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# Application of a weighted Grey-Taguchi method for optimizing recycled aggregate concrete mixtures

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ABSTRACT

Assessment of the optimal mixture is an important issue to obtain desired quality. This paper integrates grey relational analysis and an objective weighting technique into the Taguchi method to propose the weighted Grey-Taguchi method. This method can be employed to assess the optimal mixture with multiple responses. In the application of this method, water/cement ratio, volume ratio of recycled coarse aggregate, replacement by river sand, content of crushed brick, and cleanliness of aggregate are selected as control factors with responses of slump, slump-flow, resistivity (7-day, 14-day, 28-day), ultrasonic pulse velocity (7-day, 14-day, 28-day), and compressive strength (7-day, 14-day, 28-day) to assess the optimal mixture of recycled aggregate concrete. Results demonstrate and verify that the optimal mixture has a water/cement ratio of 0.5, a volume fraction of recycled coarse aggregate of 42.0%, 100% replacement of river sand, 0% crushed brick, and water-washed aggregates.



## 1. Introduction

The assessment of an optimal mixture for obtaining desired quality is an important issue in the field of material engineering. The problem of optimal mixture assessment can be described as $y = f(x_1, x_2, \ldots, x_n)$, in which $y$ denotes the key response used to represent quality, and $x_1$ to $x_n$ are the control factors that will mainly affect the performance of the response. If each control factor has three input levels, $3^n$ mixtures (factor level combinations) are required for a full factorial design to determine the optimal mixture by the traditional design of experiment (DOE) [1].

Since DOE, which can be mainly divided into full factorial design and fractional factorial design (the latter to decrease the number of experiments), was developed, it has been widely employed in various mixture proportion studies, such as mineral aggregates [2], high strength concrete beams [3], and aerostatic bearings [4]. However, two problems, namely large time/cost requirements for experiments and complex calculations resulting from full factorial design and fractional factorial design, respectively, will be encountered in practice. Therefore, a Taguchi method [5] employing an orthogonal array and signal-to-noise ratio (S/N ratio) analysis was proposed to improve the effectiveness and efficiency of DOE by reducing the time/cost of experiments and obtaining robust evaluations. Taguchi methods have been successfully employed to solve mixture proportion problems of epoxy-$TiO_2$ particulate filled functionally graded composites [6], pulsed current gas tungsten arc welding [7], and laser welding [8].

Both conventional DOE and Taguchi methods can only consider a single response at a time. But, in practice, the presentation of quality ought to be considered in various responses, i.e., the problem of optimal mixture assessment ought to be described as $(y_1, y_2, \ldots, y_m) = f(x_1, x_2, \ldots, x_n)$ where $y_1$ to $y_m$ are different responses used to represent quality. To solve the optimal mixture problems with multiple responses by DOE and Taguchi methods, the optimal mixture for each response is frequently assessed individually, and then the overall optimal mixture is determined by engineering experience or cross analysis. This approach still cannot deal with too many responses at the same time, because of the increasing complexity of the calculations and the possibility of erroneous judgments.

Therefore, a Grey-Taguchi method employing grey relational analysis in the Taguchi method [5] has been proposed to effectively solve the optimal mixture problem with multiple responses. The technique of grey relational analysis can provide a comprehensive index, i.e., grey relational grade (GRG), to represent the complete performance of all responses $(y_1, y_2, \ldots, y_m)$. Recently, this Grey-Taguchi method has been utilized to solve optimal mixture problems with multiple responses in various fields, such as thin-film sputtering processes [9], submerged arc welding process parameters in hardfacing [10], and laser butt welding parameters [11].

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To enhance the reasonability of the comprehensive index produced by grey relational analysis, this paper proposes a weighting technique for determining appropriate weights on responses. Generally, weighting techniques can be established in subjective or objective ways. Subjective weighting techniques are based on engineering judgments, and two common subjective techniques are the analytic hierarchy process (AHP) and the analytic network process (ANP) [12]. However, this paper proposes an objective weighting technique based on the maximum deviation, i.e., larger weights are assigned to responses which can distinguish the differences between mixtures more effectively. Such an objective technique can not only provide weights on responses but also detect inefficient responses to further reduce time/cost of experiments.

Finally, this paper merges the proposed weighting technique into the Grey-Taguchi method to establish a weighted Grey-Taguchi method to solve optimal mixture problems with multiple responses. In the application of the weighted Grey-Taguchi method, this paper utilizes a set of experimental data on concrete made with recycled aggregates (recycled aggregate concrete) to assess the optimal mixture. In the experimental data, water/cement ratio, volume ratio of recycled coarse aggregate, replacement by river sand, content of crushed brick, and cleanliness of aggregate are selected as control factors on the eleven responses of slump, slump flow, resistivity (at 7, 14, and 28 days), ultrasonic pulse velocity (at 7, 14, and 28 days), and compressive strength (at 7, 14, and 28 days) of recycled aggregate concrete. Full response design and reduced response design (resulting from the weighting technique) are compared to demonstrate the effectiveness of the weighting technique. A verification experiment is also performed to determine the applicability and correctness of the weighted Grey-Taguchi method for recycled aggregate concrete with multiple responses.

### 1.1. Orthogonal array and S/N ratio

Orthogonal arrays and S/N ratios are two main components of the Taguchi method. An orthogonal array is used to reduce testing time/cost. If an experiment has 15 control factors with two levels, all possible $n = 2^{15} = 32{,}768$ mixtures are required to test for assessing the optimal mixture by using a full factorial design of experiment. By using the orthogonal array $L_{16}2^{15}$, only 16 mixtures are required to estimate the optimal mixture versus the optimal determination via the full factorial design of experiment. To drastically reduce the number of tests while still gaining significant insight on important factors and optimal settings, Taguchi recommended the use of eighteen basic orthogonal fractional factorial arrays known as the standard orthogonal arrays [5].

On the analysis side, Taguchi advocated the S/N ratio as a single indicator that jointly and simultaneously considers the average value and standard deviation of test results to determine the relative importance of the factors under study. The S/N ratio can be categorized into three types as follows. Selection of the appropriate S/N ratio depends on the features of responses.

