves demonstrated in Fig. 3 are complete until failure. Torsional behaviour of beams RL0, THsL0 and THL0 displayed in Fig. 4 is also complete until failure, while the response of the fibrous beams in Fig. 4 and the torsional curves of the beams of group R in Fig. 5 are truncated.

Furthermore, the measured torsional moment at cracking, T_{cr}, the angle of twist per unit length at cracking, ϑ_{Tcr}, the initial pre-cracking torsional stiffness, K, the post-cracking ultimate torsional moment, T_u, and the corresponding angle of twist per unit length, ϑ_{Tu}, of the beams of groups P, L and R are presented in Tables 2–4, respectively. It is mentioned that specimens without conventional steel reinforcement (beams of group P) and plain concrete specimens with longitudinal only reinforcement (beams RL0, THsL0 and THL0 of group L) did not exhibit increasing post-cracking response. Thus, their maximum torsional moment capacities were equal to the torsional moment at cracking: $T_{max} = T_{cr}$. For all the other specimens: $T_{max} = T_u$.

From the values of the elastic torsional stiffness, K (Tables 2–4) and the slopes of elastic parts of the behavioural curves (Figs. 3–5) it is deduced that the initial pre-cracking torsional rigidity remained practically unaffected by the amount of steel fibres and conventional reinforcement used. The values of the cracking torsional moment, T_{cr}, of the steel fibre concrete beams are higher than the corresponding values of the control specimens (without fibres). This increase on the cracking torsional strength is essential

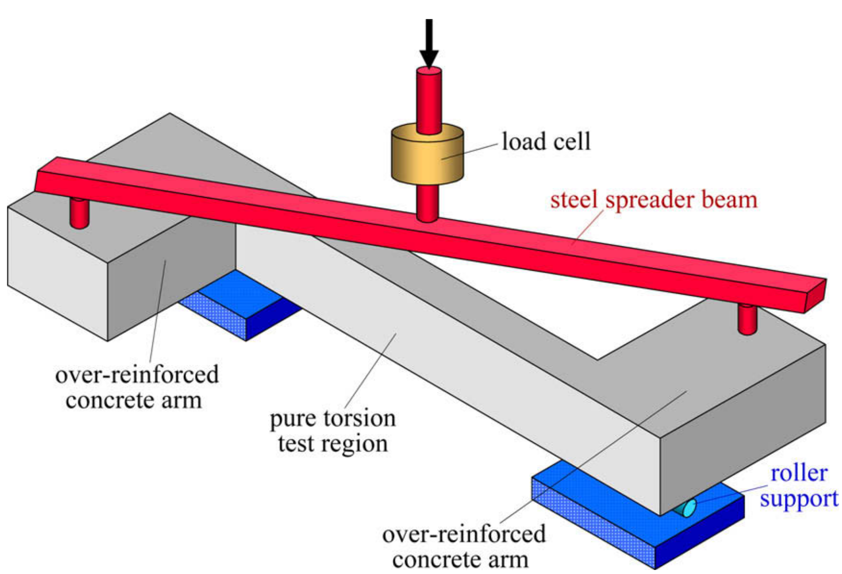

Fig. 2. Test setup.

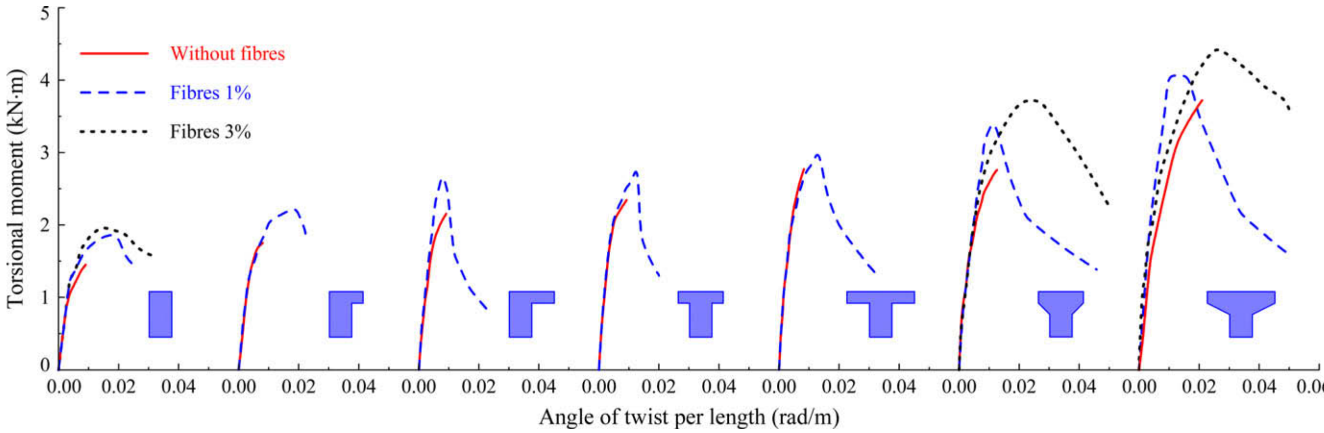

Fig. 3. Experimental curves of the beams of group P (without conventional steel reinforcement).

