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Packing density measurement and modelling of fine aggregate and mortar

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ABSTRACT

A newly developed wet packing method has been applied to measure the packing densities of blended fine aggregates (each a mixture of fine aggregates of different sizes) and mortars (each a mixture of cement and blended fine aggregate) under the wet condition, with or without superplasticizer added, and with or without compaction applied. For the blended fine aggregates, the conventional dry packing method has also been employed to measure their packing densities under the dry condition and the results show that the packing density of fine aggregate is generally higher under wet condition than under dry condition. For both the blended fine aggregates and mortars, the measured packing densities have been compared to the predicted packing densities by two existing packing models. Good agreement between the measured and predicted packing densities has been achieved with the mean absolute error being 2.1% for the blended aggregates and 1.1% for the mortars. This is the first time that the packing densities of mortar samples are directly measured and compared to predictions by packing models to verify the applicability of the wet packing method and the packing models.

1. Introduction

The packing of particles has been attracting the interest of researchers from various disciplines for over a century. This is not surprising because it is an important factor governing the properties of materials formed in many branches of industry, such as ceramics processing [1] and powder metallurgy [2]. As concrete mixes are composed of particles as well, the packing of the constitutive particles should also be playing certain roles in their behaviour. In fact, research on concrete has, for several decades, revealed that the packing of the particle system has significant effects on the rheological properties of concrete mixes [3–14]. However, the research carried out so far is still not sufficient to allow accurate measurement and prediction of the packing density and rheology of cement paste, mortar and concrete. For this reason, the first author has been advocating, in recent years, more research on particle packing. As will be explained in the following, through better understanding of the packing of the particle system in concrete, we may be able to optimize the mix compositions for the achievement of higher performance at basically no extra cost.

The concept of particle packing has already been applied to the aggregate proportioning for optimum concrete mix design. Early in the 1960s, Powers [3] had suggested that a concrete mix may be considered a mixture of aggregate particles and cement paste. He conceived that the paste trapped inside the voids in the bulk volume of aggregate is not effective in lubricating the concrete mix and that only the excess paste (the paste in excess of that needed to fill up the voids) is effective in lubricating the concrete mix. Hence, if the packing density of the aggregate is increased, the amount of paste needed to fill up the voids will be reduced and consequently, for the same amount of paste, there will be more excess paste to improve the workability, or for the same required workability, the amount of paste may be reduced. Based on this excess paste theory, one of the performance criteria for aggregate proportioning has been set as the achievement of a high packing density.

Following the geometric similarity principle, the excess paste theory may be extended to postulate that in a cement paste or mortar, the water trapped inside the voids between the constitutive particles is not effective in lubricating the solid–water mixture and that only the excess water (the water in excess of that needed to fill up the voids) is effective in lubricating the solid–water mixture. Hence, the rheological performance of cement paste and mortar may be improved by increasing the packing density of the constitutive particles. For example, by incorporating condensed silica fume to fill up the voids, Lange et al. [4] have increased the packing density of the cementitious materials to reduce the water demand and the water/cementitious materials (W/CM) ratio so as to improve the strength and durability of the mortar produced. Likewise, Kwan [5] has found that at a W/CM ratio lower than 0.28, the addition of condensed silica fume to fill up the voids and increase the packing density can improve the workability of the concrete mix. More recently, Xie et al. [6] have blended cement with an ultra-pulverized fuel ash to increase the packing density and produce a higher performance concrete.

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Extensive research on how the packing density, excess paste and excess water affect the rheology of concrete has been carried out [7–10]. However, the packing densities of cement paste, mortar and concrete have rarely been directly measured (herein, the packing density of a solid–water mixture means the packing density of the solid particles in the mixture). For fine and coarse aggregates, there are standard test methods for measuring the packing density under dry condition but for cementitious materials, there is up to now no generally accepted test method for measuring the packing density under dry or wet condition. As a result, it remains a difficult task to measure the packing densities of cement paste, mortar and concrete. To resolve this problem, Wong and Kwan [11] have recently developed a wet packing method for measuring the packing density of cementitious materials under wet condition. This method has been applied to study the effect of packing density on rheology of cement paste [12] and extended for application to fine aggregate [13].

On the other hand, packing models have been developed for predicting the packing density of polydisperse solid particles (solid particles of different sizes mixed together), as will be depicted hereafter. These could be useful tools for predicting the packing densities of cement paste, mortar and concrete, and for future packing density optimization. But, before they can be applied to cement paste, mortar and concrete, they need to have their applicability to each kind of such solid–water mixtures verified. In a recent study [14], the packing densities of cementitious materials containing cement, pulverized fuel ash and condensed silica fume have been measured using the wet packing method and the measured results compared to the predicted values by the existing packing models. The measured and predicted packing densities agreed very well with each other, thus verifying the applicability of both the wet packing method and the existing packing models to cementitious materials (or, in other words, cement paste).

So far, the applicability of the wet packing method and the existing packing models to mortar containing both cementitious materials and fine aggregate has not yet been verified. In this study, the packing densities of a number of blended fine aggregate and mortar samples have been measured using the wet packing method and the measured results compared to the predicted results by the existing packing models to see if the independently measured and predicted results would agree with each other. Such comparison would help to identify the deficiencies, if there are any, and cross-check the accuracy of the wet packing method and the existing packing models when applied to mortar. This is an important step for further development of the wet packing method and the packing models, which are indispensable tools for studying the effects of packing and for incorporating the concept of packing into mix design methods for high-performance concrete.

