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Plastic viscosity of fresh concrete – A critical review of predictions methods

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ABSTRACT

Rheological properties of fresh concrete, namely plastic viscosity and yield stress, are critical for the concrete industry because they affect placement and workability. Moreover, these rheological properties influence the productivity and quality of concrete, including mechanical properties and durability. Therefore proper characterization of these properties is needed to control the quality of fresh concrete and ensure sustainability of concrete structures.

Fundamental and phenomenological rheological models have been proposed in the literature for characterizing the behaviour of fresh concrete. Establishing a model for predicting the plastic viscosity of concrete based on its composition will be extremely valuable for the concrete industry. This paper provides a critical review of the most prevailing models in concrete technology as well as models proposed in the literature for predicting the plastic viscosity of dense suspensions to a total of eight models. Review has revealed that Mahmoodzadeh and Chidiac models based on the cell method provides a higher degree of correlation to the experimental data as well as a more consistent and reliable predictions in comparison to the models currently proposed in the literature for concrete and/or dense suspensions.

1. Introduction

Rheology, defined as "the study of deformation and flow", provides a measure between shear stress and rate of deformation. The corresponding constitutive equation can be employed to describe mathematically the flow of fresh concrete. Concrete composed of cement particles, aggregates, water and air, can be characterized as suspended solid particles (aggregate) in viscous media (cement paste) [1–5]. Numerous constitutive equations have been proposed to characterize the rheology of fresh concrete as suspensions, but only Bingham model and Herschel and Bulkley (HB) model have received wide acceptance. For normal concrete, experimental data have confirmed that the flow of fresh concrete follows Bingham's material model, i.e.:

$$\tau = \tau_0 + \eta \times \dot{\gamma} \qquad (1)$$

In which τ is the shear stress (Pa), τ_0 the yield stress (Pa), η the plastic viscosity (Pa s), and $\dot{\gamma}$ the shear strain rate (1/s). τ_0 and η are referred to as Bingham material properties with the first property providing a measure of the shear stress required to initiate flow and the second one a measure of the material resistance to flow after the material begins to flow. These two rheological properties are therefore needed to quantitatively characterize the flow of fresh concrete [1].

Quantitative characterization of the rheological properties is important to the sustainability of the concrete construction industry for the following reasons: (1) workability of fresh concrete forms one of the bases of concrete mixture design for quality control purposes – establishing a quantitative measure for workability will mitigate material waste by properly controlling the quality of fresh concrete as a priori; (2) flow behaviour of fresh concrete impacts the quality of concrete hardened properties [2,3] – establishing a quantitative measure for workability will mitigate the premature failure of the concrete materials and concrete structures due to inadequate flow properties of the concrete mixture; and (3) concrete placement which includes transportation, pumping, casting and vibration, is affected by the plastic viscosity and yield stress of fresh concrete – establishing a quantitative measure for workability will provide the tools to design concrete mixture with flow properties suitable for the specificity of the job and with the least placement cost. To illustrate the potential of such metric, one can review the lessons learned from the newly developed concrete technologies, namely high performance concrete (HPC) and self-consolidating concrete (SCC). Their successful use depends on proper characterization of their rheological properties. SCC needs to flow under its own weight and fill areas that are heavily congested with steel reinforcements without any segregation [5]. HPC, whose mixture possesses a very low water to cement ratio, needs to have adequate flow properties (workability) to fill the various forming system configurations without segregation and without entrapping air voids. [5]. These examples demonstrate the need for developing a quantitative characterization of the

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rheological properties of fresh concrete that can be incorporated in the design and control of concrete mixtures. Towards that need, a study was carried out to evaluate models reported in the literature for predicting the plastic viscosity of fresh concrete. Subsequently, models with acceptable predictions can be incorporated in the design of concrete mixtures to overcome the limitations of current design methods which only consider slump. Slump, which has been correlated to yield stress, is not a sufficient measurement for characterizing the flow properties of fresh concrete.

The most common approach adopted for quantifying the rheological properties of fresh concrete is to experimentally measure shear stress versus shear strain rate using concrete rheometer. By assuming that the flow of fresh concrete obeys Bingham model, an estimate of the yield stress and plastic viscosity is obtained [1–5]. Other researchers have attempted to quantify the plastic viscosity of fresh concrete based on its composition, specifically the work of Roshavelov [4] and Ferraris and deLarrard [5] to name a few. However, these attempts have received limited success due to the limitations of the models, i.e., the proposed models did not consider particles interactions which is needed for concrete given the high concentration of particle; and due to the need for model validations. This paper provides a critical review of the models proposed in the literature for quantifying the plastic viscosity of fresh concrete as well as the models proposed for quantifying the plastic viscosity of concentrated suspensions intended for chemical and material engineering applications. It should be noted that the input parameters for all the models evaluated are volumetric fraction of solid material which is referred to as packing density (φ) and maximum packing density of the whole mixture ($\varphi_{\max}$). These input parameters are a function of concrete composition. Experimental data reported in the literature was used to evaluate the predictive capabilities of these models in quantifying the plastic viscosity of fresh concrete.

2. Rheological models – plastic viscosity

Two different approaches have been postulated for modeling the plastic viscosity of fresh concrete: phenomenological models and fundamental models. The bases for their development are reviewed briefly. Phenomenological models are founded on observations. The most promising model in this category for fresh concrete is the one proposed by Ferraris and deLarrard [5] and Ferraris et al. [6]. They postulated that the plastic viscosity is only a function of the packing density to the maximum packing density of the whole mixture ($\varphi_{\max}$). Using regression analyses, they developed the following model for estimating the plastic viscosity of fresh concrete [5,6].

$$\eta = \exp\left\{26.75 \times \left(\frac{\varphi}{\varphi_{\max}} - 0.7448\right)\right\} \tag{2}$$

The predictive capabilities and usefulness of phenomenological models, which includes Ferraris and deLarrard, are limited by the experimental data. Therefore, the use of Ferraris and deLarrard model can lead to erroneous predictions as the bounds of the model, namely, $\frac{\varphi}{\varphi_{\max}}$, φ and φ_{max} were not defined. Fundamental models proposed to quantify plastic viscosity are based on the science of rheology and fluid mechanics. These models are divided into two groups. The first group includes the models that are prevailing in concrete technology, whereas the second group compiles the models proposed to quantify the plastic viscosity of concentrated suspensions in solvent, typically used for other engineering applications. For the first group, three models were selected for this review as they are considered representative of the recent models put forward in the concrete literature, namely Murata and Kikukawa [7], Hu and deLarrard [8] and Roshavelov [4]. For the second group, the models were classified into four sub-groups: generalized models, analogous approach, cell method, and average method. A complete list of the models evaluated is given in Appendix A. The fundamentals corresponding to these models are briefly discussed and only results of the models that have provided the best predictions are included in this review.