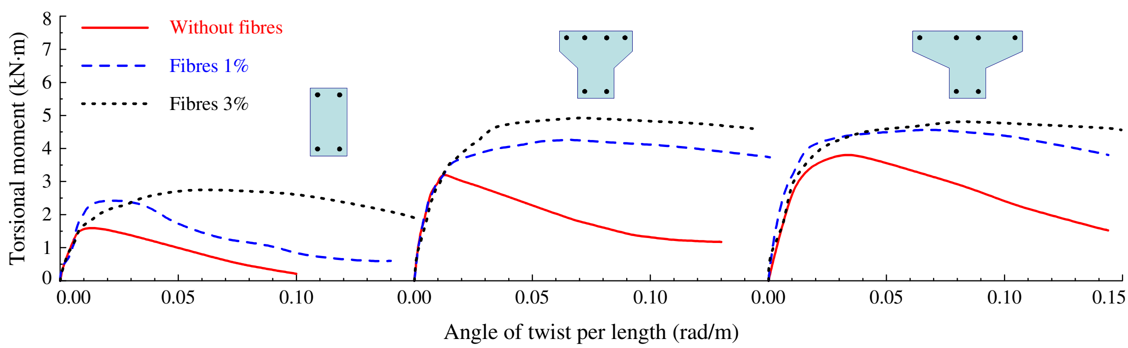

Fig. 4. Experimental curves of the beams of group L (with longitudinal bars).

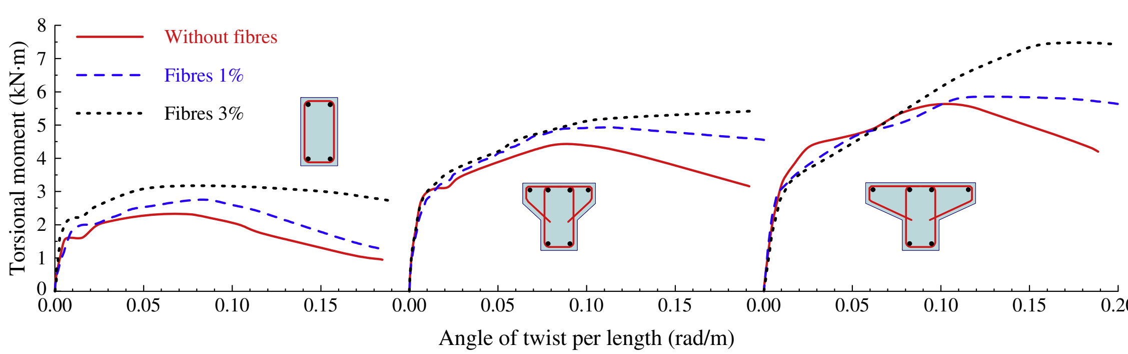

Fig. 5. Experimental curves of the beams of group R (conventionally reinforced beams with longitudinal bars and stirrups).

for the beams without conventional steel reinforcement (specimens of group P, see also Fig. 3 and Table 2). In these cases steel fibres are the only steel reinforcement and seem capable to provide enhanced torque capacities, especially in the cases of the beams with high volume of fibres ($V_f = 3\%$).

After concrete cracking, the favourable effect of steel fibres is clearly indicated to the post-cracking response of the beams with longitudinal only reinforcement (specimens of group L). Steel fibre concrete beams demonstrated further increase on the torsional strength, whereas the corresponding plain concrete beams (control specimens) did not exhibit increasing post-cracking response (Fig. 4 and Table 3). Furthermore, the values of the ultimate torsional moment, T_u, of the conventionally reinforced fibre concrete beams with bars and stirrups are considerably higher than the values of the corresponding control specimens (Fig. 5 and Table 4). These improvements on the torque capacities were observed to both rectangular and flanged beams. Nevertheless, Fig. 6 demonstrates the percentage increase on the ultimate torsional moment due to the use of fibres and indicates that this increase seems to be reduced in the cases of the flanged beams with respect to the rectangular beams.

Fig. 6 also shows that the addition of steel fibre volume fractions 1% and 3% to the rectangular beams caused an increase of the torsional strength that ranges from 18% to 64% and from 35% to 86%, respectively. Further, the addition of steel fibres with $V_f = 1\%$ and 3% to the T-flanged beams with $b_f / b_w = 2$ caused an

Table 2
Test results of beams of group P (without longitudinal bars and stirrups).

Beam code name	V_f (%)	T_{max} = T_{cr} (kN m)	ϑ_{Tmax} = ϑ_{Tcr} (rad/m)	K (kN m^2/rad)	ϑ_{85Tmax} (rad/m)	μ_{T85max} = μ_{T85cr}
RP0	–	1.45	0.009	284	–	(1.00)
RP1	1	1.84	0.019	324	0.023	1.21
RP3	3	1.95	0.014	341	0.028	2.00
LsP0	–	1.75	0.008	401	–	(1.00)
LsP1	1	2.21	0.019	454	0.022	1.19
LP0	–	2.15	0.009	600	–	(1.00)
LP1	1	2.65	0.008	610	0.010	1.25
TsP0	–	2.34	0.009	641	–	(1.00)
TsP1	1	2.72	0.012	645	0.013	1.08
TP0	–	2.77	0.008	881	–	(1.00)
TP1	1	2.96	0.013	891	0.016	1.23
THsP0	–	2.75	0.013	855	–	(1.00)
THsP1	1	3.39	0.011	895	0.015	1.36
THsP3	3	3.69	0.021	870	0.037	1.76
THP0	–	3.72	0.018	1525	–	(1.00)
THP1	1	4.05	0.015	1571	0.021	1.40
THP3	3	4.42	0.026	1537	0.047	1.81

μ_T = 1.00.