1. The smaller-the-better (STB) type

$$SN_{STB} = -10\log_{10}\left(\frac{1}{t}\sum_{i=1}^{m} y^2(i)\right). \tag{1}$$

2. The larger-the-better (LTB) type

$$SN_{LTB} = -10\log_{10}\left(\frac{1}{t}\sum_{i=1}^{m} \frac{1}{y^2(i)}\right). \tag{2}$$

3. The nominal-the-better (NTB) type

$$SN_{NTB} = 10\log_{10}\left(\frac{1}{t}\sum_{i=1}^{t=m} (y(i)-v)^2\right). \tag{3}$$

In the above-mentioned equations, $m$, $y(i)$, and $v$ are the number of mixtures, the testing result of the $i$th test, and the target value of the response, respectively.

### 1.2. Grey relational analysis

Grey relational analysis can be used to consider multiple responses at the same time and then to provide a comprehensive index to represent the evaluation of responses. Grey relational analysis has been widely employed in various fields and has thus demonstrated its applicability [13–15].

Pre-processing of the raw data matrix is required to satisfy the comparability (non-dimension, scaling, and polarization) among responses before conducting grey relational analysis. The raw data matrix, $D$, is illustrated as follows.

$$D = \begin{bmatrix} x_0(1) & x_0(2) & \dots & x_0(m) \\ x_1(1) & x_1(2) & \dots & x_1(m) \\ x_2(1) & x_2(2) & \dots & x_2(m) \\ \dots & \dots & \dots & \dots \\ x_n(1) & x_n(2) & \dots & x_n(m) \end{bmatrix}.$$

in which $x_0$ is the reference set, and $x_1$ to $x_n$ are the comparison set. Each set is composed of $m$ responses, and $x_i(j)$ represents the evaluation of the $i$th series on the $j$th response. The reference series can be composed of measured data or assumed data based on the requirements of evaluation. The raw data matrix can be pre-processed by Eq. (4) (the smaller-the-better type) or Eq. (5) (the larger-the-better type) depending on the feature of response [16] (Eq. (5) is used in the present study)

$$r_i(j) = -\frac{x_i(j)}{oB} + 2. \tag{4}$$

$$r_i(j) = \frac{x_i(j)}{oB}. \tag{5}$$

$OB$ indicates the object value of responses. In the smaller-the-better type of response, such as cost, $OB$ can be defined as the minimum value of the response. In the larger-the-better type of response, such as benefit, $OB$ can be defined as the maximum value of the response. $r_o(j)$ and $r_i(j)$ are the pre-processed values of $x_o(j)$ and $x_i(j)$, respectively. The difference between $r_o(j)$ and $r_i(j)$ can be calculated as $\Delta_{0i}(j) = |r_0(j) - r_i(j)|$ and then the difference matrix, $\Delta$, is constructed as follows.

$$\Delta = \begin{bmatrix} \Delta_{01}(1) & \Delta_{01}(2) & \dots & \Delta_{01}(m) \\ \Delta_{02}(1) & \Delta_{02}(2) & \dots & \Delta_{02}(m) \\ \dots & \dots & \dots & \dots \\ \Delta_{0n}(1) & \Delta_{0n}(2) & \dots & \Delta_{0n}(m) \end{bmatrix},$$

The grey relational coefficient, $\varepsilon_{0i}(j)$, between $r_o(j)$ and $r_i(j)$ is defined as

$$\varepsilon_{0i}(j) = \frac{\Delta min + \rho\Delta max}{\Delta_{oi}(j) + \rho\Delta max}, \tag{6}$$

in which $\rho$ is the identification coefficient ($\rho \in (0,1]$ and usually set as 0.5). $\Delta min = \min_{\forall i}\min_{\forall j}\Delta_{0i}(j)$ and $\Delta max = \max_{\forall i}\max_{\forall j}\Delta_{0i}(j)$. The grey relational coefficient is used to measure the closeness between $r_o(j)$ and $r_i(j)$ in the space of pre-processed data. Thus, the grey relational coefficient matrix, $\varepsilon$, is constructed as follows.

$$\varepsilon = \begin{bmatrix} \varepsilon_{01}(1) & \varepsilon_{01}(2) & \dots & \varepsilon_{01}(m) \\ \varepsilon_{02}(1) & \varepsilon_{02}(2) & \dots & \varepsilon_{02}(m) \\ \dots & \dots & \dots & \dots \\ \varepsilon_{0n}(1) & \varepsilon_{0n}(2) & \dots & \varepsilon_{0n}(m) \end{bmatrix},$$

Finally, the grey relational grade, $g_{0i}$, between $r_0$ and $r_i$ can be obtained by using Eq. (7)

$$g_{0i} = \sum_{j=1}^{m} w'(j)\varepsilon_{0i}(j) \tag{7}$$

in which $w'(j)$ represents the normalized non-negative weight assigned to the $j$th response and $\sum_{j=1}^{m} w'(j)$ is equal to 1. Grey relational grade is regarded as the sum of weighted grey relational coefficients and can be then used to determine the priority of comparison series.

### 1.3. Weighting technique

As shown in Eq. (7), $w(j)$ is an important component to obtain reasonable grey relational grades. The following weighting technique proposed in this paper is constructed by the concept of maximum deviation. Note that the definition of deviation in this paper is used to measure the degree of dispersion, which is different from the standard deviation used in statistics and probability theory.