2.1. Murata and Kikukawa

Murata and Kikukawa [7] implemented Roscoe's [9] equation to quantify the plastic viscosity of concrete, and proposed the following methodology:

(1) Calculate the plastic viscosity of cement paste by postulating that cement particles are suspended in water, i.e. there are no physical or chemical interactions between the cement particles and water. Then by recognizing that Roscoe's equation was developed with the premise that the particles are solid, spherical, and identical in shape and size, they proposed an extension to account for the irregularly shaped and non-uniform size of the particles. They proposed the following relation

$$^{i}\eta_r = \frac{^{i}\eta}{^{i}\eta_0} = \left(1 - \frac{^{i}\varphi}{^{i}C}\right)^{-^{i}k} \tag{3}$$

where $^{i}k = {^{i}k_1}{^{i}\varphi} + {^{i}k_2}$, superscript "$i$" is set equal to 1 for cement paste, $^{1}\eta_r$ becomes the relative plastic viscosity of cement paste, $^{1}\eta$ the plastic viscosity of cement paste, $^{1}\eta_0$ the plastic viscosity of water, $^{1}\varphi$ the volumetric concentration of cement, ^{1}C the percentage of absolute volume of cement, and ^{1}k the coefficient of agglomerated cement particles. Coefficients $^{1}k_1$ and $^{1}k_2$ are constant and are found through regression.

(2) Plastic viscosity of mortar is then established using the same premise stipulated for the cement paste in Eq. (3) with superscript "i" is set equal to 2 for mortar, $^{2}\eta_r$ becomes the relative plastic viscosity of mortar, $^{2}\eta$ the plastic viscosity of mortar, $^{2}\eta_0$ the plastic viscosity of cement paste, ^{2}C the solid volume ratio of fine aggregate, $^{2}\varphi$ the volumetric concentration of fine aggregate, and ^{2}k a linear function of the fineness modulus.

(3) Plastic viscosity of concrete is similarly obtained with superscript "i" is set equal to 3 for concrete, $^{3}\eta_r$ becomes the relative plastic viscosity of concrete, $^{3}\eta$ the plastic viscosity of concrete, $^{3}\eta_0$ the plastic viscosity of mortar, ^{3}C the solid volume ratio of coarse aggregate, and $^{3}\varphi$ the volumetric concentration of coarse aggregate.

2.2. Hu and deLarrard

For multimodal spherical suspensions, Farris [10] stated that it is possible to ignore the interaction of different classes of particle size in situation where the size ratio of spheres is greater than 1/10. Accordingly, he proposed the following functional form:

$$\eta_r = H(\varphi_1)H(\varphi_2)\ldots H(\varphi_N) \tag{4a}$$

where φ_i are the concentration of each size, and $(H\varphi_i)$ the relative plastic viscosity of fluid containing particle size classes from 1 to i to plastic viscosity containing 1 to $i-1$. Hu and deLarrard [8] incorporated this theory along with Krieger–Dougherty [11] equation to simulate the flow of HPC. They developed the following equations:

$$\eta = \eta_0(1+k_s p_s)\left(1-\frac{\varphi_F}{\alpha_F}\right)^{-2.5\alpha_F}\left(1-\frac{\varphi_C}{\alpha_C}\right)^{-k\alpha_c}\left(1-\frac{\varphi_G}{\alpha_G}\right)^{-k\alpha_G} \quad (4b)$$

$$\phi_F = \frac{V_F}{V_0+V_F}, \quad \phi_C = \frac{V_C}{V_0+V_F+V_C}, \quad \phi_G = \frac{V_G}{V_0+V_F+V_C+V_G}, \quad (4c)$$

$$\varphi_{x\max} = 1-0.45\left(\frac{d_x}{D_x}\right)^{0.19} \quad (4d)$$

where k_s and k are found by curve fitting, η_0 the plastic viscosity of water, p_s the proportion of superplasticizer as fraction of its saturating dosage, φ_x the volume concentration, V_x the partial volumes, $\varphi_{x\max}$ the maximum packing density and, d_x and D_x the sieve sizes corresponding to 10% and 90%, respectively, of the material concerned. Subscript x in Eq. (4d) can be replaced by 0, F, C and G for water, silica-fume, cement and aggregate, respectively.

2.3. Roshavelov

In 1951, Mooney [12] put forward a model that permits the inclusion of particles interactions by means of crowding theory. It was assumed that the interaction between particles can be captured by a simple geometric crowding factor but did not provide a methodology for developing such interaction functions. By adopting Mooney's theorem, Roshavelov [4] proposed a crowding factor that is based on some geometric argument. The proposed approach was found to have good predictions of the plastic viscosity only at the maximum shear rate.

The second group of fundamental models includes those that were developed for dense suspensions. Terms dense or high concentrations refer to the suspensions in which the particle size is greater than the average particle separation. For non-Brownian systems, low Reynolds number and large Peclet number, the relative viscosity is found to be a function of concentration [13], where

$$\eta_r = f(\varphi) \quad (5a)$$

Concrete is classified as dense suspensions with particles of varying sizes and shapes. Accordingly, maximum packing density ($\varphi_{\max}$) was introduced to account for non-mono-sized particles, where

$$\eta_r = f(\varphi, \varphi_{\max}) \quad (5b)$$

2.4. Generalized models

Models which have been generalized from other material models, have been proposed to quantify the plastic viscosity of concentrated suspensions [14,15]. In this study, the model put forward by Sudduth [15] is presented,

$$\ln(\eta/\eta_0) = \left(\frac{\eta_i}{k}\right)\left(\frac{1}{\xi-1}\right)((1-k\varphi)^{1-\xi}-1), \quad k=1/\varphi_{\max} \quad \text{For}: \xi\neq 1 \quad (6a)$$

$$\eta = \eta_0(1-k\varphi)^{-\eta_i/k}, \quad k=1/\varphi_{\max} \quad \text{For}: \xi = 1 \quad (6b)$$

where η_i is the intrinsic viscosity and is a function of the particle shape (For spherical particles, $\eta_i = 5/2$), ξ the interaction factor and is found by regression analysis using experimental data. For ξ equals to 0, 1 and 2 respectively, Eqs. (6) yields the Arrhenius equation, the Krieger–Dougherty equation, and Mooney equation. Accordingly, Sudduth formulation is the generalized form of Arrhenius, Krieger–Dougherty and Mooney equations.

2.5. Analogous approach

The theoretical treatment to obtain the rheological properties of concentrated suspensions can be divided into two main categories, hydrodynamic approach and analogous approach. For the later one, it is postulated that physical properties such as diffusion coefficient, modulus of elasticity, and thermal conductivity have the same form of constitutive equations, and are therefore treated analogously. In this method, the plastic viscosity is treated as a field property [16]. By taking into account the fact that the plastic viscosity of aggregate is much greater than that of water, the following equation has been developed based on Fan and Boccaccini model [16].

$$\eta_r = C + \varphi^m + \frac{F_s^2}{\varphi - \varphi^m} \quad (7)$$

where C, F_S and m are constant.