Table 3
Test results of beams of group L (with longitudinal bars – without stirrups: ρ_t = 0).

Beam code name	ρ_ℓ (%)	V_f (%)	Elastic response until first cracking			Post-cracking		T_{max} (kN m)	ϑ_{Tmax} (rad/m)	μ_T	ϑ_{85Tmax} (rad/m)	μ_{T85max}	μ_{T85cr}
			T_{cr} (kN m)	ϑ_{Tcr} (rad/m)	K (kN m^2/rad)	T_u (kN m)	ϑ_{Tu} (rad/m)						
RL0	1.57	–	1.47	0.015	300	–	–	1.47	0.015	1.00	0.028	1.87	1.87
RL1	1.57	1	1.80	0.012	301	2.41	0.021	2.41	0.021	1.75	0.041	1.95	3.42
RL3	1.57	3	2.00	0.016	293	2.73	0.055	2.73	0.055	3.44	0.119	2.16	7.44
THsL0	1.62	–	3.18	0.015	850	–	–	3.18	0.015	1.00	0.033	2.20	2.20
THsL1	1.62	1	3.57	0.016	855	4.26	0.066	4.26	0.066	4.13	0.162	2.45	10.13
THsL1	1.62	3	3.70	0.018	858	4.92	0.070	4.92	0.070	3.89	0.190	2.71	10.56
THL0	1.24	–	3.80	0.032	1535	–	–	3.80	0.032	1.00	0.062	1.94	1.94
THL1	1.24	1	4.30	0.033	1536	4.56	0.068	4.56	0.068	2.06	0.138	2.03	4.18
THL3	1.24	3	4.20	0.041	1541	4.80	0.074	4.80	0.074	1.80	0.166	2.24	4.05

Table 4
Test results of beams of group R (with longitudinal bars and stirrups).

Beam code name	ρ_ℓ (%)	ρ_t (%)	V_f (%)	Elastic response until first cracking			Post-cracking		μ_T	ϑ_{85Tmax} (rad/m)	μ_{T85max}	μ_{T85cr}
				T_{cr} (kN m)	ϑ_{Tcr} (rad/m)	K (kN m^2/rad)	T_u = T_{max} (kN m)	ϑ_{Tu} = ϑ_{Tmax} (rad/m)				
RR0	1.57	0.63	–	1.60	0.015	305	2.31	0.073	4.87	0.096	1.32	6.40
RR1	1.57	0.63	1	2.00	0.016	329	2.73	0.088	5.50	0.120	1.36	7.50
RR3	1.57	0.63	3	2.21	0.014	350	3.15	0.101	7.21	0.200	1.98	14.29
THsR0	1.62	0.71	–	3.10	0.017	850	4.41	0.084	4.94	0.151	1.80	8.88
THsR1	1.62	0.71	1	3.28	0.021	836	4.93	0.114	5.43	0.250	2.19	11.90
THsR1	1.62	0.71	3	3.50	0.020	851	5.50	0.250	12.50	1.851	7.40	92.55
THR0	1.24	0.66	–	4.34	0.026	1581	5.60	0.100	3.85	0.153	1.53	5.88
THR1	1.24	0.66	1	4.40	0.030	1585	5.78	0.140	4.67	0.250	1.79	8.33
THR3	1.24	0.66	3	4.30	0.035	1578	7.41	0.156	4.46	1.716	11.00	49.03

increase of the torsional strength that ranges from 12% to 34% and from 25% to 55%, respectively. Furthermore, for the case of the T-flanged beams with $b_f/b_w = 3$ the addition of 1% and 3% steel fibre volume fraction caused an increase of the torsional strength that ranges from 3% to 20% and from 19% to 32%, respectively.

3.2. Cracking behaviour and failure modes

Control specimens (beams without steel fibres) exhibited typical torsional failures modes (see also photographs of Fig. 7): (i) Beams without bars and stirrups showed an abrupt collapse by crushing of concrete and separated in two parts (failure mode "A"). (ii) Beams with longitudinal bars only also failed in a similar manner exhibiting brittle torsional failure; they develop an intense single helical crack, but the bars prevented the splitting of the specimens in two parts (failure mode "B"). (iii) Beams with conventional reinforcement including bars and stirrups showed an increasing post-cracking behaviour along with the formation of a number of helical and diagonal cracks. In these cases as the torque increased some of these cracks became excessively wide and beams failed by transverse steel yielding before concrete crushing (failure mode "C").