The weight vector, $w$, is assumed as $w = [w(1), w(2), \ldots, w(m)]$, in which $w(j) \geqslant 0$ and $\sum_{j=1}^{m} w(j)^2 = 1$. For a specific response j, the unit deviation, $D_i(j)$, can be defined as

$$D_i(j) = \sum_{k=1}^{n} d_i(k) \sum_{k=1}^{n} \sqrt{[r_i(j) - r_k(j)]^2}, \tag{8}$$

in which $r_i(j)$ is the normalized value of $x_i(j)$ and $d_i(k)$ is the distance between $r_i(j)$ and $r_k(j)$. Then, the weighted deviation of the $j$th response, $D(j)$, can be calculated as $D(j) = \sum_{i=1}^{n}\sum_{k=1}^{n} w(j) d_i(k)$. The total weighted deviation of all responses can be also calculated as $D = \sum_{j=1}^{m} D(j)$. The aim of this weighting technique is to assign larger weights to responses which can distinguish the differences between mixtures more effectively. For this reason, this paper utilizes the following optimization equation to identify the weight vector which can maximize the total deviations.

$$\begin{cases} \max D = \sum\limits_{j=1}^{m} \sum\limits_{i=1}^{n} \sum\limits_{k=1}^{n} w(j) d_i(k) \\ s.t. w(j) \geqslant 0, \sum\limits_{j=1}^{m} w(j)^2 = 1 \end{cases}. \tag{9}$$

A Lagrange function is employed to solve Eq. (9) as

$$L(w, \lambda) = \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{k=1}^{n} w(j) d_i(k) + \lambda \left[ \sum_{j=1}^{m} w(j)^2 - 1 \right]. \tag{10}$$

Calculating the gradients of Eq. (10) as,

$$\begin{cases} \frac{\partial L}{\partial w(j)} = \sum\limits_{i=1}^{n} \sum\limits_{k=1}^{n} d_i(k) + 2\lambda \sum\limits_{j=1}^{m} w(j) = 0 \\ \frac{\partial L}{\partial \lambda} = \sum\limits_{j=1}^{m} w(j)^2 - 1 = 0 \end{cases}. \tag{11}$$

then, $w(j)$ can be calculated as

$$w(j) = \frac{\sum_{i=1}^{n}\sum_{k=1}^{n} d_i(k)}{\sqrt{\sum_{j=1}^{m} \left[\sum_{i=1}^{n}\sum_{k=1}^{n} d_i(k)\right]^2}}. \tag{12}$$

$w(j)$ ought to be further transformed into $w'(j)$ to satisfy the requirement of normalization, i.e., $w'(j) = \frac{w(j)}{\sum_{j=1}^{m} w(j)}$. Hence, the weight of the $j$th response, $w'(j)$, can be defined as

$$w'(j) = \frac{\sum_{i=1}^{n}\sum_{k=1}^{n} d_i(k)}{\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k=1}^{n} d_i(k)}. \tag{13}$$

## 2. Recycled aggregate concrete

Due to the flourish of urban construction caused by the fast development of industry, many old or degraded buildings need to be demolished or reconstructed according to urban renewal plans. Thus, large amounts of construction waste or demolition waste are produced, with concrete waste accounting for the highest proportion. Traditionally, most concrete waste was sent to a landfill. However, owing to the limitations of available urban landfill areas, the conservation of natural resources, and the prevention of environmental pollution, use of recycled concrete aggregate whenever possible has become an important issue, especially with the urgent requirements of sustainability. In related studies, recycled concrete aggregate has been widely employed as construction materials, such as base or sub-base materials [17–20]. Furthermore, the properties of concrete made with recycled aggregates have also been studied to assess their flexibility and applicability [21–23].

The constituents of the concrete used in this study included the ASTM C 150 Type I Portland cement, crushed recycled coarse and fine concrete aggregates, river sand, and crushed recycled brick. The crushed recycled coarse and fine concrete aggregates were generated through the demolition of Portland cement concrete elements of reinforced concrete buildings collected from different construction sites in Taiwan. These concrete rubbles were crushed by a 3/4-in. jaw crusher and then screened to the desired gradation for both coarse and fine aggregates using the conventional sieve analysis process.

Three tests were run on each mixture and their responses were recorded. The size of each cylindrical concrete specimen is $\oint$ $100 \times 200$ mm, and the mixing of concrete satisfies the requirements of the ACI 211.1 specification.

## 3. The weighted Grey-Taguchi procedure

The weighted Grey-Taguchi method is employed in this paper to assess the optimal mixture with multiple responses for recycled aggregate concrete. The procedure of the weighted Grey-Taguchi method is illustrated as Fig. 1 and stated as follows.

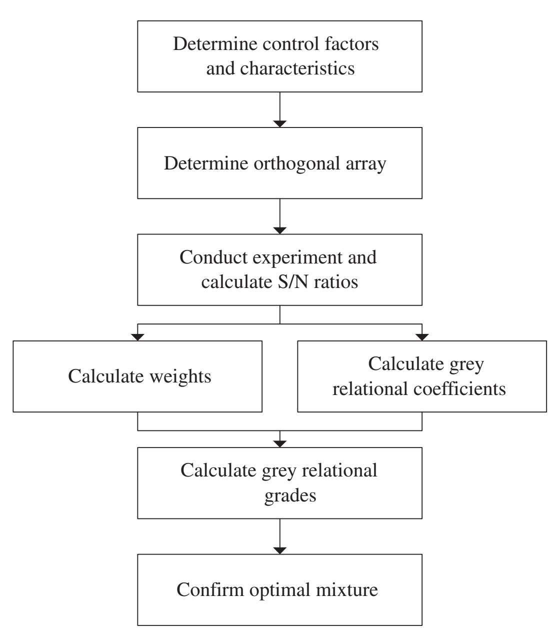


**Fig. 1.** Flow chart of the weighted Grey-Taguchi method.

**Table 1**
Designation of control factors and factor levels.

| Designation | Control factors | Level 1 | Level 2 |
|---|---|---|---|
| A | Water/cement ratio | 0.5 | 0.7 |
| B | Volume ratio of recycled coarse aggregate | 42.00% | 40.40% |
| C | Replacement of river sand | 0% | 100% |
| D | Content of crushed brick | 5% | 0% |
| E | Cleanliness of aggregate | as-is | water-washed |

**Table 2**
Response variables.

| Designation | Responses |
|---|---|
| $y_1$ | Slump (cm) |
| $y_2$ | Slump-flow (cm) |
| $y_{31}$ | 7-day resistivity (KΩ cm) |
| $y_{32}$ | 14-day resistivity (KΩ cm) |
| $y_{33}$ | 28-day resistivity (KΩ cm) |
| $y_{41}$ | 7-day ultrasonic pulse velocity (m/s) |
| $y_{42}$ | 14-day ultrasonic pulse velocity (m/s) |
| $y_{43}$ | 28-day ultrasonic pulse velocity (m/s) |
| $y_{51}$ | 7-day compressive strength (MPa) |
| $y_{52}$ | 14-day compressive strength (MPa) |
| $y_{53}$ | 28-day compressive strength (MPa) |

*Step 1*. Determine control factors and responses.