2. Packing density measurement

In the Belgian Standard NBN B11-206 [15], British Standard BS 812: part 2 [16], European Standard EN 1097-3 [17] and European Standard EN 1097-4 [18], test methods for measuring the packing densities of fine and coarse aggregates under the dry condition have been provided. Although the test methods in different standards differ slightly in certain details, they are basically the same in the sense that they all measure the bulk density of the solid particles under the dry condition and determine the packing density as the ratio of bulk density to solid density. All these test methods may be categorized as the dry packing method. Since any compaction applied can significantly increase the packing density, the packing density is dependent on the form and amount of compaction applied and the compacted packing density is generally higher than the uncompacted packing density.

Up to now, the above dry packing method is the only standardized method for packing density measurement. In 2004, DeSchutter and Poppe [19] measured the packing density of sand by the dry packing method and correlated the water demand of cement–sand mortar to the packing density of the sand. In 2007, Miyake and Matsushita [20] measured the packing density of all the constitutive particles in cement–sand mortar (inclusive of the cementitious materials and sand) using the dry packing method and found that the workability of mortar is governed mainly by the water film thickness to particle size ratio, which may be determined from the water content of the mortar and the packing density of the constitutive particles.

However, the dry packing method has the major problem that when applied to very fine particles, such as those of cementitious materials, it tends to underestimate the packing density because of agglomeration of the particles arising from the presence of inter-particle forces [21]. Furthermore, in the context of cement paste, mortar or concrete, the possible effects of the mixing water and chemical admixtures have all been excluded. To avoid these shortcomings, some researchers have chosen to measure the packing density of cementitious materials under the wet condition. For instance, DeLarrard [8] determined the packing density from the water content needed to produce a thick paste whereas Dewar [22] determined the packing density from the water content needed to form a cement paste at standard consistence. These indirect methods are based on the faith that when the cement paste is at the prescribed consistence, the water content is just enough to fill up the voids between the cement grains. In actual fact, however, such faith has never been proven.

The authors are of the view that the packing density under the wet condition should better be measured directly as the maximum solid concentration of the solid–water mixture that can be achieved, as exemplified by the so called "wet packing method" for cementitious materials developed by the authors' research team [11]. Basically, this wet packing method mixes the solid particles with water at varying water content, determines the solid concentration of each solid–water mixture so formed and takes the maximum solid concentration achieved as the packing density of the solid particles. More recently, it was found that with just minor modifications, this method is applicable also to fine aggregate [13].

In the present study, the wet packing method is extended for application to mortar containing both cementitious materials and fine aggregate. As details of the method have been presented before [11,13], only the salient points pertinent to mortar are presented herein. To measure the packing density of mortar, the constitutive solids, namely, the cementitious materials and fine aggregate, are first mixed together under dry condition. Then, the dry solid mixture is added with water at different water/solid ratios (note: every such ratio referred to hereafter is by volume and denoted as W/S ratio). For each mortar sample formed at a known W/S ratio, the bulk density is measured by filling the mortar into a cylindrical mould of known volume V and weighing the mass M of the mortar inside the mould. From the bulk density M/V, the solid concentration ϕ of the mortar may be determined as:

$$\phi = \frac{M/V}{\rho_w u_w + \rho_\alpha R_\alpha + \rho_\beta R_\beta + \rho_s R_s} \tag{1}$$

where ρ_w is the density of water, ρ_α and ρ_β are, respectively, the densities of the cementitious material α and the cementitious material β, ρ_s is the density of the fine aggregate, u_w is the W/S ratio, and R_α, R_β and R_s are, respectively, the volumetric ratios of α, β and fine aggregate to the total solids content (note: there needs not be any limit to the types of cementitious materials and if there is another cementitious material γ with a density of ρ_γ and a volumetric ratio of R_γ, its presence can be accounted for simply by adding the term $\rho_\gamma R_\gamma$ to the denominator at the right hand side).

The above procedure of mixing the dry solid mixture with water at a certain W/S ratio and determining the solid concentration ϕ of the mortar sample so formed is repeated at different W/S ratios. To obtain the maximum solid concentration as the packing density of the mortar, it is recommended to start with a W/S ratio high enough to form a saturated solid–water mixture (for the case of mortar, a W/S ratio of 0.8 should be high enough) and then reduce the W/S ratio successively until the solid concentration ϕ has reached a maximum value and then dropped.

3. Packing theory and models

In theory, the packing density of monosize spheres can reach 0.74, if the spheres are carefully arranged in an ordered pattern. In reality, however, the spheres tend to be randomly arranged with a packing density of only 0.60 when un-vibrated or 0.64 when vibrated [23]. In general, the packing density of monosize particles is governed by two factors: the packing condition and particle shape [24]. Regarding the packing condition, as the particles are vibrated or compacted to overcome the friction between them so that they could rearrange to a closer packing, the packing density increases. Regarding the particle shape, the packing density is generally higher for more spherical particles and lower for less spherical (or more angular) particles. Taking the sphericity (the ratio of the surface area of a sphere having the same volume as the particle to the actual surface area of the particle) as a measure of how close the particle shape is to a perfect sphere [25], the packing density is an increasing function of the sphericity of the particles [26].