The steel fibre beams exhibited different failure mechanisms since fibres prevented crack growth and crack propagation in concrete. The main benefit of steel fibre reinforcement is to establish a crack-control mechanism and to provide for a pseudo-ductility in the post-cracking behaviour. The improvement of the first-crack strength due to steel fibres is usually a secondary consequence

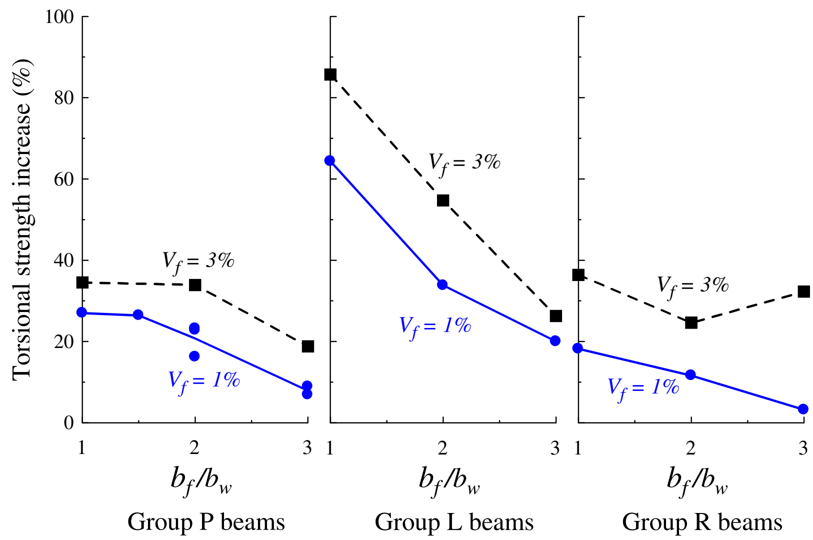

Fig. 6. Increase of the torsional strength of the tested beams due to the use of steel fibres.

which is related to the fibres quantity more than to the fibres efficiency. For low volume fibre content this improvement in the first-crack strength is of minor importance.

Fibrous beams without bars and stirrups displayed a typical failure of steel fibre concrete under tension since steel fibre pull-out was the main strength bearing mechanism. It is emphasized that fibres prevented the sudden failure of these specimens by controlling crack growth, especially in beams with high volume of fibres (3%) (failure mode "D"). Fibrous concrete beams with steel bars without stirrups developed dense diagonal and helical cracks and failed due to steel fibres debonding (failure mode "E"). Finally, conventionally reinforced beams with bars and stirrups containing steel fibres exhibited a highly ductile post-cracking behaviour, while the developed cracks were narrower and more closely spaced with a higher volume fraction of fibres used (failure mode "F"). Another characteristic of the failure modes "E" and "F" is that increasing the volume of fibres the number of cracks also increases, yet resulting in a significantly reduced crack width. Cracking patterns of typical tested beams at the end of their loading due to failure or approximately at the 85% of the maximum torsional moment (assumed as the end of the reliable response range) are illustrated in the photographs of Fig. 7.

3.3. Contribution of steel fibres to the post-elastic response

Test results indicated that the increase of the steel fibre fraction results in a significant improvement of the post-elastic behaviour of the beams, especially for the response branch after the maximum torsional moment. For a thorough understanding and evaluation of the fibres' contribution on the response and for comparison reasons of test results in terms of ductility, the following indices are introduced:

$$\mu_T = \frac{\vartheta_{Tmax}}{\vartheta_{Tcr}} \tag{2}$$

$$\mu_{T85max} = \frac{\vartheta_{85Tmax}}{\vartheta_{Tmax}} \tag{3}$$

$$\mu_{T85cr} = \frac{\vartheta_{85Tmax}}{\vartheta_{Tcr}} \tag{4}$$

where ϑ_{Tmax}: the rotation at the maximum torsional moment; ϑ_{85Tmax}: the rotation at the point of 85% of the maximum torsional moment, assumed as the end of the reliable response range.

Values of the above mentioned indices for the examined beams are presented in Tables 2–4, for the beams of groups P, L and R, respectively. From these values it is obvious that the primary advantage of the use of steel fibres is the significant improvement of the post-cracking torsional response, increasing the torque rotation capabilities of the conventionally reinforced beams without considerable loss of their strength.

3.4. Steel fibres as the only shear torsional reinforcement

The use of non-conventional fibrous reinforcement in torsional beams as the only shear reinforcement instead of common stirrups is an interesting alternative that has been proposed for special cases such as in strengthening works [25]. Recent experimental works also indicated that the addition of steel fibres in reinforced concrete beams under shear allows the reduction, and in special cases the total replacement, of the required stirrups [4–6]. This reduction of stirrups is suggested in cases where design criteria recommend high steel ratio of shear reinforcement, which leads to small stirrup spacing and, therefore, the use of steel fibres is more convenient.

The possibility of the use of steel fibres as the only shear reinforcement in torsional beams is also discussed in this experimental research. For this reason, comparisons of the torsional behavioural curves of (i) the non-fibrous reinforced concrete beams with bars and stirrups with (ii) the fibrous beams with longitudinal reinforcement and steel fibres with volume fraction 1% and 3% are presented in Fig. 8a, b and c.

Despite the rather limited experimental data curves shown in Fig. 8 (nine tested beams) these comparisons indicate a potential for using steel fibres as a replacement for stirrups in torsional beams. The experimental curves of Fig. 8 show that this alternative could be achieved under specific circumstances with adequate quantities of steel fibres and this replacement seems to be more efficient in rectangular beams than in flanged ones.