Five two-level control factors including (1) water/cement ratio, (2) volume ratio of recycled coarse aggregate, (3) replacement of river sand, (4) content of crushed brick, and (5) cleanliness of aggregate which are labelled by A, B, C, D, and E, respectively, are considered in this paper, as shown in Table 1. On the other hand, eleven responses of slump, slump-flow, resistivity (7-day, 14-day, 28-day), ultrasonic pulse velocity (7-day, 14-day, 28-day), and compressive strength (7-day, 14-day, 28-day) are considered as the quality responses of recycled aggregate concrete, as shown in Table 2.

*Step 2*. Determine orthogonal array.

In addition to the specified five control factors, ten interaction factors ($A \times B$, $A \times C$, $A \times D$, $A \times E$, $B \times C$, $B \times D$, $B \times E$, $B \times E$, $C \times D$, $C \times E$, $D \times E$) were also considered in this paper. The total degrees of freedom required are $5 \times (2–1) + 10 \times (2–1) \times (2–1) = 15$. Hence, the $L_{16}2^{15}$ orthogonal array was utilized, as shown in Table 3. In classical design of experiment, this would be equivalent to a $2^{5-1}$ fractional factorial design [1]. These sixteen mixtures of recycled aggregate concrete can be used to identify the optimal mixture. The arrangement of these sixteen mixtures is shown in Table 4.

*Step 3*. Conduct experiment and calculate S/N ratios.

After Step 2, the experiment can be conducted. Three testing results of each mixture on each response are collected. Then, the S/N ratio of the three testing results can be calculated. Because the specified eleven responses are all assumed to be the larger-the-better type, Eq. (2) is adopted to calculate the S/N ratios. In the following steps, $sn_i(j)$ represents the S/N ratio of $i$th mixture on the $j$th response.

*Step 4*. Calculate the weights of responses.

All the computed S/N ratios can be collected as $SN = [sn_i(j)]_{n \times m}$, where $SN$ is composed of $n$ mixtures and $m$ responses and illustrated as follows.

$$SN = \begin{bmatrix} sn_1(1) & sn_1(2) & \dots & sn_1(m) \\ sn_2(1) & sn_2(2) & \dots & sn_2(m) \\ \dots & \dots & \dots & \dots \\ sn_n(1) & sn_n(2) & \dots & sn_n(m) \end{bmatrix}, \tag{14}$$

An ideal series of S/N ratios, denoted as $sn_0$, can be assumed at the same time, in which for fixed response $j$, $sn_0(j)$ is the maximum value among $sn_i(j)$, i.e., $sn_0(j) = \max_{\forall i} sn_i(j)$. Each column of $SN$ has to be normalized to provide comparability. Eq. (5) is adopted, because all S/N ratios are of the larger the better type, to transform $sn_i(j)$ into $r_i(j)$, where $OB = sn_0(j)$; thus, the weight $w(j)$ of response j can be determined by Eqs. (8) and (13).

*Step 5*. Calculate grey relational coefficients.

The calculations of grey relational coefficients are based on the difference between $r_0(j)$ and $r_i(j)$, where $r_0(j)$ is the normalized value of $sn_0(j)$ and is equal to 1. Grey relational coefficients can be calculated by Eq. (6).

*Step 6*. Calculated grey relational grades.

By inputting the calculated weights of responses and grey relational coefficients into Eq. (7), the grey relational grade of each mixture can be obtained. Afterward, the priority of the selected mixtures can be sorted via their grey relational grades. A larger grey relational grade means better comprehensive performance.

**Table 3**
Orthogonal array $L_{16}2^{15}$ with factor assignment for the experiment.

| No. | A | B | $A \times B$ | C | $A \times C$ | $B \times C$ | $D \times E$ | D | $A \times D$ | $B \times D$ | $C \times E$ | $C \times D$ | $B \times E$ | $A \times E$ | E | Designation of mixture |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | A1B1C1D1E1 |
| 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | A1B1C1D2E2 |
| 3 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | A1B1C2D1E2 |
| 4 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | A1B1C2D2E1 |
| 5 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | A1B2C1D1E2 |
| 6 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | A1B2C1D2E1 |
| 7 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | A1B2C2D1E1 |
| 8 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 1 | 1 | 2 | 2 | A1B2C2D2E2 |
| 9 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | A2B1C1D1E2 |
| 10 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | A2B1C1D2E1 |
| 11 | 2 | 1 | 2 | 2 | 1 | 2 | 1 | 1 | 2 | 1 | 2 | 2 | 1 | 2 | 1 | A2B1C2D1E1 |
| 12 | 2 | 1 | 2 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 1 | 2 | 1 | 2 | A2B1C2D2E2 |
| 13 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | A2B2C1D1E1 |
| 14 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | A2B2C1D2E2 |
| 15 | 2 | 2 | 1 | 2 | 1 | 1 | 2 | 1 | 2 | 2 | 1 | 2 | 1 | 1 | 2 | A2B2C2D1E2 |
| 16 | 2 | 2 | 1 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 1 | 2 | 2 | 1 | A2B2C2D2E1 |