When it comes to a polydisperse particle system, the packing density would be governed also by the particle size distribution of the particles [23,24]. Consider the simplest case of a binary mix of particles of two distinct sizes. When a small amount of smaller particles is added to the larger particles, the smaller particles would fill the voids between the larger particles and thereby increase the packing density (filling effect) while when a small amount of larger particles is added to the smaller particles, the larger particles would occupy a solid volume within the bulk and porous volume of the smaller particles and thereby also increase the packing density (occupying effect). Hence, blending of different size particles together could increase the packing density. However, if the particles are not of distinct size, there would be particle interactions such that the smaller particles would loosen the packing of the larger particles in contact with each smaller particle (loosening effect) or the larger particles would act like walls and reduce the packing of the smaller particles in contact with each larger particle (wall effect).

To cater for multiple mixes of different size particles, the above binary packing model has been extended to a variety of packing models, most of which are based on the linear packing theory [27] and may thus be classified as linear packing models. The linear packing theory postulates that among the multiple components (each comprising of all the particles of a certain size) mixed together, there is at least one component that is fully packed and dominant (dominant in the sense that it divides the other components into smaller size and larger size components, which fill voids and occupy solid volumes, respectively). With each component assumed to be dominant in turn, a series of packing density functions (each corresponding to one component assumed to be dominant) may be obtained, from which the minimum may be taken as the predicted packing density of the multiple mixes of particles. In the 1980s, this theory has been refined to account for particle interactions between similar size particles by Stovall et al. [24] and Yu and Standish [28].

The afore-mentioned packing density functions, which require the input of the packing density of each component and the particle size distribution of the particle system, are given by:

$$\gamma_i = \frac{\beta_i}{1 - \sum_{j=1}^{i-1}[1 - \beta_i + w(r)\beta_i(1 - 1/\beta_j)]y_j - \sum_{j=i+1}^{n}[1 - l(r)\beta_i/\beta_j]y_j} \tag{2}$$

where γ_i is the predicted packing density if component i is dominant, β_i and β_j are the packing densities of components i and j, y_j is the volumetric fraction of component j, r is the size ratio between the components i and j (evaluated as the ratio of the size of the smaller size component to the size of the larger size component), and $l(r)$ and $w(r)$ are the interaction functions accounting for loosening and wall effects, respectively. The interaction functions given by the different packing models are not quite the same. In this study, two most advanced and popular packing models are employed for packing density prediction. They are the one developed in 1996 by Yu et al. [29] (referred to hereafter as Model A) and the one developed in 1999 by DeLarrard [8] (referred to hereafter as Model B).

In Model A, the interaction functions accounting for the particle interactions between component i and component j are given by:

$$l(r) = 1 - (1 - r)^{3.3} - 2.8r(1 - r)^{2.7} \tag{3}$$

$$w(r) = 1 - (1 - r)^{2.0} - 0.4r(1 - r)^{3.7} \tag{4}$$

To account for the effects of particle shape on the particle interactions, the size of each component for the determination of the size ratio r is expressed in terms of an equivalent packing diameter d_p calculated using the following formula [26]:

$$d_p = \frac{d_v}{\psi^{2.785} exp[2.946(1 - \psi)]} \tag{5}$$

in which d_v is the diameter of a sphere having the same volume and ψ is the sphericity of the component.

In Model B, the interaction functions accounting for the particle interactions between component i and component j are given by:

$$l(r) = \sqrt{1 - (1 - r)^{1.02}} \tag{6}$$

$$w(r) = 1 - (1 - r)^{1.50} \tag{7}$$

This model does not consider the effects of particle shape on the particle interactions. Instead, this model includes a compaction index K in the formulation to take into account the effect of compaction. A number of K-values for different degrees of compaction have been suggested [8]. When perfect compaction is applied, the value of K is set equal to infinity and the predicted packing density is just taken as that given by Eq. (2).

To highlight the differences between Model A and Model B, their interaction functions are shown and compared graphically in Figs. 1 and 2. More mathematical details of the two packing models can be found in the original references.

4. Testing program

A testing program comprising of two parts: Part A and Part B, was launched. In Part A, fine aggregate samples were measured for their packing densities under different test conditions and using different test methods. Altogether, six test conditions, designated as D1, D2, W1, W2, W3 and W4, had been applied for the packing density tests, as summarized in Table 1. Under the conditions D1 and D2, the aggregate samples were tested using the dry packing method stipulated in British Standard BS 812: part 2 [16] with no compaction applied and with compaction applied, respectively. Under the conditions W1, W2, W3 and W4, the aggregate

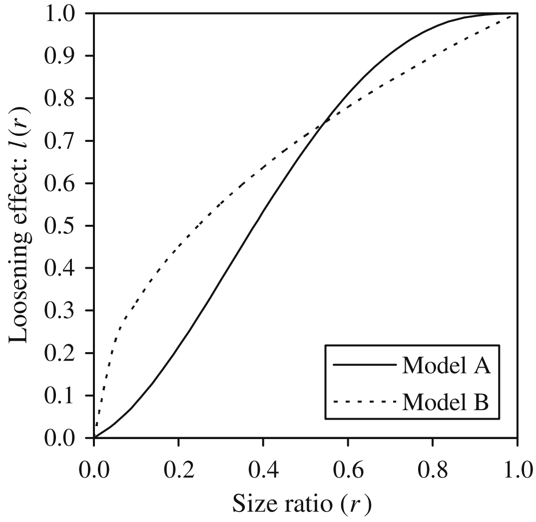

Fig. 1. Interaction function for loosening effect.