2.6. Cell method

The cell method is widely used for characterizing plastic viscosity in chemical and material engineering applications. It postulates that a spherical cell of fluid containing a particle in the center is the representative volume of the suspension at the microscopic level, and that the cell is subjected to actions at its boundary. The corresponding boundary value problems defined by:

$$\eta_0 \nabla^2 u = \nabla p$$
$$\text{subjected to } \nabla \cdot u = 0 \quad (8)$$

yield the solution to the flow problem. The relative viscosity is then obtained by equating the energy dissipation of the cell to the energy dissipated in the fluid of a cell with the same volume. The solution to the partial differential equation leads to Einstein's equation when the boundary of the cell is at infinity. Simha [17] and Happel [18], pioneers of this method, proposed two different cell descriptions,

$$\text{Simha [17]}: y(\varphi) = \frac{(\varphi/\varphi_{\max})^{1/3}}{2-(\varphi/\varphi_{\max})^{1/3}} \quad (9a)$$

$$\text{Happel [18]}: y(\varphi) = (\varphi)^{1/3} \quad (9b)$$

In which the function $y(\varphi)$ is defined as the ratio of the radius of the cell to the radius of the particle. Frankel and Acrivos [19] opted to also include the lubrication theory in their model. Moreover, they assumed that the cell size takes a cubic arrangement.

$$\text{Frankel and Acrivos [19]}: y(\varphi) = (\varphi/\varphi_{\max})^{1/3} \quad (9c)$$

Jeffrey and Acrivos [20] criticized the cell method by. pointing out that the selection of the shape of the cell and the boundary conditions are arbitrary, and therefore the quantitative significance is questionable. In addition, the method assumes that all the cells are equidistant which is not always true. In 2006, Zholkovskiy et al. [21] proposed a revised formulation that overcomes the postulated boundary conditions requirement, and therefore overcoming the main weakness of the cell method. They also argued that Simha's proposed cell radius contradicts the main assumption of the cell method. They adopted the cell size proposed by Happel in Eq. (9b). Their formulation yielded

$$\eta_r = 1 + \eta_i y^3 \frac{4(1-y^7)}{4(1+y^{10})-25y^3(1+y^4)+42y^5} \quad (10)$$

Mahmoodzadeh and Chidiac [22] revised Zholkovskiy et al. formulation for quantifying the plastic viscosity of fresh concrete. In the revised formulation, the cell size was modified to account for the maximum packing density. Moreover, it postulates that (a) the grading of particles is important, and (b) the particle size cannot be equal to that of the cell size even when the packing density is equal to the maximum packing density. Accordingly, only the cell models proposed by Simha and by Frankel and Acrivos can be incorporated in the revised formulation. Mahmoodzadeh and Chidiac models take the following form:

$$\text{Using Simha}: y(\varphi) = \frac{(\varphi/\varphi_{\max})^{1/3}}{2(1+K) - (\varphi/\varphi_{\max})^{1/3}} \tag{11a}$$

$$\text{Using Frankel and Acrivos}: y(\varphi) = (\varphi/\varphi_{\max})^{1/3}(1-K) \tag{11b}$$

where K is a function of the concrete mixture and is defined as follows:

$$\begin{cases} K = 0.006 \times \frac{\text{Cement}}{\text{Water}} & \text{Without HRWRA} \\ K = 3.8 \times \frac{\text{HRWRA}}{\text{Cement}} \times \frac{\text{Water}}{\text{Cement+FineSand+Sand}} & \text{With HRWRA} \end{cases} \tag{12}$$

HRWRA stands for high range water reducer admixture. Only, the results obtained from the second model (Eq. (11b)) are included in the review.

2.7. Average method

From hydrodynamic point of view, two different approaches have been proposed in the literature to find the effective properties of suspension. The first one is based on equalizing the dissipated energy in the suspension to the dissipated energy in the fluid with the effective properties. This method yields only the effective properties (plastic viscosity) not the full constitutive equation. The second approach relates the average stress tensor of the suspension to the average rate of strain tensor [20]. This method is based on the work of Batchelor [23] and Batchelor and Green [24], in which the bulk stress tensor ($\bar{T}$) is expressed by

$$\bar{T} = -\bar{p}I + 2\eta\bar{D} + \bar{T}_P \tag{13}$$

where $\bar{p}$ is the hydrostatic stress, $\bar{D}$ the bulk rate of stress tensor that is observed macroscopically, and $\bar{T}_P$ the contribution of the particles to the stress tensor. Equation (13) can be re-worked using Batchelor's work to yield

$$\bar{T} = -\bar{p}I + 2\eta_0 f(\varphi)\bar{D} \tag{14}$$

By comparing Eq. (13) with Eq. (14), the following expression can be derived:

$$\eta_r = \frac{\eta}{\eta_0} = f(\varphi) \tag{15}$$

Martynov and Syromyasov. [25] applied the average method to model the flow of a viscous liquid with suspended mono-size, rigid spheres. Following Batchelor's work, and solving the creep equation in a cell, they developed a closed-form solution to calculate the relative plastic viscosity, where

$$\eta/\eta_0 = 1 + \eta_i\varphi\left[1 + \frac{15\beta_2}{2\pi}\varphi + \left(\frac{15\beta_2}{2\pi}\varphi\right)^2 + \frac{14}{3}\omega\left(\frac{3}{4\pi}\varphi\right)^{5/3}\right] \tag{16}$$

In which $\beta_2 = -0.3594$ and ω is a weight function.

3. Evaluation methods

Experimental data reported in the literature [5] was used to quantitatively evaluate the predictive capabilities of the plastic viscosity models for fresh concrete. The data included 19 concrete mixtures without HRWRA and 17 concrete mixtures with HRWRA. However, the source of the coarse and fine aggregates for all 36 concrete mixtures was the same.

The plastic viscosity measurements were obtained using the BTRHEOM, a parallel plate concrete rheometer. It should be noted that there is no standard test method for measuring the plastic viscosity of concrete, and that the reported experimental measurements are only applicable to BTRHEOM given that previous evaluation of concrete rheometers has revealed that the measurements of plastic viscosity are different for different types of concrete rheometer [26]. Based on the authors' experiences with BTRHEOM, the coefficient of variance for stable concrete mixes is in the range of 10–15% and that the errors are higher for mixes with low plastic viscosity due to some segregation and for mixes with high plastic viscosity due to added stiffness.

Three tests were used to assess the model predictions of the plastic viscosity. The first test provided a global assessment by calculating the variance of the error term. For the second test, the covariance and correlation were calculated to determine the extent to which the models co-vary. And for the third test, the predictive trends of the models were determined with varying the packing density as an indirect assessment of the particle interaction contribution to the plastic viscosity models.