**Table 4**
Sixteen sets of mixture of recycled aggregate concrete.

| No. | Designation of mixture | Water/cement ratio | Volume ratio of recycled coarse aggregate (%) | Replacement of river sand (%) | Content of crushed brick (%) | Cleanliness of aggregate |
|---|---|---|---|---|---|---|
| 1 | A1B1C1D1E1 | 0.5 | 42.00 | 0 | 5 | as-is |
| 2 | A1B1C1D2E2 | 0.5 | 42.00 | 0 | 0 | water-washed |
| 3 | A1B1C2D1E2 | 0.5 | 42.00 | 100 | 5 | water-washed |
| 4 | A1B1C2D2E1 | 0.5 | 42.00 | 100 | 0 | as-is |
| 5 | A1B2C1D1E2 | 0.5 | 40.40 | 0 | 5 | water-washed |
| 6 | A1B2C1D2E1 | 0.5 | 40.40 | 0 | 0 | as-is |
| 7 | A1B2C2D1E1 | 0.5 | 40.40 | 100 | 5 | as-is |
| 8 | A1B2C2D2E2 | 0.5 | 40.40 | 100 | 0 | water-washed |
| 9 | A2B1C1D1E2 | 0.7 | 42.00 | 0 | 5 | water-washed |
| 10 | A2B1C1D2E1 | 0.7 | 42.00 | 0 | 0 | as-is |
| 11 | A2B1C2D1E1 | 0.7 | 42.00 | 100 | 5 | as-is |
| 12 | A2B1C2D2E2 | 0.7 | 42.00 | 100 | 0 | water-washed |
| 13 | A2B2C1D1E1 | 0.7 | 40.40 | 0 | 5 | as-is |
| 14 | A2B2C1D2E2 | 0.7 | 40.40 | 0 | 0 | water-washed |
| 15 | A2B2C2D1E2 | 0.7 | 40.40 | 100 | 5 | water-washed |
| 16 | A2B2C2D2E1 | 0.7 | 40.40 | 100 | 0 | as-is |

*Step 7*. Confirm optimal mixture.

The result of Step 6 indicates the best mixture among the selected sixteen mixtures in the orthogonal array. However, the overall optimal mixture ought to be confirmed by main effects table, main effects plot, analysis of variance (ANOVA), and contribution ratio. Both main effects table and main effects plot are used to represent the effects of each control factor on the grey relational grade at different levels. The calculation of effect is an absolute value in the Taguchi method, which is used to measure the impact of changes in factor levels. The overall optimal mixture can be then identified. The purpose of ANOVA is to investigate which control

**Table 5**
Average values of responses for the sixteen mixtures.

| No. | Designation of mixture | Slump (cm) | Slump flow (cm) | Resistivity (KΩ cm) | | | Ultrasonic pulse velocity (m/s) | | | Compressive strength (MPa) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 7-day | 14-day | 28-day | 7-day | 14-day | 28-day | 7-day | 14-day | 28-day |
| 1 | A1B1C1D1E1 | 17.50 | 37.00 | 7.57 | 7.97 | 7.93 | 2837 | 2893 | 2723 | 17.79 | 20.94 | 22.91 |
| 2 | A1B1C1D2E2 | 15.50 | 40.00 | 6.50 | 9.43 | 9.00 | 2753 | 3253 | 3013 | 17.54 | 24.89 | 25.24 |
| 3 | A1B1C2D1E2 | 18.00 | 35.00 | 7.73 | 9.55 | 9.30 | 2817 | **3347** | **3193** | 18.35 | 22.04 | 28.88 |
| 4 | A1B1C2D2E1 | 18.00 | 32.00 | 7.43 | 9.17 | 9.20 | 2910 | 3130 | 2873 | 23.16 | 25.94 | 30.17 |
| 5 | A1B2C1D1E2 | 9.50 | 20.00 | 7.23 | 8.07 | 8.53 | 3040 | 3155 | 3057 | 21.96 | 26.22 | 29.91 |
| 6 | A1B2C1D2E1 | 14.00 | 26.00 | 6.50 | 7.20 | 7.43 | 2843 | 3120 | 2893 | 17.02 | 18.92 | 20.26 |
| 7 | A1B2C2D1E1 | 10.50 | 20.00 | 7.07 | 7.10 | 7.80 | **3047** | 2933 | 2837 | 23.64 | 29.35 | 33.59 |
| 8 | A1B2C2D2E2 | 5.00 | 20.00 | 9.03 | 8.63 | **10.13** | 2750 | 3277 | 3007 | **28.36** | **33.79** | **36.16** |
| 9 | A2B1C1D1E2 | 10.00 | 23.00 | 7.47 | 9.03 | 9.10 | 2573 | 3003 | 2707 | 13.22 | 17.17 | 18.48 |
| 10 | A2B1C1D2E1 | **20.00** | 56.00 | 6.40 | 8.47 | 8.67 | 2550 | 2620 | 2840 | 5.85 | 7.57 | 9.74 |
| 11 | A2B1C2D1E1 | 15.00 | **60.00** | 7.83 | 8.20 | 8.33 | 2383 | 3015 | 3043 | 10.54 | 13.88 | 17.64 |
| 12 | A2B1C2D2E2 | 9.00 | 20.00 | **9.20** | **10.33** | 9.53 | 2747 | 3050 | 2910 | 21.55 | 23.98 | 27.67 |
| 13 | A2B2C1D1E1 | 16.00 | 35.00 | 6.67 | 6.77 | 7.70 | 2677 | 2556 | 2937 | 7.88 | 9.98 | 12.91 |
| 14 | A2B2C1D2E2 | 19.00 | 43.00 | 5.70 | 6.93 | 7.93 | 2567 | 2783 | 2863 | 8.76 | 11.92 | 14.32 |
| 15 | A2B2C2D1E2 | 11.50 | 36.00 | 7.07 | 7.37 | 7.80 | 2907 | 3000 | 2723 | 11.87 | 14.63 | 19.86 |
| 16 | A2B2C2D2E1 | 16.00 | 33.00 | 6.63 | 7.03 | 7.43 | 2750 | 3103 | 2800 | 11.81 | 16.22 | 20.42 |