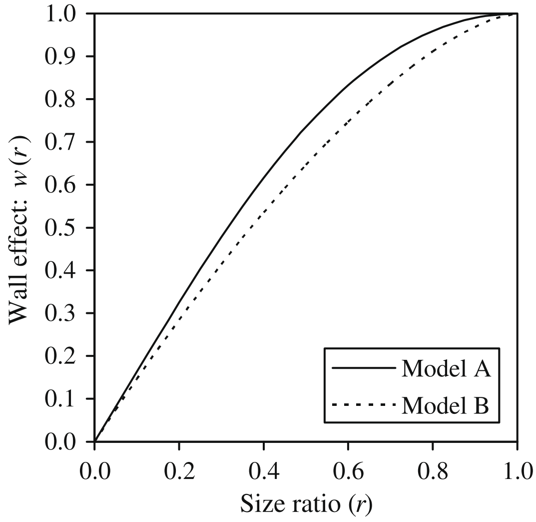

Fig. 2. Interaction function for wall effect.

Table 1
Test conditions.

Test condition	Packing method	Compaction applied	Superplasticizer added
D1	Dry	Uncompacted (BS 812: part 2: 1995)	Nil
D2		Compacted using a tamping rod (BS 812: part 2: 1995)	Nil
W1	Wet	Uncompacted	Nil
W2		Uncompacted	Added
W3		Compacted using a jolting table	Nil
W4		Compacted using a jolting table	Added

Table 2
Four size classes of crushed rock fine aggregate.

Size class	Sieve size range	
	Lower sieve size (sieve retained on)	Upper sieve size (sieve passed through)
S1	75 μm	150 μm
S2	150 μm	300 μm
S3	300 μm	600 μm
S4	600 μm	1.18 mm

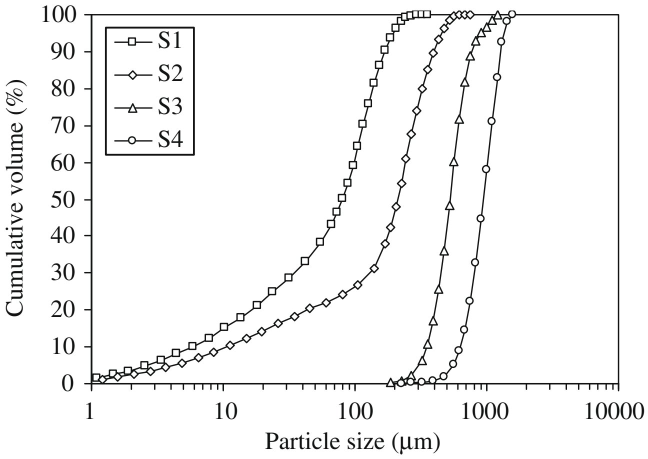

Fig. 3. Particle size distributions of aggregate size classes S1–S4.

Table 3
Mix proportions of the blended fine aggregates.

Blended aggregate no.	Volume fractions of different size classes (%)			
	S1	S2	S3	S4
B0	0	0	0	100
B1	0	0	40	60
B2	0	25	25	50
B3	25	25	25	25

samples were tested using the newly developed wet packing method with or without superplasticizer (SP) added and with or without compaction applied, as depicted in Table 1. For the last two conditions, the compaction was applied by subjecting the steel container filled with the aggregate sample to 90 blows on a jolting table.

The fine aggregate tested was crushed rock fine of granite rock origin. From the fine aggregate, four size classes of aggregate particles were produced by sieving. Each size class was a portion of the aggregate particles retained on a certain sieve after sieving. The four size classes so produced were named as S1, S2, S3 and S4, which were, respectively, the portions of aggregate particles retained on the sieves of size 75 μm, 150 μm, 300 μm and 600 μm, as listed in Table 2. Their particle size distributions, measured by the laser diffraction method, are presented in Fig. 3. It should be noted that due to agglomeration of the fine particles, there were some particles finer than 75 μm in the size class S1 and some particles finer than 150 μm in the size class S2. In addition, three blended mixes of aggregate, named as B1, B2 and B3, were also produced by blending the size class S4 with different proportions of smaller size classes, as listed in Table 3. For comparison, the non-blended aggregate obtained directly from the size class S4 was named as B0.

The relative density of the fine aggregate was measured to be 2.61 under saturated and surface dry condition and 2.59 under oven dried condition. All the aggregate samples had been oven dried before the packing density tests. For the wet packing tests, the water absorption of the aggregate was taken into account in the calculation of the packing density results. The SP added to the solid–water mixtures under the conditions W2 and W4 was a third-generation polycarboxylated-based superplasticizer with a solid content of 20% and a relative density of 1.03. According to the supplier, the recommended dosage of this SP in terms of liquid mass should be 0.5–3.0% of the mass of cement. In this context, the SP dosage was based on the mass of aggregate and the highest recommended dosage of 3.0% was employed.