The variance of the error term (σ) was calculated in accordance with

$$\sigma^2 = \frac{\sum_{j=1}^{n}(\eta_{j\text{Experimental}} - \eta_{j\text{Model}})^2}{n-q} \tag{17}$$

where the term $(n-q)$ represents the model's degrees of freedom, n the number of data points and q the number of fitting parameters. To assess the models, the experimental data were divided into two sets. The first set was used for determining the coefficient(s) for the model using statistical regressive analyses, and the second set was used for testing the predictive capabilities of the models. Ten data points were used for the regression, and nine points to assess the models predictions for concrete without HRWRA, and seven points to assess the predictions for concrete with HRWRA. To evaluate the soundness of the proposed number of experimental test points, Ferraris and deLarrard [5] phenomenological model, which was fitted to the same data, was first tested. The results from the regression analysis and model predictions for concrete without HRWRA and with HRWRA are shown in Fig. 1. The plots indicate the same distribution of errors for both the predictive and regression analyses, and the errors are found to be similar to the ones reported by Ferraris and deLarrard [5]. The variance of the error calculated from the data obtained using the regression analysis and those obtained using the model for concrete without HRWRA is 31 Pa s and 42 Pa s, respectively, and 143 Pa s and 142 Pa s for concrete containing HRWRA.

4. Comparative analyses

Figs. 1–4 provide a visual comparison of the results obtained using regression analysis and models predictions with the experimental data. These models are found to yield different plastic viscosity predictions. Tables 1 and 2 give the number of parameters associated with each model as well as the standard error calculated for the regression analysis and model evaluation corresponding to concrete without HRWRA and with HRWRA, respectively. The results show that the errors for the model evaluation are similar to those calculated for the regression analysis. This indicates that the number of test data used to calibrate these models is adequate for the number of parameters.

Closer examination of the results in Table 1 show that Mahmoodzadeh and Chidiac models has yielded the best results followed by the models developed by Zholkovskiy, Martynov and Syromyasov, Sudduth, Fan and Boccaccini, Hu and deLarrard, Ferraris and deLarrard, and Murata and Kikukawa. Moreover, the second best model, i.e. Zholkovskiy, has a standard error that is 11% greater than that of Mahmoodzadeh and Chidiace model when

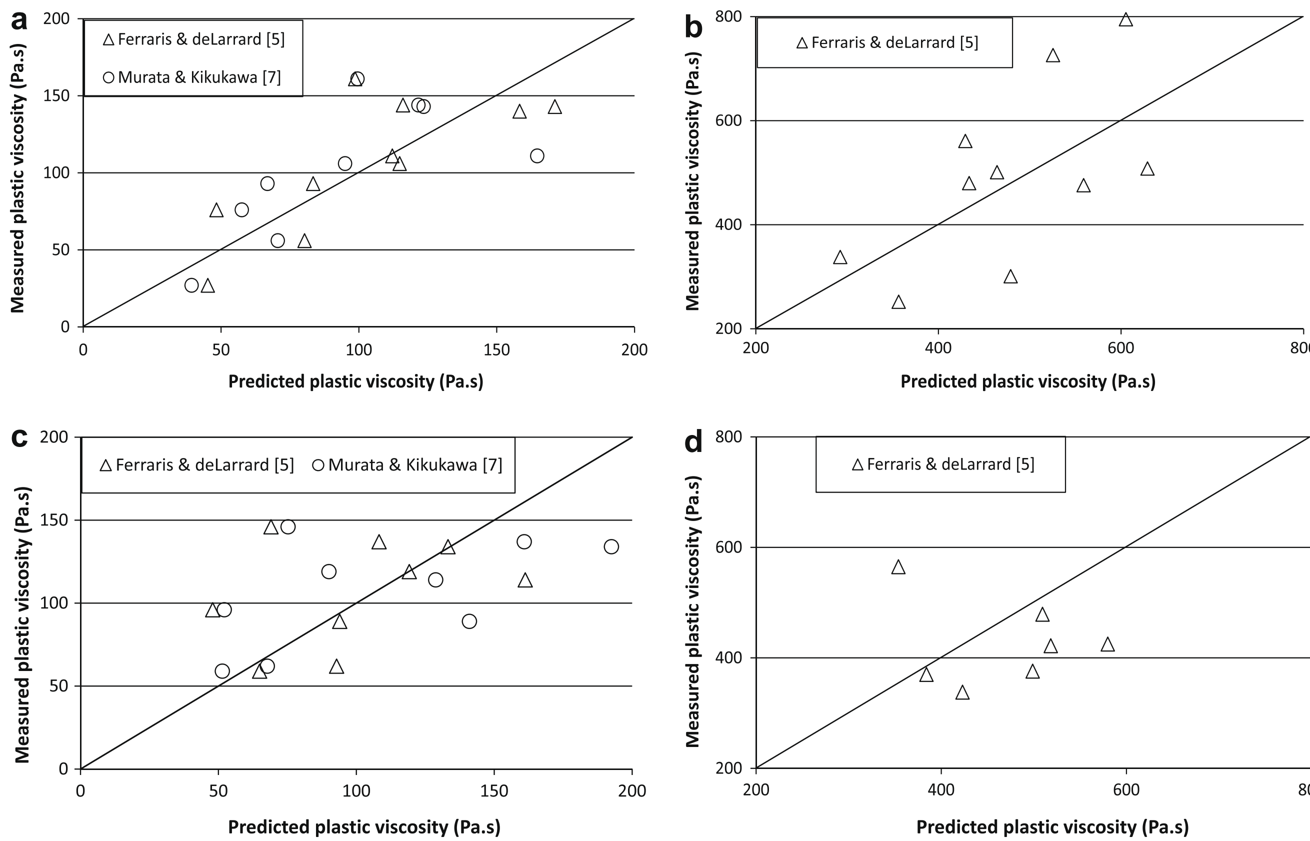

Fig. 1. Plastic viscosity according to Ferraris and deLarrard [5] and Murata and Kikukawa [7]: (a) from regression analysis for concrete without HRWRA; (b) from regression analysis for concrete with HRWRA; (c) model predictions for concrete without HRWRA; (d) model predictions for concrete with HRWRA.