**Table 6**
Computed S/N ratios of recycled aggregate concrete mixtures.

| No. | Designation of mixture | Slump | Slump flow | Resistivity | | | Ultrasonic pulse velocity | | | Compressive strength | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 7-day | 14-day | 28-day | 7-day | 14-day | 28-day | 7-day | 14-day | 28-day |
| 1 | A1B1C1D1E1 | 24.82 | 31.36 | 17.53 | 18.02 | 17.98 | 69.01 | 69.20 | 68.53 | 25.00 | 26.40 | 27.18 |
| 2 | A1B1C1D2E2 | 23.80 | 32.04 | 16.14 | 19.39 | 19.05 | 68.49 | 70.24 | 69.54 | 24.88 | 27.79 | 28.00 |
| 3 | A1B1C2D1E2 | 25.11 | 30.87 | 17.73 | **20.71** | 19.33 | 68.76 | **70.48** | **70.05** | 24.96 | 26.84 | 29.21 |
| 4 | A1B1C2D2E1 | 25.11 | 30.09 | 17.42 | 19.24 | 19.27 | 69.05 | 69.87 | 68.79 | 27.28 | 28.27 | 29.59 |
| 5 | A1B2C1D1E2 | 19.53 | 26.00 | 17.18 | 18.13 | 18.62 | **69.65** | 69.94 | 69.69 | 26.83 | 28.37 | 29.51 |
| 6 | A1B2C1D2E1 | 22.88 | 28.29 | 16.26 | 17.14 | 17.32 | 69.05 | 69.88 | 69.23 | 24.61 | 25.45 | 26.08 |
| 7 | A1B2C2D1E1 | 20.40 | 25.93 | 16.95 | 17.01 | 17.82 | 69.64 | 69.14 | 69.05 | 27.47 | 29.35 | 30.52 |
| 8 | A1B2C2D2E2 | 13.61 | 25.93 | 19.11 | 18.46 | **20.07** | 68.64 | 70.29 | 69.56 | **29.05** | **30.57** | **31.16** |
| 9 | A2B1C1D1E2 | 19.99 | 27.18 | 17.43 | 19.12 | 19.16 | 68.13 | 69.53 | 68.49 | 22.41 | 24.68 | 25.16 |
| 10 | A2B1C1D2E1 | **26.00** | 34.96 | 16.00 | 18.54 | 18.75 | 68.08 | 68.30 | 69.05 | 15.33 | 17.57 | 19.76 |
| 11 | A2B1C2D1E1 | 23.48 | **35.55** | 17.87 | 18.24 | 18.41 | 67.46 | 69.55 | 69.66 | 20.45 | 22.84 | 24.91 |
| 12 | A2B1C2D2E2 | 18.89 | 26.00 | **19.26** | 20.26 | 19.58 | 68.54 | 69.65 | 68.95 | 26.67 | 27.60 | 28.82 |
| 13 | A2B2C1D1E1 | 24.07 | 30.87 | 16.35 | 16.60 | 17.69 | 68.55 | 68.02 | 69.35 | 17.90 | 19.96 | 22.22 |
| 14 | A2B2C1D2E2 | 25.57 | 32.66 | 15.05 | 16.80 | 17.94 | 68.09 | 68.85 | 69.13 | 18.80 | 21.51 | 23.11 |
| 15 | A2B2C2D1E2 | 21.17 | 31.12 | 16.98 | 17.34 | 17.76 | 69.27 | 69.52 | 68.53 | 21.49 | 23.28 | 25.96 |
| 16 | A2B2C2D2E1 | 24.07 | 30.36 | 16.28 | 16.93 | 17.38 | 68.60 | 69.82 | 68.93 | 21.42 | 24.19 | 26.18 |

factors significantly affect the responses. ANOVA is accomplished by separating the total variability of the grey relational grades, which is measured by the sum of the squared deviations from the total mean of the grey relational grades, into contributions by each control factor and the error. The contribution ratio by each control factor in the total sum of the squared deviations can be used to evaluate the importance of a control factor change on the performance of all responses.

## 4. Results and discussion

### 4.1. Full responses design

Table 5 shows the average values of the specified eleven responses for the selected sixteen mixtures while Table 6 contains their corresponding S/N ratios. Scanning Table 6 for the largest value (shaded) in each of the 11 columns, it is seen that the reference/

**Table 7**
Pre-processed values of S/N ratios on full responses.

| No. | $y_1$ | $y_2$ | $y_{31}$ | $y_{32}$ | $y_{33}$ | $y_{41}$ | $y_{42}$ | $y_{43}$ | $y_{51}$ | $y_{52}$ | $y_{53}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $r_0$ | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 1 | 0.95 | 0.88 | 0.91 | 0.87 | 0.90 | 0.99 | 0.98 | 0.98 | 0.86 | 0.86 | 0.87 |
| 2 | 0.92 | 0.90 | 0.84 | 0.94 | 0.95 | 0.98 | 0.99 | 0.99 | 0.86 | 0.91 | 0.90 |
| 3 | 0.97 | 0.87 | 0.92 | **1.00** | 0.96 | 0.99 | **1.00** | **1.00** | 0.86 | 0.88 | 0.94 |
| 4 | 0.97 | 0.85 | 0.90 | 0.93 | 0.96 | 0.99 | 0.99 | 0.98 | 0.94 | 0.92 | 0.95 |
| 5 | 0.75 | 0.73 | 0.89 | 0.88 | 0.93 | **1.00** | 0.99 | 0.99 | 0.92 | 0.93 | 0.95 |
| 6 | 0.88 | 0.80 | 0.84 | 0.83 | 0.86 | 0.99 | 0.99 | 0.99 | 0.85 | 0.83 | 0.84 |
| 7 | 0.78 | 0.73 | 0.88 | 0.82 | 0.89 | 0.99 | 0.98 | 0.99 | 0.95 | 0.96 | 0.98 |
| 8 | 0.52 | 0.73 | 0.99 | 0.89 | **1.00** | 0.99 | 0.99 | 0.99 | **1.00** | **1.00** | **1.00** |
| 9 | 0.77 | 0.76 | 0.91 | 0.92 | 0.95 | 0.98 | 0.99 | 0.98 | 0.77 | 0.81 | 0.81 |
| 10 | **1.00** | 0.98 | 0.83 | 0.89 | 0.93 | 0.98 | 0.97 | 0.99 | 0.53 | 0.57 | 0.63 |
| 11 | 0.90 | **1.00** | 0.93 | 0.88 | 0.92 | 0.97 | 0.99 | 0.99 | 0.70 | 0.75 | 0.80 |
| 12 | 0.73 | 0.73 | **1.00** | 0.98 | 0.98 | 0.98 | 0.99 | 0.98 | 0.92 | 0.90 | 0.93 |
| 13 | 0.93 | 0.87 | 0.85 | 0.80 | 0.88 | 0.98 | 0.97 | 0.99 | 0.62 | 0.65 | 0.71 |
| 14 | 0.98 | 0.92 | 0.78 | 0.81 | 0.89 | 0.98 | 0.98 | 0.99 | 0.65 | 0.70 | 0.74 |
| 15 | 0.81 | 0.88 | 0.88 | 0.84 | 0.89 | 0.99 | 0.99 | 0.98 | 0.74 | 0.76 | 0.83 |
| 16 | 0.93 | 0.85 | 0.85 | 0.82 | 0.87 | 0.98 | 0.99 | 0.98 | 0.74 | 0.79 | 0.84 |