In Part B, the packing density of mortar (in other words, the packing density of the solid particles in the mortar) was investigated. Since there was no applicable dry packing method, the packing densities of the mortar samples were measured using only the wet packing method. In total, twelve mortar mixes containing cement as the only cementitious material and crushed granite rock fine with a maximum size of 1.18 mm as the fine aggregate were produced for packing density tests. These mixes were designed to have the same cement to fine aggregate ratio by solid volume of 3:4 but different fine aggregate proportions, as depicted in Table 4.

For the mortar mixes, the cement used was an ordinary Portland cement of strength class 52.5 N, which was tested to have a relative density of 3.11 and a Blaine fineness of 338 m^2/kg. Its particle size distribution, measured by the laser diffraction method, is plotted in Fig. 4. On the other hand, the same fine aggregate and SP as those in Part A were used. All the fine aggregate had been oven dried before the packing density tests and the water absorption of the aggregate was taken into account in the calculation of the packing density results. Regarding the SP added to each mortar sample, the highest recommended dosage of 3.0% of the mass of cement was employed. No compaction was applied to the mortar samples. This corresponded to the test condition W2.

Table 4
Fine aggregate proportions of the mortar mixes.

Mortar mix no.	Volume fractions of different size classes in fine aggregate (%)			
	S1	S2	S3	S4
M1	25	25	25	25
M2	20	20	20	40
M3	0	25	25	50
M4	25	25	0	50
M5	25	25	50	0
M6	0	15	35	50
M7	15	0	35	50
M8	15	35	0	50
M9	0	40	60	0
M10	40	0	60	0
M11	40	0	0	60
M12	50	0	0	50

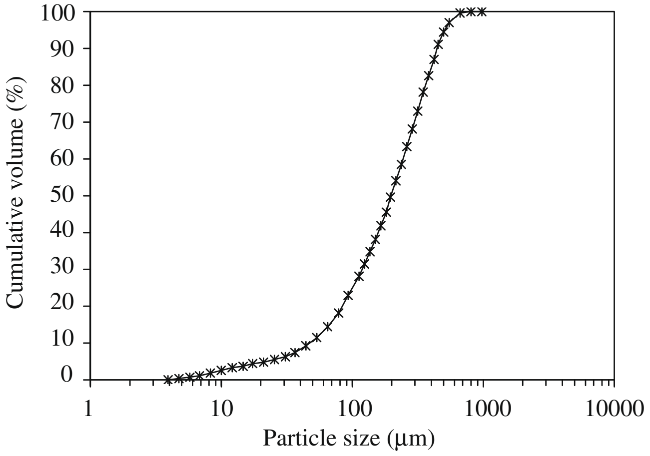

Fig. 4. Particle size distribution of cement.

5. Results and discussion

5.1. Packing densities measurement of fine aggregate

The dry packing densities of the four size classes measured under conditions D1 and D2 are presented in the first and second rows of Table 5. Under D1 (uncompacted condition), the packing densities of S1, S2, S3 and S4 were 0.447, 0.468, 0.476 and 0.489, respectively, while under D2 (compacted condition), the packing densities of S1, S2, S3 and S4 were 0.559, 0.548, 0.537 and 0.533, respectively. From these results, it is evident that the increase in packing density due to compaction is generally more significant for a smaller size class. This may be explained as follows. When uncompacted, a smaller size class has more serious agglomeration and thus a lower packing density. However, when compacted, since agglomeration has been overridden by compaction and a smaller size class has a wider size range (see Fig. 3), a smaller size class has a higher packing density. Because of the relatively lower packing density when uncompacted and the relatively higher packing density when compacted, a smaller size class tends to have a larger increase in packing density due to compaction.

The wet packing density of each aggregate sample was obtained by plotting the solid concentration of the aggregate-water mixture formed against the W/S ratio by volume, as shown in Fig. 5 for S1 under condition W4, and determining the maximum solid concentration from the graph plotted as the packing density of the aggregate. Following this procedure, the wet packing densities of the four size classes under conditions W1, W2, W3 and W4 have been

Table 5
Measured results for packing density of the four size classes of fine aggregate.

Test condition	Size class			
	S1	S2	S3	S4
D1	0.447	0.468	0.476	0.489
D2	0.559	0.548	0.537	0.533
W1	0.554	0.544	0.535	0.521
W2	0.575	0.554	0.549	0.535
W3	0.592	0.570	0.560	0.558
W4	0.604	0.581	0.575	0.572

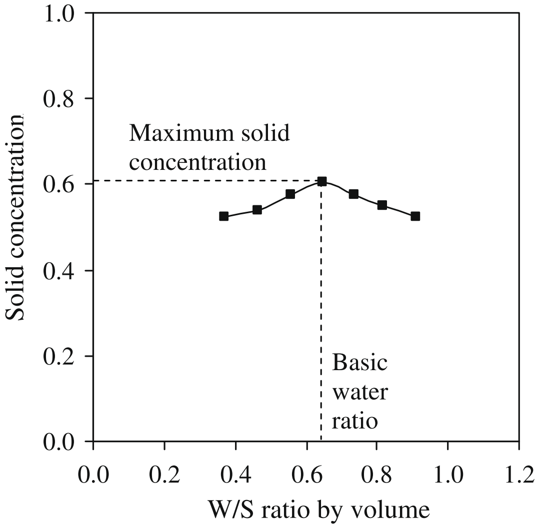

Fig. 5. Wet packing results for S1 under condition W4.