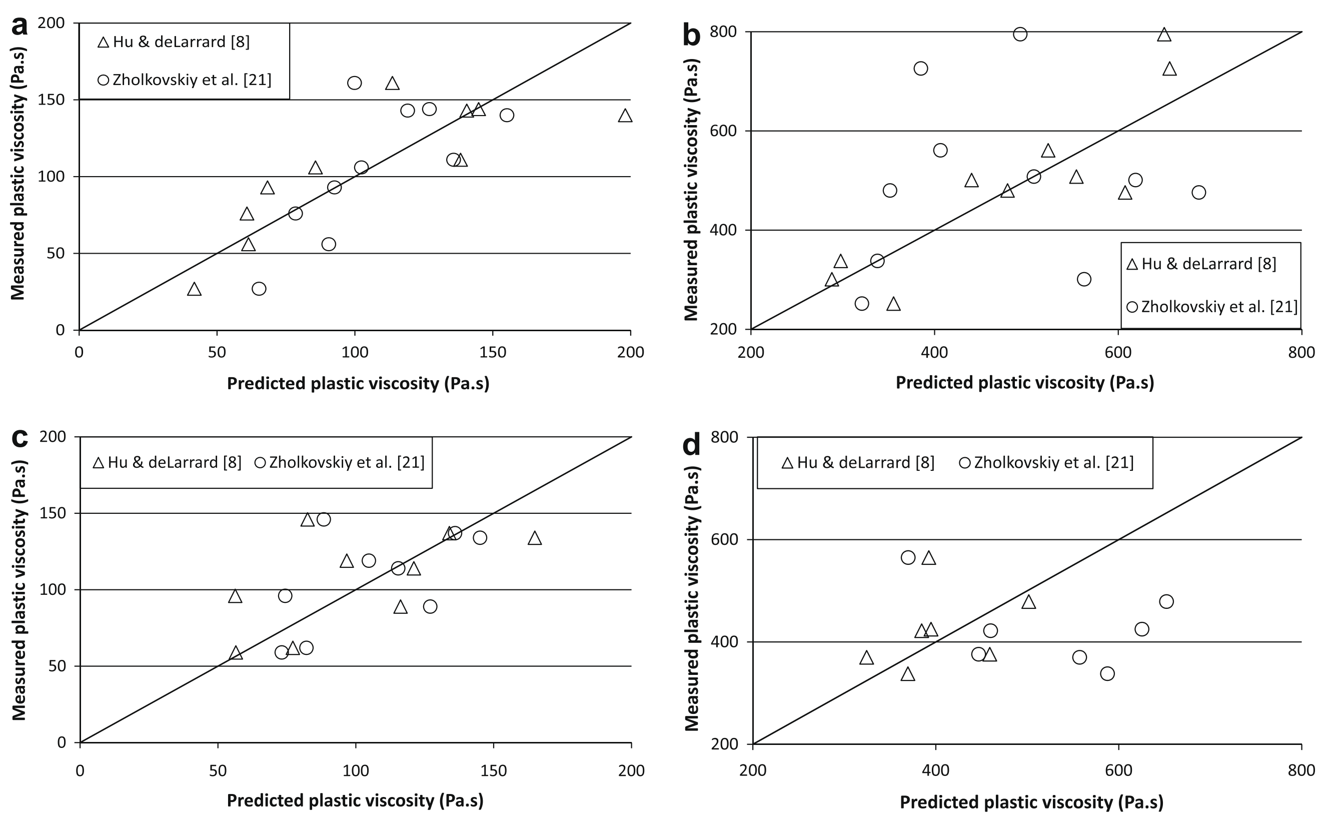

Fig. 2. Plastic viscosity according to Hu and deLarrard [8] and Zholkovskiy et al. [21]: (a) from regression analysis for concrete without HRWRA; (b) from regression analysis for concrete with HRWRA; (c) model predictions for concrete without HRWRA; (d) model predictions for concrete with HRWRA.

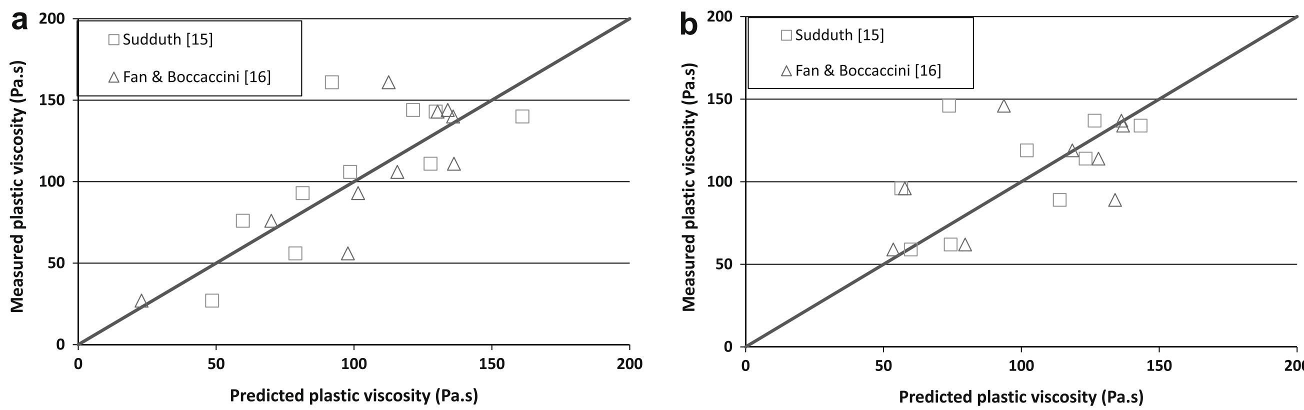

Fig. 3. Plastic viscosity according to Sudduth [15] and Fan and Boccaccini [16]: (a) from regression analysis for concrete without HRWRA; (b) model predictions for concrete without HRWRA.