It is also mentioned that in order to completely substitute the conventional shear reinforcement with steel fibres, it is necessary the fibrous concrete beams to obtain at least equal performances in terms of strength and ductility with the corresponding conventionally reinforced beams. Regarding the strength, it is possible to provide the same value if adequate percentage of fibres is added. However, the ductility resources differ in the case of use of fibres comparing to the ones of conventional steel reinforcement, because in the first case it is related to the gradual pull-out of fibres, whereas in the second it is related to the plastic deformation of the steel reinforcement. For this reason, the complete substitution is

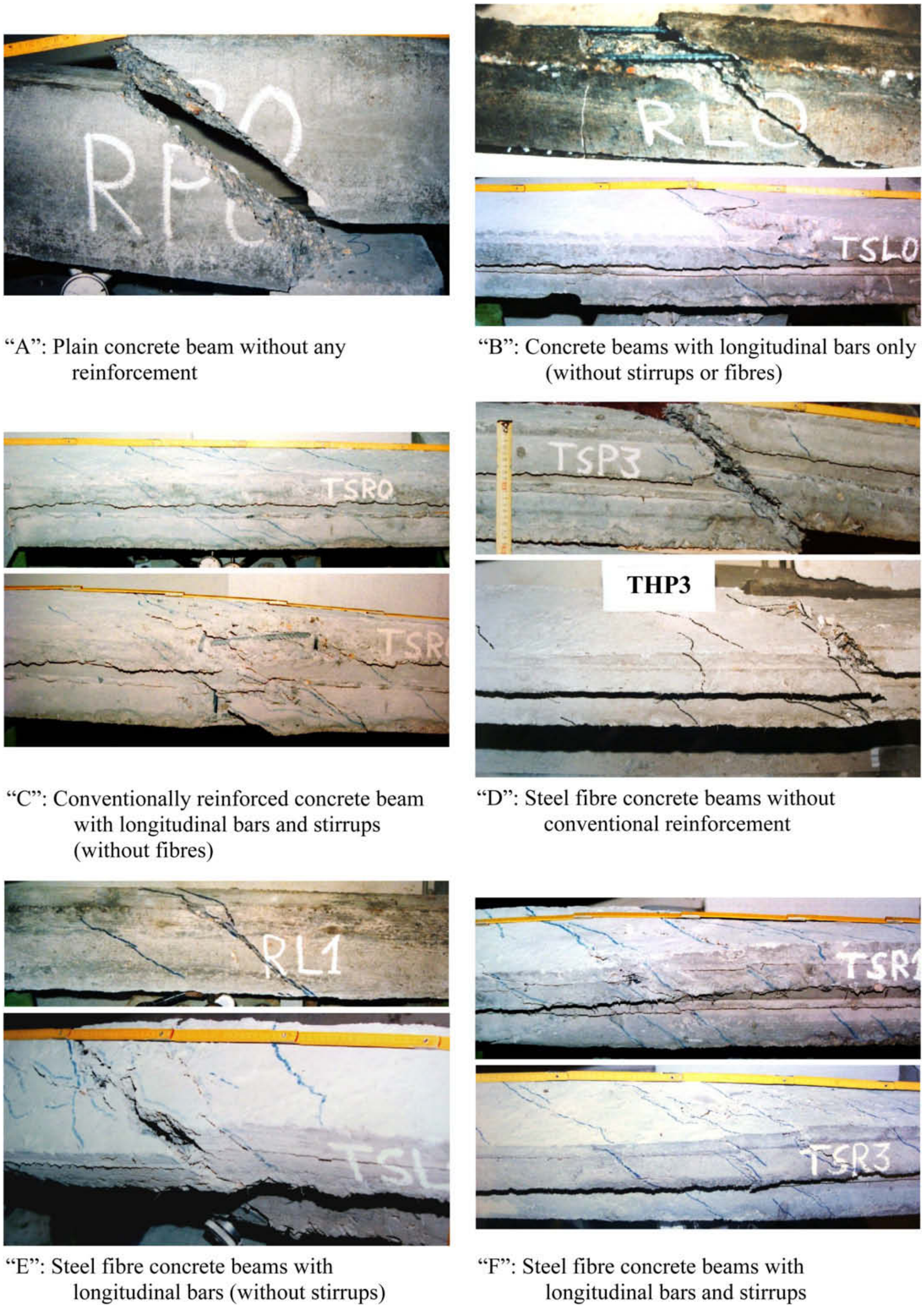

Fig. 7. Failure modes of typical tested specimens.

not efficacious when high amplitude of the shear crack width occurs.