determined, as presented in the third to sixth rows of Table 5. It is seen that under W1 (uncompacted and with no SP added), the packing density was within 0.521–0.554, while under W2 (uncompacted and with SP added), the packing density was within 0.535–0.575. With compaction applied, the packing density increased to within 0.558–0.592 under W3 (compacted and with no SP added) and to within 0.572–0.604 under W4 (compacted and with SP added). These results reveal that the addition of SP has a slight beneficial effect on the packing density of fine aggregate. More importantly, the packing density of fine aggregate is significantly higher and less sensitive to compaction under wet condition than under dry condition.

The dry and wet packing densities of the non-blended aggregate B0 and the three blended aggregates B1, B2 and B3 under conditions D1, D2, W1, W2, W3 and W4 are presented in Table 6. As expected, the packing densities of the blended aggregates varied with the test conditions in a similar manner as for the non-blended aggregate. With regard to the effect of blending, which substantially widened the size range of the aggregate, the packing density increased remarkably in the order of B0, B1, B2 and B3 under each test condition. This may be attributed to the filling effect of the smaller size particles added to the larger size particles, which is generally greater when the size range is wider. Hence, with the size range of the particles widened by blending several aggregates of widely different sizes together, the packing density can be remarkably increased.

5.2. Packing densities modelling of fine aggregate

To predict the packing densities of the blended aggregates B1, B2 and B3 using Eq. (2), it is necessary first of all to determine the packing density β_i and volumetric fraction y_i of each component i. The packing density of each component may be obtained from the measured packing densities for S1, S2, S3 and S4 presented in Table 5. However, the size classes S1, S2, S3 and S4 were not exactly monosized. For instance, S1 contained particles ranging from 1 to 150 μm. Because of the finite size ranges, the measured packing densities for S1, S2, S3 and S4 were not really the packing densities of monosize particles. To obtain the packing density of monosize particles, for each size class, the size range was divided into several components, each with the largest size not larger than 1.1 times the smallest size so that the particles contained therein may be considered monosized. Then, assuming that the several components in the same size class have the same packing density, the packing density of the monosize particles was evaluated by back calculation from the measured packing density of the size class being considered. On the other hand, the volumetric fraction of each component was obtained from the overall particle size distribution of the blended aggregate, which can be evaluated simply as the weighted average of the particle size distributions of the various size classes blended together.

When Model A was applied, the interaction functions given by Eqs. (3) and (4) were employed. In theory, the sphericity of every component was required for the evaluation of the size ratio as per Eq. (5). However, since all aggregate samples were from the same source, the aggregate particles in different components should have more or less the same shape and hence the same sphericity. Hence, when evaluating the size ratio between any two components, the effect of sphericity was cancelled out and no sphericity was really required. When Model B was applied, the interaction functions given by Eqs. (6) and (7) were employed. In theory, the compaction index K was required to take into account the effect of compaction. To this end, a number of K-values had been tried but it was found that a K-value of infinity would yield the closest agreement between the measured and predicted packing density results. Hence, a K-value of infinity was adopted. In actual fact, the authors are of the view that the effect of compaction should have been fully accounted for when measuring the packing density of each size class and therefore there should be no necessity to account for the effect of compaction again by means of any reduced K-value.

Table 6
Measured results for packing density of the blended fine aggregates.

Test condition	Blended aggregate no.			
	B0	B1	B2	B3
D1	0.489	0.512	0.541	0.561
D2	0.533	0.551	0.590	0.638
W1	0.521	0.554	0.586	0.622
W2	0.535	0.564	0.603	0.641
W3	0.558	0.581	0.621	0.661
W4	0.572	0.586	0.628	0.670

The predicted packing densities of the three blended aggregates B1, B2 and B3 by the two packing models are listed in Table 7 in conjunction with the measured packing densities for comparison. It is seen that as for the measured packing densities, the predicted packing densities also increased in the order of B1, B2 and B3. The comparison of the measured results with the predictions by Model A reveals that the absolute difference between the measured and predicted packing densities is at most 2.1% for B1 and B2 and at most 5.1% for B3. Likewise, the comparison of the measured results with the predictions by Model B reveals that the absolute difference between the measured and predicted packing densities is at most 1.8% for B1 and B2 and at most 3.4% for B3. In all cases, the largest difference is associated with B3, which contained S1. This was probably due to errors or fluctuations in the particle size distribution of S1 (note that S1 actually contained some very fine particles, the amount of which was dependent on the sieving process during production). On the whole, for Model A, the average difference and average absolute difference between the measured and predicted results are only 1.5% and 2.1%, respectively, while for Model B, the average difference and average absolute difference between the measured and predicted results are only 0.7% and 1.3%, respectively. Hence, both packing models should be sufficiently accurate under the test conditions covered in the present study, albeit Model B appears to be slightly more accurate.