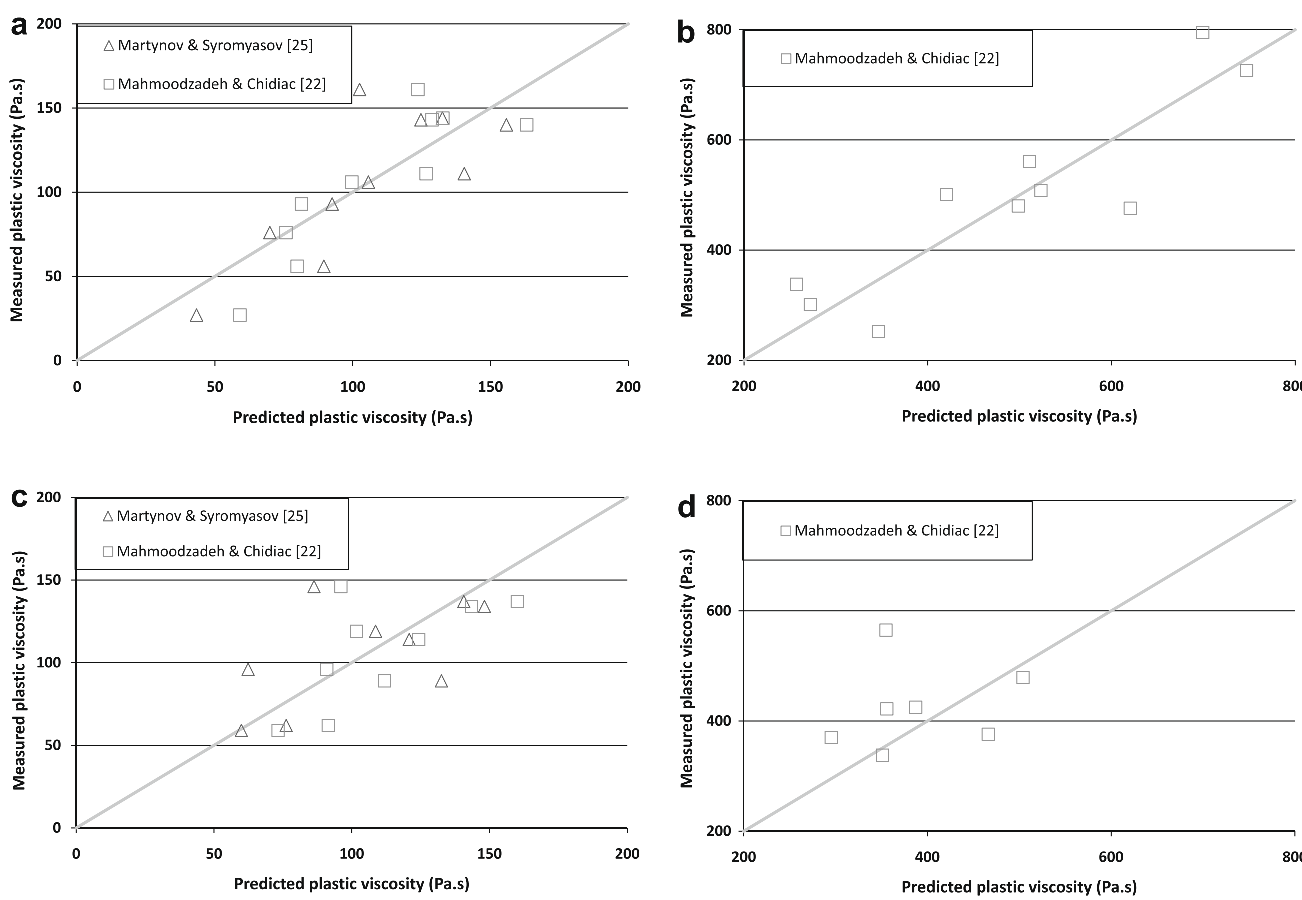

Fig. 4. Plastic viscosity according to Martynov and Syromyasov [24] and Mahmoodzadeh and Chidiac [22]: (a) From regression analysis for concrete without HRWRA; (b) from regression analysis for concrete with HRWRA; (c) model predictions for concrete without HRWRA; (d) model predictions for concrete with HRWRA.

predicting the plastic viscosity of concrete without HRWRA. From Table 2, one observes that Mahmoodzadeh and Chidiac proposed model yields the lowest standard error followed by the Hu and deLarrard, Ferraris and deLarrard and Zholkovskiy model. The percent difference in the error between Hu and deLarrard's model and Mahmoodzadeh and Chidiac model is 4%. However to better assess the correlation between the experimental data and the models predictions, all the experimental data were used for this analysis and the results are given in Tables 3 and 4 for concrete without HRWRA and with HRWRA, respectively. The corresponding 95% confidence bounds are also given in Tables 3 and 4. The results of Table 3 show that Mahmoodzadeh and Chidiac proposed model provides the

Table 1
Summary of standard error obtained from models calibration and evaluation for concrete without HRWRA and number of fitting parameters.

Model	Number of parameters	Error (Pa s)	
		Regression	Evaluation
Ferraris and deLarrard	2	31	42
Murata and Kikukawa	4	52	54
Hu and deLarrard	3	34	37
Sudduth	2	31	34
Fan and Boccaccini	3	27	34
Zholkovskiy et al.	1	32	30
Martynov and Syromyasov	2	29	32
Mahmoodzadeh and Chidiac	2	23	27

Table 2
Summary of standard error obtained from models calibration and evaluation for concrete with HRWRA and number of fitting parameters.

Model	Number of parameters	Error (Pa s)	
		Regression	Evaluation
Ferraris and de Larrard	2	143	142
Murata and Kikukawa	–	–	–
Hu and deLarrard	4	102	119
Sudduth	–	–	–
Fan and Boccaccini	–	–	–
Zholkovskiy et al.	1	218	206
Martynov and Syromyasov	–	–	–
Mahmoodzadeh and Chidiac	2	84	114

Table 3
Degree of correlation between models and experimental data for concrete without HRWR.

Models	Correlation	Confidence bounds (95%)	
		Lower bound	Upper bounds
Ferraris and deLarrard	0.62	0.24	0.84
Murata and Kikukawa	0.61	0.21	0.83
Hu and deLarrard	0.75	0.44	0.9
Sudduth	0.67	0.31	0.86
Fan and Boccaccini	0.74	0.43	0.89
Zholkovskiy et al.	0.66	0.29	0.86
Martynov and Syromyasov	0.7	0.36	0.88
Mahmoodzadeh and Chidiac	0.79	0.53	0.92

Table 4
Degree of correlation between models and experimental data for concrete with HRWRA.

Models	Correlation	Confidence bounds (95%)	
		Lower bound	Upper bounds
Ferraris and deLarrard	0.47	−0.01	0.78
Murata and Kikukawa	–	–	–
Hu and deLarrard	0.83	0.58	0.94
Sudduth	–	–	–
Fan and Boccaccini	–	–	–
Zholkovskiy et al.	−0.05	−0.52	0.44
Martynov and Syromyasov	–	–	–
Mahmoodzadeh and Chidiac	0.82	0.57	0.93

best correlation, 0.79, with 0.53 and 0.92 as the 95% confidence bounds. Hu and deLarrard is found to be the second best with a correlation value of 0.75 and 95% confidence bounds of 0.44 and 0.90. Although the correlation obtained for the two models is found to be comparable, Hu and deLarrard model requires the calibration of three parameters in comparison to two for Mahmoodzadeh and Chidiac model.

For closer assessment of the errors, the percent difference between the models predictions and the experimental data is shown in Fig. 5. Only the results from models with a correlation factor of 0.7 or higher are shown and for concrete without HRWRA. These results show that Mahmoodzadeh and Chidiac model produces more consistent predictions in comparison to the other three models. The error is found to be less than 25% for all the data with the exception of three data points corresponding to plastic viscosity values of 56, 62 and 146 Pa s. However, when comparing with the other three models, it was found that Martynov and Syromyasov model has five predictions with error greater than 25%, followed by Fan and Boccaccini with six predictions and then Hu and deLarrard with nine. Martynov and Syromyasov model predictions with error more than 25% is found to correspond to plastic viscosity values of 56, 62, 146, 89 and 161 Pa s, Fan and Boccaccini model predictions correspond to plastic viscosity values of 56, 62, 146, 89, 161 and 96 Pa s, whereas Hu and deLarrard model predictions correspond to plastic viscosity values of 62, 146, 89, 96, 93, 111, 119, 134 and 140 Pa s. These results indicate that Mahmoodzadeh and Chidiac model, Martynov and Syromyasov model and Fan and Boccaccini model predictions of plastic viscosity are somewhat consistent, i.e. Martynov and Syromyasov model predictions of plastic viscosity that are greater than 25% included the data points from Mahmoodzadeh and Chidiac model and Fan and Boccaccini included those from Martynov and Syromyasov. Moreover, the errors in the predictions of Hu and deLarrard model (phenomenological model) are not consistent with the other three models (based on fundamental principles).