4. Analytical results

4.1. Fibrous beams without conventional reinforcement

The classical elastic Saint Venant's theory to the torsion problem, although it properly describes the elastic behaviour, fails to predict the ultimate torsional strength of concrete elements. Analytical solutions of the governing equation deduced by this approach are based on the assumption that brittle failure occurs when the maximum developing tensile stress reaches the material maximum tensile strength. By ignoring the post-cracking tension softening phenomenon of the material, they consistently underestimate the ultimate torsional strength of the element. For plain concrete members the ultimate torque has been experimentally found to be roughly up to 50% greater than predicted ones [7,26]. Further-

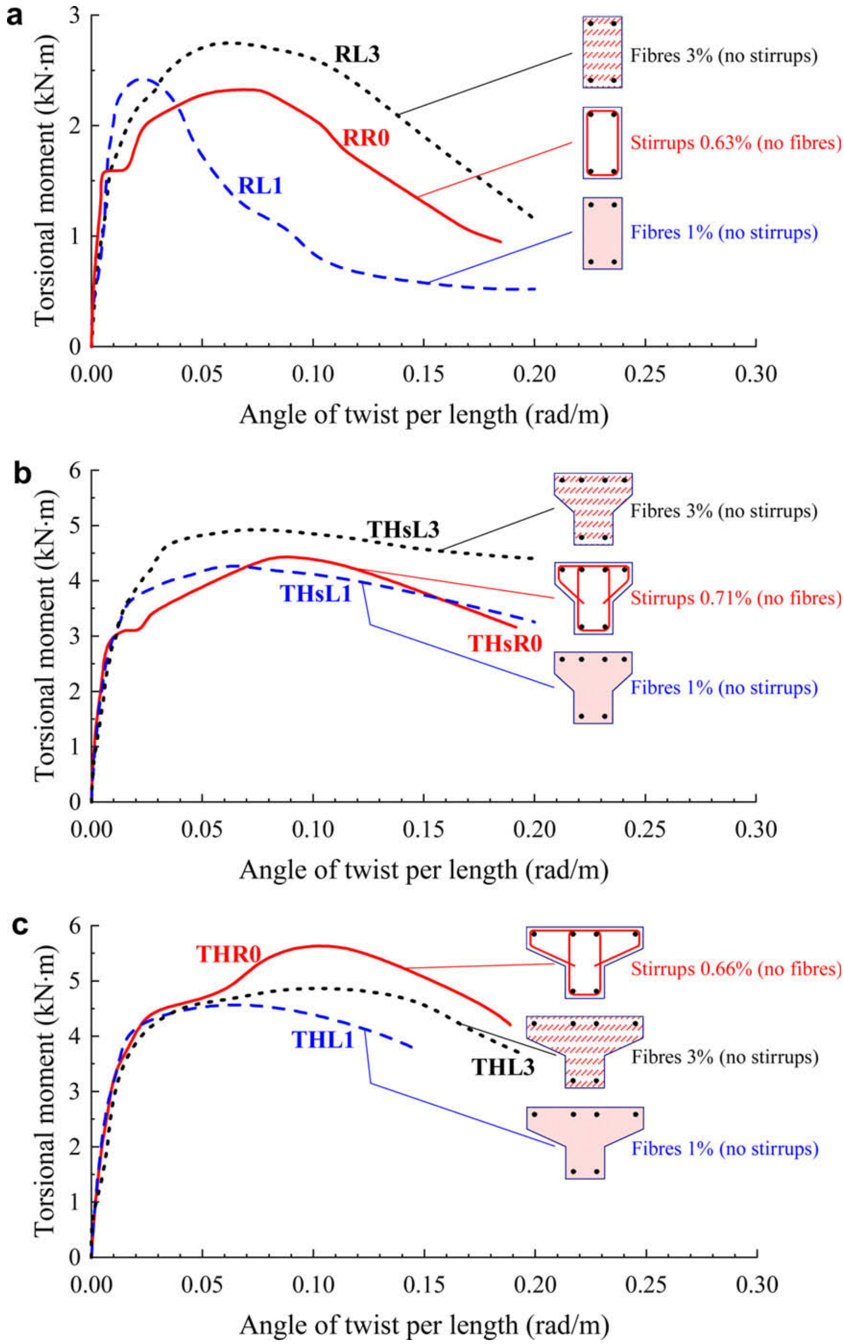

Fig. 8. Investigation of the use of steel fibres as the only shear torsional reinforcement instead of stirrups.

more, in the case of steel fibre concrete elements the post-cracking strength of the composite material represents an even more important part of the element strength and energy absorption capacity.

Hsu has proposed the skew bending theory for the prediction of the torsional strength of plain concrete beams [7]. Other subsequent researchers have proposed modified versions of this theory in order to predict the torsional strength of steel fibre concrete members [9–11]. Relations yielded by these efforts are presented and used for the prediction of the torsional strength of beams tested in the present work:

$$-\text{ Craig et al. [9]}: \quad T_{u,sb} = 140.23\left(\frac{b^2}{645.16}+10\right)h^3\sqrt{f'_{cf}} \tag{5a}$$

$$T_{u,sb} = \frac{b^2h}{3}(0.85f_{rf}) \tag{5b}$$

$$-\text{ Mansur \& Paramasivam [10]}: \quad T_{u,sb} = \frac{b^2h}{3}(0.71f_{rf}) \tag{6}$$

$$-\text{ Wafa et al. [11]}: \quad T_{u,sb} = \frac{b^2h}{3}(0.71f_{ctf,split}) \tag{7}$$

where f'_{cf}, f_{rf}, and $f_{ctf,split}$ are the compression, the rupture and the splitting strength of the composite mixture (steel fibre concrete), respectively.

Recently, a new efficient numerical algorithm that employs a smeared crack approach for the analysis of plain concrete elements in torsion has been proposed by Karayannis [27]. This analytical model is initially based on Saint Venant's theory and utilizes a special numerical technique that can be regarded as a combination of the numerical techniques of finite elements and finite differences [8]. This scheme can easily be applied to elements with practically

Table 5
Comparisons between experimental and analytical results for beams of group P.