5.3. Packing density measurement of mortar

The wet packing density of each mortar sample was obtained by plotting the solid concentration of the cement–aggregate–water mixture formed against the W/S ratio by volume, as shown in Fig. 6 for M4, and determining the maximum solid concentration from the graph plotted as the packing density of the mortar. Following this procedure, the wet packing densities of the twelve mortar mixes M1–M12 have been determined, as presented in the second column of Table 8. The results reveal that the wet packing densities of the mortar mixes formed with different fine aggregate proportions ranged from 0.715 to 0.762. The maximum difference in packing density due to the variation in fine aggregate proportions was around 7%. Such a difference may appear small but actually has great effect on the water demand of the mortar. At a packing density of 0.715, the voids content is 1 − 0.715 = 0.285 while at a packing density of 0.762, the voids content is 1 − 0.762 = 0.238 (the voids content is the ratio of the volume of voids to the bulk volume of the solid particles). The corresponding difference in voids content is therefore about 20%. Since the voids have to be filled up with water or otherwise there will be unfilled air voids in the mortar, the voids content may be taken as a measure of the water demand.

Table 7
Comparison with predicted packing densities for the blended fine aggregates.

Test condition	Blended aggregate no.	Measured packing density	Model A		Model B	
			Predicted packing density	Difference (%)	Predicted packing density	Difference (%)
D1	B1	0.512	0.501	−2.1	0.503	−1.8
	B2	0.541	0.541	+0.0	0.534	−1.3
	B3	0.561	0.576	+2.7	0.565	+0.7
D2	B1	0.551	0.553	+0.4	0.555	+0.7
	B2	0.590	0.602	+2.0	0.595	+0.8
	B3	0.638	0.659	+3.3	0.648	+1.6
W1	B1	0.554	0.545	−1.6	0.547	−1.3
	B2	0.586	0.594	+1.4	0.587	+0.2
	B3	0.622	0.654	+5.1	0.643	+3.4
W2	B1	0.564	0.559	−0.9	0.561	−0.5
	B2	0.603	0.608	+0.8	0.601	−0.3
	B3	0.641	0.669	+4.4	0.658	+2.7
W3	B1	0.581	0.577	−0.7	0.580	−0.2
	B2	0.621	0.627	+1.0	0.620	−0.2
	B3	0.661	0.685	+3.6	0.675	+2.1
W4	B1	0.586	0.592	+1.0	0.594	+1.4
	B2	0.628	0.641	+2.1	0.634	+1.0
	B3	0.670	0.697	+4.0	0.688	+2.7

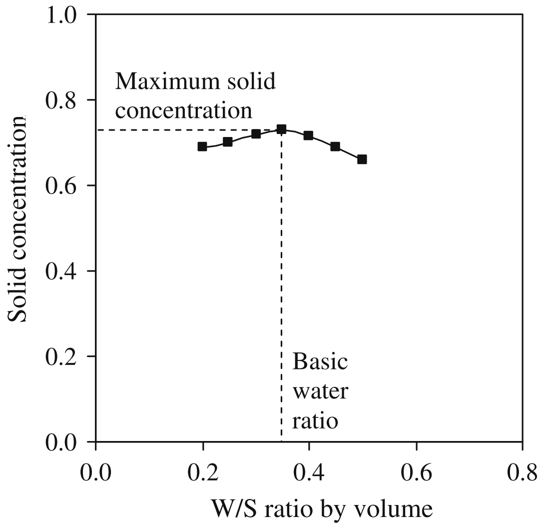

Fig. 6. Wet packing results for mortar mix M4.

5.4. Packing density modelling of mortar

As in the previous case of blended aggregates, to predict the packing density of the 12 mortar mixes M1–M12 using Eq. (2), it is necessary first to determine the packing density β_i and volumetric fraction y_i of each component i in the cement and fine aggregate. Since the cement was not monosized, the size range of the cement was divided into many components, each with the largest size not larger than 1.1 times the smallest size so that each component may be considered monosized. Then, assuming that the various components have the same packing density, the packing density of each component was evaluated by back calculation from the measured packing density of the cement. For this purpose, the packing density of the cement has been measured as 0.609. Likewise, for each size class of the fine aggregate, the size range was divided into components, each with the largest size not larger than 1.1 times the smallest size and the packing density of each component was evaluated by back calculation from the measured packing density of the size class. On the other hand, the volumetric fraction of each component was obtained from the overall particle size distribution of the cement–aggregate mixture.

Table 8
Comparison with predicted packing densities for the mortar mixes.