The correlations among the models were calculated to determine the extent to which the models co-vary and the results are given in Table 5. It is found that the models predictions correlate better with each other in comparison to the experimental results with the exception of Mahmoodzadeh and Chidiac and Hu and deLarrard. It is also found that the predictions obtained from Mahmoodzadeh and Chidiac correlates best with those obtained from Hu and deLarrard and vice versa ($R^2 = 0.96$). The predictions from Martynov and Syromyasov are found to correlate best with those obtained from Sudduth and Zholkovskiy ($R^2 = 0.99$). Predictions from Fan's and Boccaccini model and Murata and Kikukawa's model are found to correlate best with those of Martynov and Syromyasov ($R^2 = 0.94$) and Hu and deLarrard

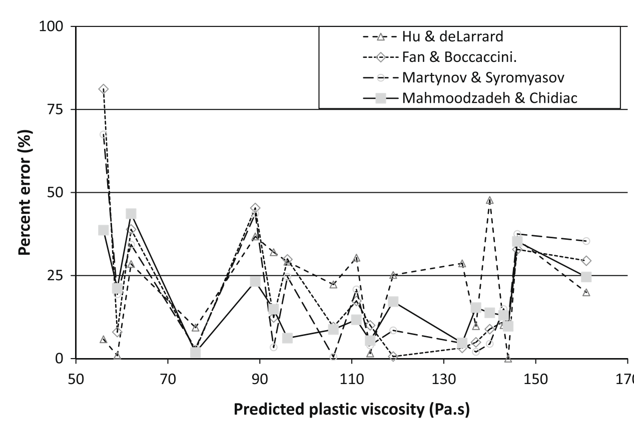

Fig. 5. Error in percent difference between experimental data and model predictions for concrete without HRWRA.

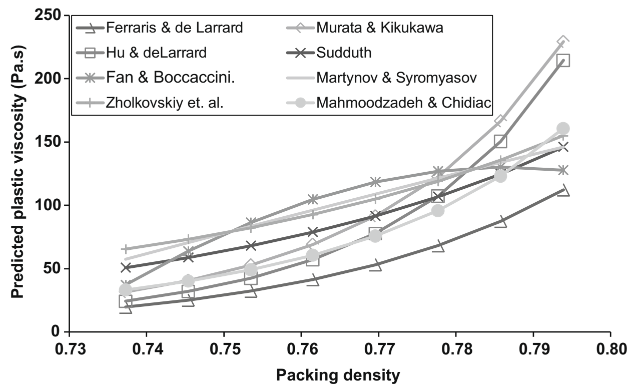

Fig. 6. Predicted plastic viscosity versus packing density.

(R^2 = 0.95). These results show that the two models that are found to provide the highest correlation with the experimental data are also found to have best correlation in their predictions. Another important observation is the correlation between Mahmoodzadeh and Chidiac and the predictions obtained using Murata and Kikukawa (R^2 = 0.92), Sudduth (R^2 = 0.92), Martynov and Syromyasov (R^2 = 0.91) and Zholkovskiy (R^2 = 0.92). The same observation is noted for Hu and deLarrard. However, the degree of correlation obtained among models predictions does not appear to have any link to the degree of correlation between the models predictions and the experimental data.

Predicted plastic viscosities for different values of packing density are shown in Fig. 6. Curves obtained for Fan and Boccaccini and Martynov and Syromyasov are found to differ from the other models. Comparing the results obtained from Hu and deLarrard with those of Mahmoodzadeh and Chidiac, one observes that the two models yield comparable values of plastic viscosity when the packing density is less than or equal to 0.78. The model predictions diverge significantly when the packing density is greater than 0.78. Similar trend is observed for Murata and Kikukawa model and Ferraris and deLarrard model. This difference in model predictions is attributed to particle interactions which become more pronounced as packing density of the mixture increases.

5. Concluding remarks

This review indicates that the prevailing models used for predicting the plastic viscosity of concrete, with the exception of Mahmoodzadeh and Chidiac model, are based on theories that were not intended for a medium to high concentration of suspended particles such as concrete. Accordingly, these models do not consider particle interactions and therefore assume as a priori a low concentration of spherical solid particles. The impact of these limitations is apparent when the results of Figs. 5 and 6 were examined. The review has also revealed that rheological models developed on the basis of fundamental principles, is a necessary requirement but not a sufficient one for obtaining good results. Understanding of the flow behaviour of fresh concrete is also needed in order to develop a comprehensive model.

Results of Fig. 5 clearly show that the degree of correlation between model predictions and the experimental data can be misleading. Mahmoodzadeh and Chidiac model and Hu and deLarrard model were found to yield similar degree of correlation. However examination of the percent error for each prediction has shown that the two models are not the same and that Mahmoodzadeh and Chidiac models is the only ones that is consistent and reliable in predicting the plastic viscosity. Moreover, the errors in the predicted plastic viscosity obtained using Mahmoodzadeh and Chidiac model when compared to the experimental data are high only for low and high values of plastic viscosities. This is the range where the error in the BTRHEOM measurements is also expected to be high for the following reasons: (1) for high slump concrete, BTRHEOM measurements of shear stress and strain rate are not linear and do not obey Bingham's model used by BTRHEOM to estimate the rheological properties, and (2) for low slump concrete, BTRHEOM measurements are found problematic. The same observation cannot be made for Hu and deLarrard model although a reasonable degree of correlation with the experimental data was obtained. The good fit has been attributed to the higher number of fitting parameters – three in comparison to two for Mahmoodzadeh and Chidiac model.

Although all the models were used to predict the plastic viscosity of fresh concrete using the composition of the mixture, only Mahmoodzadeh and Chidiac model is found to yield results that are consistent and comparable to the experimental ones. This model although still requires further testing, can be used by the concrete industry for designing concrete mixture instead of the traditional slump measurement.

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Table 5
Degree of correlation among the models for concrete without HRWRA.

	Ferraris and deLarrard	Murata and Kikukawa	Hu and deLarrard	Sudduth	Fan and Boccaccini	Martynov and Syromyasov	Zholkovskiy et al.	Mahmoodzadeh and Chidiac
Ferraris and deLarrard	1.00	0.72	0.74	0.85	0.82	0.82	0.79	0.76
Murata and Kikukawa	0.72	1.00	0.95	0.93	0.75	0.90	0.94	0.92
Hu and deLarrard	0.74	0.95	1.00	0.94	0.78	0.91	0.94	0.96
Sudduth	0.85	0.93	0.94	1.00	0.90	0.99	1.00	0.92
Fan and Boccaccini	0.82	0.75	0.78	0.90	1.00	0.94	0.88	0.81
Martynov and Syromyasov	0.82	0.90	0.91	0.99	0.94	1.00	0.99	0.91
Zholkovskiy et al.	0.79	0.94	0.94	1.00	0.88	0.99	1.00	0.92
Mahmoodzadeh and Chidiac	0.76	0.92	0.96	0.92	0.81	0.91	0.92	1.00
Experimental	0.62	0.61	0.75	0.67	0.74	0.70	0.66	0.79

Appendix A. Models evaluated that relate the plastic viscosity to the concentration of the suspension