**Table 8**
Deviations of mixtures on full responses.

| No. | $y_1$ | $y_2$ | $y_{31}$ | $y_{32}$ | $y_{33}$ | $y_{41}$ | $y_{42}$ | $y_{43}$ | $y_{51}$ | $y_{52}$ | $y_{53}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.68 | 1.18 | 0.76 | 0.78 | 0.62 | 0.12 | 0.14 | 0.14 | 1.72 | 1.5 | 1.3 |
| 2 | 1.48 | 1.34 | 0.92 | 1.09 | 0.65 | 0.11 | 0.17 | 0.11 | 1.71 | 1.66 | 1.36 |
| 3 | 1.77 | 1.12 | 0.84 | 1.91 | 0.75 | 0.11 | 0.22 | 0.2 | 1.72 | 1.52 | 1.54 |
| 4 | 1.77 | 1.13 | 0.73 | 1.02 | 0.72 | 0.12 | 0.12 | 0.11 | 2.27 | 1.79 | 1.64 |
| 5 | 2.28 | 1.79 | 0.70 | 0.77 | 0.57 | 0.22 | 0.13 | 0.13 | 2.11 | 1.82 | 1.61 |
| 6 | 1.53 | 1.33 | 0.86 | 0.97 | 0.94 | 0.12 | 0.12 | 0.09 | 1.71 | 1.50 | 1.31 |
| 7 | 1.97 | 1.81 | 0.70 | 1.02 | 0.66 | 0.22 | 0.15 | 0.08 | 2.35 | 2.20 | 2.00 |
| 8 | 5.42 | 1.81 | 1.69 | 0.79 | 1.24 | 0.10 | 0.18 | 0.11 | 3.11 | 2.76 | 2.28 |
| 9 | 2.10 | 1.52 | 0.73 | 0.97 | 0.68 | 0.15 | 0.12 | 0.15 | 1.86 | 1.55 | 1.48 |
| 10 | 2.21 | 2.29 | 1.01 | 0.81 | 0.59 | 0.16 | 0.29 | 0.08 | 4.45 | 4.04 | 3.57 |
| 11 | 1.48 | 2.52 | 0.91 | 0.77 | 0.57 | 0.28 | 0.11 | 0.13 | 2.27 | 1.90 | 1.54 |
| 12 | 2.57 | 1.79 | 1.79 | 1.60 | 0.90 | 0.10 | 0.11 | 0.09 | 2.07 | 1.62 | 1.46 |
| 13 | 1.51 | 1.12 | 0.83 | 1.27 | 0.72 | 0.10 | 0.34 | 0.09 | 3.21 | 2.95 | 2.46 |
| 14 | 1.98 | 1.51 | 1.70 | 1.14 | 0.62 | 0.16 | 0.19 | 0.08 | 2.84 | 2.34 | 2.12 |
| 15 | 1.79 | 1.14 | 0.70 | 0.91 | 0.68 | 0.15 | 0.12 | 0.14 | 1.99 | 1.79 | 1.33 |
| 16 | 1.51 | 1.12 | 0.85 | 1.06 | 0.90 | 0.10 | 0.12 | 0.09 | 2.01 | 1.61 | 1.30 |
| Sum | 33.03 | 24.50 | 15.74 | 16.88 | 11.83 | 2.34 | 2.63 | 1.85 | 37.39 | 32.55 | 28.30 |

**Table 9**
Differences between $r_0(j)$ and $r_i(j)$ on full responses.

| No. | $y_1$ | $y_2$ | $y_{31}$ | $y_{32}$ | $y_{33}$ | $y_{41}$ | $y_{42}$ | $y_{43}$ | $y_{51}$ | $y_{52}$ | $y_{53}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.05 | 0.12 | 0.09 | 0.13 | 0.10 | 0.01 | 0.02 | 0.02 | 0.14 | 0.14 | 0.13 |
| 2 | 0.08 | 0.10 | 0.16 | 0.06 | 0.05 | 0.02 | 0.01 | 0.01 | 0.14 | 0.09 | 0.10 |
| 3 | 0.03 | 0.13 | 0.08 | **0.00** | 0.04 | 0.01 | **0.00** | **0.00** | 0.14 | 0.12 | 0.06 |
| 4 | 0.03 | 0.15 | 0.10 | 0.07 | 0.04 | 0.01 | 0.01 | 0.02 | 0.06 | 0.08 | 0.05 |
| 5 | 0.25 | 0.27 | 0.11 | 0.12 | 0.07 | **0.00** | 0.01 | 0.01 | 0.08 | 0.07 | 0.05 |
| 6 | 0.12 | 0.2 | 0.16 | 0.17 | 0.14 | 0.01 | 0.01 | 0.01 | 0.15 | 0.17 | 0.16 |
| 7 | 0.22 | 0.27 | 0.12 | 0.18 | 0.11 | 0.01 | 0.02 | 0.01 | 0.05 | 0.04 | 0.02 |
| 8 | 0.48 | 0.27 | 0.01 | 0.11 | **0.00** | 0.01 | 0.01 | 0.01 | **0.00** | **0.00** | **0.00** |
| 9 | 0.23 | 0.24 | 0.09 | 0.08 | 0.05 | 0.02 | 0.01 | 0.02 | 0.23 | 0.19 | 0.19 |
| 10 | **0.00** | 0.02 | 0.17 | 0.11 | 0.07 | 0.02 | 0.03 | 0.01 | 0.47 | 0.43 | 0.37 |
| 11 | 0.10 | **0.00** | 0.07 | 0.12 | 0.08 | 0.03 | 0.01 | 0.01 | 0.30 | 0.25 | 0.20 |
| 12 | 0.27 | 0.27 | **0.00** | 0.02 | 0.02 | 0.02 | 0.01 | 0.02 | 0.08 | 0.10 | 0.07 |
| 13 | 0.07 | 0.13 | 0.15 | 0.20 | 0.12 | 0.02 | 0.03 | 0.01 | 0.38 | 0.35 | 0.29 |
| 14 | 0.02 | 0.08 | 0.22 | 0.19 | 0.11 | 0.02 | 0.02 | 0.01 | 0.35 | 0.30 | 0.26 |
| 15 | 0.19 | 0.12 | 0.12 | 0.16 | 0.11 | 0.01 | 0.01 | 0.02 | 0.26 | 0.24 | 0.17 |
| 16 | 0.07 | 0.15 | 0.15 | 0.18 | 0.13 | 0.02 | 0.01 | 0.02 | 0.26 | 0.21 | 0.16 |