Mortar mix no.	Measured packing density	Model A						Model B	
		$\psi_{\text{Agg.}} = 0.6$		$\psi_{\text{Agg.}} = 0.7$		$\psi_{\text{Agg.}} = 0.8$			
		Predicted packing density	Difference (%)	Predicted packing density	Difference (%)	Predicted packing density	Difference (%)	Predicted packing density	Difference (%)
M1	0.725	0.735	+1.4	0.731	+0.8	0.729	+0.6	0.728	+0.4
M2	0.737	0.743	+0.8	0.739	+0.3	0.737	0.0	0.736	−0.1
M3	0.756	0.761	+0.7	0.759	+0.4	0.758	+0.3	0.756	0
M4	0.730	0.736	+0.8	0.733	+0.4	0.731	+0.1	0.730	0
M5	0.719	0.734	+2.0	0.730	+1.5	0.727	+1.1	0.726	+1.0
M6	0.762	0.765	+0.4	0.763	+0.1	0.762	0.0	0.760	−0.3
M7	0.751	0.755	+0.5	0.753	+0.3	0.751	0.0	0.750	−0.1
M8	0.734	0.743	+1.2	0.740	+0.8	0.738	+0.5	0.736	+0.3
M9	0.734	0.753	+2.6	0.750	+2.2	0.748	+1.9	0.745	+1.5
M10	0.719	0.728	+1.3	0.724	+0.7	0.721	+0.3	0.720	+0.1
M11	0.727	0.731	+0.6	0.728	+0.1	0.725	−0.3	0.725	−0.3
M12	0.715	0.721	+0.8	0.717	+0.3	0.714	−0.1	0.714	−0.1

Note: $\psi_{\text{Agg.}}$ is the assumed sphericity of the fine aggregate when applying Model A.

When applying Model A, the sphericity of the particles was required. For the cement, the actual specific surface area in terms of Blaine fineness was 338 m^2/kg whereas the specific surface area calculated from the particle size distribution with the particles assumed to be spherical was 242 m^2/kg. Hence, the sphericity of the cement was determined as 242/338 = 0.716. For the fine aggregate, however, the sphericity could not be determined because the actual specific surface area was not known. To overcome this difficulty, several values ranging from 0.6 to 0.8 were assumed in turn for the sphericity of the fine aggregate to study the possible outcome. The packing densities so predicted are tabulated in the third to eighth columns of Table 8. It is seen that regardless of what sphericity value has been assumed, the predicted packing densities agree fairly well with the measured results. At assumed sphericity values of 0.6, 0.7 and 0.8, the maximum absolute differences between the measured and predicted packing densities of the mortar mixes are only 2.6%, 2.2% and 1.9%, respectively, and the average absolute differences between the measured and predicted packing densities are only 1.1%, 0.7% and 0.4%, respectively. Such small differences indicate that the actual effects of sphericity on the interaction functions are insignificant and thus may be neglected to avoid the difficulty of measuring the sphericities of the cement and fine aggregate. In this regard, it should be borne in mind that the effect of sphericity on the packing density of each component has been fully accounted for in the packing density measurement of the cement and each size class of the fine aggregate.

When applying Model B, several *K*-values were attempted but it was found again that a *K*-value of infinity would yield the closest agreement between the measured and predicted packing density results. Hence, a *K*-value of infinity was adopted. The packing densities so predicted are tabulated in the ninth to tenth columns of Table 8. On the whole, the predicted packing densities agree very well with the measured results. Basically, the maximum absolute difference and average absolute difference between the measured and predicted packing densities of the mortar mixes are only 1.5% and 0.4%, respectively. Comparing the packing density predictions by the two packing models, it appears that Model B is slightly more accurate than Model A, but as both models give reasonably acceptable predictions of the packing densities of all the mortar mixes tested, both models should be considered sufficiently accurate for practical applications.

6. Conclusions

The wet packing method previously developed for measuring the packing density of cementitious materials has been extended for application to crushed rock fine aggregate and mortar. It has been applied to measure the packing densities of sieved fractions of fine aggregate and blended fine aggregate. For comparison, the packing densities of the fine aggregate samples were also measured using the conventional dry packing method. The results reveal that the packing density of fine aggregate is significantly higher and less sensitive to compaction under wet condition than under dry condition. Moreover, the addition of superplasticizer has a slight beneficial effect on the packing density. Hence, the dry packing method, which tends to underestimate the packing density, is not really suitable for measuring the packing density of fine aggregate. The wet packing method has also been applied to measure the packing densities of mortar mixes with different fine aggregate proportions. The results reveal that the fine aggregate proportions have significant effects on the packing density of mortar. This is the first time that the packing density of mortar is directly measured.

Two existing packing models have been employed to analyze and predict the packing densities of the fine aggregate and mortar samples. They were the model developed by Yu et al. [29] in 1996 (Model A) and the model developed by DeLarrard [8] in 1999 (Model B). They differ in the interaction functions and in the treatment of the effects of particle shape and compaction. When Model A was applied, it was found that the effect of particle shape on particle interactions is insignificant and may therefore be neglected to avoid the difficulty of measuring the sphericity of the particles. When Model B was applied, it was found that the closest agreement with the measured results is achieved when the *K*-value, which accounts for the effect of compaction, is taken as infinity. Despite the apparent differences between them, both models yielded predicted packing densities of the fine aggregate and mortar samples in good agreement with the corresponding measured results. For the fine aggregate samples, the average absolute difference between the measured and predicted packing densities under the test conditions covered in the present study was 2.1% when Model A was employed and 1.3% when Model B was employed. For the mortar samples, the average absolute difference between the measured and predicted packing densities was 1.1% when Model A was employed and 0.4% when Model B was employed. Such good agreement verified the applicability and accuracy of the wet packing method and the two existing packing models, which are useful tools for the mix design of high-performance concrete. This is the first time that the applicability of the existing packing models to mortar is verified by measured results.

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