Beam code name	T_{exp} (kN m)	$T_{cal}^{(5a)}$ (kN m)	$\frac{T_{exp}}{T_{cal}^{(5a)}}$	$T_{cal}^{(5b)}$ (kN m)	$\frac{T_{exp}}{T_{cal}^{(5b)}}$	$T_{cal}^{(6)}$ (kN m)	$\frac{T_{exp}}{T_{cal}^{(6)}}$	$T_{cal}^{(7)}$ (kN m)	$\frac{T_{exp}}{T_{cal}^{(7)}}$	T_{cal}^{mod} (kN m)	$\frac{T_{exp}}{T_{cal}^{mod}}$
RP1	1.84	1.79	1.03	2.32	0.79	1.94	0.95	1.37	1.34	1.65	1.12
RP3	1.95	1.85	1.05	2.56	0.76	2.14	0.91	1.51	1.29	1.94	1.01
LsP1	2.21	2.24	0.99	2.71	0.82	2.26	0.98	1.59	1.39	2.25	0.98
LP1	2.65	2.81	0.94	3.63	0.73	3.04	0.87	2.14	1.24	2.74	0.97
TsP1	2.72	2.65	1.03	3.01	0.90	2.51	1.08	1.77	1.54	2.91	0.93
TP1	2.96	3.37	0.88	4.01	0.74	3.35	0.88	2.36	1.25	3.28	0.90
THsP1	3.39	2.85	1.19	3.23	1.05	2.70	1.26	1.90	1.78	3.19	1.06
THsP3	3.69	3.11	1.19	5.16	0.72	4.31	0.86	3.04	1.21	3.77	0.98
THP1	4.05	3.66	1.11	3.92	1.03	3.27	1.24	2.31	1.75	4.00	1.01
THP3	4.42	3.93	1.12	8.94	0.49	7.47	0.59	5.26	0.84	4.53	0.98
			1.05		0.80		0.96		1.36		0.99
			0.102		0.163		0.195		0.277		0.061

any cross-section because it includes numerical mapping and it has no need of extended computing power since it does not make use of any stiffness matrix calculations. The smeared crack model uses the behaviour of a crack process zone which is described by constitutive relations expressed in terms of normal stress and crack width. The total strain is considered as the sum of elastic and fracture strain components. For the determination of the fracture strain the crack process zone width is used, which is attributed to the material, the type of loading and the size of the beam [26,27].

This method has been extended and modified [20,21] in order to incorporate a stress–strain model with a non-linear post-cracking branch for the fibrous material behaviour in tension to make this method applicable to the torsion problem of steel fibre concrete beams. This approach takes into account the volume fraction of fibres, the tensile strength of concrete and parameters concerning the geometry, the orientation and the slip mechanism of the fibres. Detailed derivation of the equations and the solution technique of this analytical model can be found in Refs. [20], [21] and [27]. Extensive comparisons between analytical results yielded by these analyses and experimental data derived from a broad range of parametrical studies established the validity of these analytical models for plain and steel fibre concrete beams subjected to pure torsion or combined torsion with shear force, bending moment and axial force [21,26,28].

Based on the above mentioned analytical relationships and the analytical model of the author, predictions of the torsional moment capacities for the tested beams of group P are presented and compared with the experimental ones in Table 5. In this Table, the calculated values of $T_{cal}^{(5a)}, T_{cal}^{(5b)}, T_{cal}^{(6)}, T_{cal}^{(7)}$ and T_{mod} represent the analytically predicted ultimate torsional moments of the steel fibre beams derived from Eqs. (5a), (5b), (6), (7), and the analytical model, respectively. The mean values and the standard deviation values of the ratios of the experimental to the calculated torsional strengths are also summarized and compared in Table 5.

4.2. Rectangular fibrous beams with conventional reinforcement

The typical torque moment versus angle of twist experimental curve of a steel fibre concrete beam with longitudinal bars and stirrups (see for example the experimental curve of beam RR3 in Fig. 5) comprises two distinct regions: an elastic region up until first cracking and a post-cracking branch. The different character of the response in these regions reveals the different nature of the load resisting mechanism in each case. The pre-cracking part is characterized by the high value of the torsional stiffness and the element behaves as a homogeneous concrete element since the influence of the steel reinforcement is of minor importance. The post-cracking part is characterized by a further increase of the torque moment at a lower rate, depending on the volume of the transverse reinforcement. The consistently decreasing torsional stiffness reveals the different nature of the mechanism of resistance. It is noted that in conventionally reinforced concrete beams with bars and stirrups (without fibres), the transition from the one region to the other is usually abrupt and it could be characterized by the lack of a bearing mechanism soon after cracking and before the activation of the reinforcements and the concrete struts [7,15,17,29]. This is more or less justified by the formation of an almost horizontal plateau in the experimental curves of T versus ϑ between these two regions (see for example the experimental curves of the beams RR0, THsR0 and THR0 in Figs. 5 and 8). However, in fibrous concrete beams with bars and stirrups, the transition from the pre-cracking to the post-cracking region seems to be gradual because of the steel fibres which provide crack-bridging and significant resistance to shear across developing cracks (see also the T versus ϑ curves of the beams with steel fibres in Fig. 5).