Method	Developed by	Proposed model	Parameters
Generalized	Dabak and Yucel [14]	$\eta_r = \left[1 + \left(\frac{\eta_i \varphi \varphi_{\max}}{n(\varphi_{\max} - \varphi)}\right)\right]^n$	η_i, n
	Sudduth [15]	$\ln(\eta/\eta_0) = \left(\frac{\eta_i}{k}\right)\left(\frac{1}{\sigma - 1}\right)\left((1 - k\phi)^{1-\sigma} - 1\right)$	η_i, σ
		$k = 1/\varphi_{\max}$	
Analogous	Fan and Boccaccini [16]	$\eta^c = \eta_\alpha f_{\alpha c} + \eta_\beta f_{\beta c} + \frac{\eta_\alpha \eta_\beta F_s}{\eta_\beta f_{\alpha III} + \eta_\alpha f_{\beta III}}$	
		$f_{\alpha c} = f_\alpha^m, \quad f_{\alpha c} = f_\beta^n$	
		$\eta_r = C + \varphi^m + \frac{F_s^2}{\varphi - \varphi^m}$	C, m, F_S
	Douglas [27]	$\eta = K\left(1 - \frac{\varphi}{\varphi_{\max}}\right)^{-n}$	K, n
	Bicerano et al. [28]	$\eta_r = \left(1 - \frac{\varphi}{\varphi_{\max}}\right)^{-n} \times \lambda$	n, C_1, C_2
		$\lambda = \left[1 - C_1 \frac{\varphi}{\varphi_{\max}} + C_2\left(\frac{\varphi}{\varphi_{\max}}\right)^2\right]$	
Cell	Simha [17]	$\eta_r = \eta/\eta_0 = 1 + \eta_i \lambda \varphi$	η_i
		$\lambda = \frac{4(1 - y^7)}{[4(1 + y^{10}) - 25y^3(1 + y^4) + 42y^5]}$	
		$y = a/b = \frac{(\varphi/\varphi_{\max})^{1/3}}{(2 - (\varphi/\varphi_{\max})^{1/3})}$	
		$\eta_r \sim \frac{54}{4f^3}\left(\frac{\varphi^2}{1 - (\varphi/\varphi_{\max})^3}\right)$	f
	Happel [18]	$\eta/\eta_0 = 1 + \eta_{i2} \varphi \psi$	$\eta_{i2} = 5.5$
		$\psi = \frac{4\gamma^7 + 10 - \frac{84}{11}\gamma^2}{10(1 - \gamma^{10}) - 25\gamma^3(1 - \gamma^4)}$	
		$\gamma = a/b = \varphi^{1/3}$	
Cell	Frankel and Acrivos [19]	$\eta_r \sim C'\left[\frac{(\varphi/\varphi_{\max})^{1/3}}{1 - (\varphi/\varphi_{\max})^{1/3}}\right]$	C'
	Sengun and Probstein [29]	$\eta_r = 1 + C\left(\frac{3\pi}{8}\right)\left(\frac{\beta}{\beta + 1}\right) \times \lambda$	C
		$\lambda = \left[\frac{3 + 4.5\beta + \beta^2}{\beta + 1} - 3\left(1 + \frac{1}{\beta}\right)\ln(\beta + 1)\right]$	
		$\beta = \frac{2a}{h_0} = \frac{(\phi/\phi_{\max})^{1/3}}{1 - (\phi/\phi_{\max})^{1/3}}$	
	Malomuzh and Orlov [30]	$\eta_r = \frac{\psi(1 - \psi)}{\psi(1 - \psi) + 1 - \sqrt{1 + 2\psi^2(1 - \psi)}}$	C
		$\psi = R_0^3/R^3 \approx C\varphi$	
	Ruiz-Reina et al. [31]	$\eta(\varphi, \lambda) = 1 + \eta_i \varphi \times \lambda$	η_i
		$\lambda = \frac{4(1 - \varphi^{7/3})}{4(1 + \varphi^{10/3}) - 25\varphi(1 + \varphi^{4/3}) + 42\varphi^{5/3}}$	
	Sherwood [32]	$\eta_r = 1 + \eta_i \lambda$	η_i
		$\lambda = \frac{4(y^7 - 1)}{10y^{10} - 25y^7 + 42y^5 - 25y^3 + 4}$	
		$\eta_r \sim \frac{2}{5\varepsilon^3},\ y = a/b = \varphi^{1/3}$	
		$\eta_r = 1 + \frac{15y^7 + 20}{6y^{10} + 15y^7 - 63y^5 + 50y^3 - 8}$	
		$\eta_r \sim \frac{2}{3\varepsilon^2},\ y = a/b = \varphi^{1/3}$	
		$\eta_r = 1 + \frac{25y^7 + 10}{10y^{10} - 10y^7 - 21y^5 + 25y^3 - 4}$	
		$\eta_r \sim \frac{1}{3\varepsilon^2},\ y = a/b = \varphi^{1/3}$	
	Zholkovskiy et al. [21]	$\eta_r = 1 + \eta_i y^3 \lambda$	η_i
		$\lambda = \frac{4(1 - y^7)}{4(1 + y^{10}) - 25y^3(1 + y^4) + 42y^5}$	
		$y = a/b = \varphi^{1/3}$	
	Mahmoodzadeh and Chidiac [22]	$\eta_r = 1 + \eta_i y^3 \lambda$	η_i
		$\lambda = \frac{4(1 - y^7)}{4(1 + y^{10}) - 25y^3(1 + y^4) + 42y^5}$	
		$y(\varphi) = \frac{(\varphi/\varphi_{\max})^{1/3}}{2(1 + K) - (\varphi/\varphi_{\max})^{1/3}}$ Or $y(\varphi) = (\varphi/\varphi_{\max})^{1/3}(1 - K)$	
		$\begin{cases} K = 0.006 \times \frac{\text{Cement}}{\text{Water}} & \text{Without HRWRA} \\ K = 3.8 \times \frac{\text{HRWRA}}{\text{Cement}} & \text{With HRWRA} \\ \times \frac{\text{Water}}{\text{Cement+Fine Sand+Sand}} & \end{cases}$	
Average	Brule and Jongschaap [33]	$\eta_r = 1 + \frac{3\pi}{16}\left(\frac{1 - \varepsilon}{\varepsilon}\right)$	$\varepsilon = 1 - \left(\frac{\varphi}{\varphi_m}\right)^{1/3}$
		$\eta_r = 1 + \frac{\pi}{4}\left(\varepsilon - 1 + \ln\left(\frac{1}{\varepsilon}\right)\right)$	
	Martynov and Syromyasov [25]	$\eta/\eta_0 = 1 + \eta_i \varphi \lambda$	ρ, η_i
		$\lambda = \left[1 + \frac{15\beta_2}{2\pi}\varphi + \left(\frac{15\beta_2}{2\pi}\varphi\right)^2 + \frac{14}{3}\rho\left(\frac{3}{4\pi}\varphi\right)^{5/3}\right]$	
		$\beta_2 \approx -0.3594$	

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