Based on the this observation and aiming to the description of the entire torsional behaviour of reinforced concrete elements, the combination of two different theoretical models has already been adopted for reinforced concrete beams by Chalioris [29], [30] and for fibrous reinforced concrete beams by the authors [17] and by a recent study of Gunneswara Rao and Rama Seshu [18]. In these combined methodologies, the elastic response until the first cracking is described by specially modified approaches of the classical elastic theory, whereas the post-cracking part is described by specially extended versions of the softened truss model developed by Hsu & Mo [31]. In these modified truss models the contribution of steel fibres has been incorporated as an additional reinforcement component that contributes to the torsional resistance along with the conventional steel reinforcement. It is also noted that these combined methods can provide full analytical torsional curves for the behaviour of rectangular fibrous beams with conventional reinforcement.

The above described combined methods were used in this study in order to analytically predict the values of the torsional moment at cracking, the angle of twist per unit length at cracking, the initial

Table 6
Comparisons between experimental and analytical results for the rectangular beams of group R.

Beam code name	Elastic response until first cracking		Post-cracking response		
	$\frac{T_{cr}^{exp}}{T_{cr}}$ (kN m)	$\frac{\vartheta_{Tcr}^{exp}}{\vartheta_{Tcr}^{cal}}$ (rad/m)	$\frac{K^{exp}}{K^{cal}}$ (kN m^2/rad)	$\frac{T_u^{exp}}{T_u^{cal}}$ (kN m)	$\frac{\vartheta_{Tu}^{exp}}{\vartheta_{Tu}^{cal}}$ (rad/m)
RR0	$\frac{1.60}{1.41} = 1.13$	$\frac{0.015}{0.013} = 1.15$	$\frac{305}{298} = 1.02$	$\frac{2.31}{2.27} = 1.02$	$\frac{0.073}{0.089} = 0.82$
RR1	$\frac{2.00}{1.65} = 1.21$	$\frac{0.016}{0.014} = 1.14$	$\frac{329}{332} = 0.99$	$\frac{2.73}{2.74} = 1.00$	$\frac{0.088}{0.085} = 1.04$
RR3	$\frac{2.21}{1.93} = 1.15$	$\frac{0.014}{0.014} = 1.00$	$\frac{350}{340} = 1.03$	$\frac{3.15}{3.14} = 1.00$	$\frac{0.101}{0.087} = 1.16$

pre-cracking torsional stiffness, the post-cracking ultimate torsional moment and the corresponding angle of twist per unit length for the rectangular beams of group R (RR0, RR1 and RR3). These analytical values are presented and compared with the experimental ones in Table 6.

5. Concluding remarks

The experimental program of this study includes torsional tests of 35 rectangular and flanged beams with various steel fibre and conventional steel reinforcement arrangements. The main objective of this study is to investigate the effectiveness of the use of short steel fibres on the torsional response of concrete beams with rectangular, L- and T-shaped cross-section. Based on the test results presented herein, the following concluding remarks are drawn:

Fibrous concrete beams exhibited improved overall torsional performance with respect to the corresponding non-fibrous control beams. The main contribution of steel fibres on the torsional behaviour is mainly observed after concrete cracking.

The addition of steel fibres was essential to the beams without conventional steel reinforcement since fibres were the only reinforcement and proved capable to provide enhanced torsional moment capacities, especially in the beams with high volume of fibres ($V_f = 3\%$). The presence of fibre reinforcement even at low percentage was very effective in preventing the sudden brittle failure of both rectangular and non-rectangular beams by implying crack-control mechanism.

After concrete cracking, the favourable effect of steel fibres was clearly demonstrated to the post-cracking response of the beams with longitudinal only reinforcement. Fibre concrete beams displayed further increase on the torsional strength, whereas the corresponding control specimens did not exhibit increasing post-cracking response.

Increasing the steel fibre volume fraction a significant improvement of the post-elastic response of the beams is achieved, especially after the maximum torque moment. However, these improvements on the torque capacities seemed to be reduced in the cases of the flanged beams with respect to the rectangular ones.

The use of steel fibres as the only shear torsional reinforcement in beams without stirrups has also been investigated in this study. Experimental torque curves indicated that short steel fibres is a reliable way to increase the post-elastic twist capabilities of the beams. Furthermore, comparisons between the experimental results showed that the use of high fractions of steel fibres ($V_f = 3\%$) in torsional beams with longitudinal bars (without stirrups) provided increased strength and ductility capabilities. This improvement due to the addition of steel fibres seemed to be more effective in the beams with rectangular cross-section than in the corresponding flanged beams.

The analytical results of existing numerical approaches for predicting the torsional strength of steel fibre reinforced concrete beams have been presented and compared to the observed data to estimate the effectiveness of these models. It is emphasized that the prediction of the entire torsional behaviour of the rectangular fibrous concrete beams with longitudinal bars and stirrups has been achieved using analytical methods that combine (a) a modified approach of the classical elastic theory for the elastic response until the first cracking and (b) an extended version of the softened truss model for the post-cracking part. Analytical results yielded from this combined methodology proved to be in good compliance with the experimental ones.

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