**Table 10**
Grey relational coefficients of mixtures on full responses.

| No. | $y_1$ | $y_2$ | $y_{31}$ | $y_{32}$ | $y_{33}$ | $y_{41}$ | $y_{42}$ | $y_{43}$ | $y_{51}$ | $y_{52}$ | $y_{53}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.84 | 0.67 | 0.73 | 0.65 | 0.7 | 0.96 | 0.93 | 0.92 | 0.63 | 0.64 | 0.65 |
| 2 | 0.74 | 0.71 | 0.6 | 0.79 | 0.82 | 0.93 | 0.99 | 0.97 | 0.62 | 0.72 | 0.70 |
| 3 | 0.87 | 0.64 | 0.75 | 1.00 | 0.87 | 0.95 | 1.00 | 1.00 | 0.63 | 0.66 | 0.79 |
| 4 | 0.87 | 0.61 | 0.71 | 0.77 | 0.86 | 0.96 | 0.96 | 0.93 | 0.8 | 0.76 | 0.83 |
| 5 | 0.49 | 0.47 | 0.69 | 0.66 | 0.77 | 1.00 | 0.97 | 0.98 | 0.76 | 0.77 | 0.82 |
| 6 | 0.67 | 0.54 | 0.60 | 0.58 | 0.64 | 0.96 | 0.97 | 0.95 | 0.61 | 0.59 | 0.59 |
| 7 | 0.53 | 0.47 | 0.67 | 0.57 | 0.68 | 1.00 | 0.93 | 0.94 | 0.81 | 0.86 | 0.92 |
| 8 | 0.33 | 0.47 | 0.97 | 0.69 | 1.00 | 0.94 | 0.99 | 0.97 | 1.00 | 1.00 | 1.00 |
| 9 | 0.51 | 0.50 | 0.72 | 0.76 | 0.84 | 0.92 | 0.95 | 0.91 | 0.51 | 0.55 | 0.55 |
| 10 | 1.00 | 0.93 | 0.58 | 0.69 | 0.78 | 0.91 | 0.88 | 0.94 | 0.34 | 0.36 | 0.39 |
| 11 | 0.71 | 1.00 | 0.77 | 0.67 | 0.74 | 0.88 | 0.95 | 0.98 | 0.45 | 0.49 | 0.54 |
| 12 | 0.47 | 0.47 | 1.00 | 0.92 | 0.91 | 0.94 | 0.95 | 0.94 | 0.74 | 0.71 | 0.76 |
| 13 | 0.76 | 0.64 | 0.61 | 0.55 | 0.67 | 0.94 | 0.87 | 0.96 | 0.38 | 0.41 | 0.45 |
| 14 | 0.94 | 0.75 | 0.52 | 0.56 | 0.69 | 0.91 | 0.91 | 0.95 | 0.40 | 0.45 | 0.48 |
| 15 | 0.56 | 0.66 | 0.67 | 0.59 | 0.67 | 0.98 | 0.95 | 0.92 | 0.48 | 0.50 | 0.59 |
| 16 | 0.76 | 0.62 | 0.61 | 0.57 | 0.64 | 0.94 | 0.96 | 0.94 | 0.48 | 0.53 | 0.60 |

ideal set of S/N ratios is $sn_0$ = (26.00, 35.55, 19.26, 20.71, 20.07, 69.65, 70.48, 70.05, 29.05, 30.57, 31.16). The normalized values of S/N ratios are shown in Table 7, in which $r_0(j)$ is the normalized value of $sn_0(j)$ and is identically 1.

To calculate the weights of the responses, the deviation of each mixture for each response is determined as shown in Table 8. Then, the deviation sums of responses are calculated as $D(j)$ = (33.03, 24.50, 15.74, 16.88, 11.83, 2.34, 2.63, 1.85, 37.39, 32.55, 28.30), in which $D(j)$ is computed by $\sum_{i=1}^{16} D_i(j)$. The total deviation sum is also calculated as $\sum_{j=1}^{11} D(j) = 207.05$. Finally, the weights of responses can be calculated by Eq. (13) as $w'(j)$ = (0.16, 0.12, 0.08, 0.08, 0.06, 0.01, 0.01, 0.01, 0.18, 0.16, 0.14).

To calculate the grey relational coefficients, the differences between $r_0(j)$ and $r_i(j)$ are computed in Table 9. As shown in Table 9, $\Delta max$ and $\Delta min$ are assessed as 0.48 and 0.00, respectively. Grey relational coefficients are calculated by Eq. (6), based on Table 9 and $\rho$ = 0.5, and are shown in Table 10.

Finally, by inputting the weights of the responses and the grey relational coefficients, the grey relational grades of the sixteen mixtures can be calculated via Eq. (7) and are shown in Table 11. From Table 11, the best of the sixteen mixtures is A1B2C2D2E2 (GRG of 0.80), which means water cement ratio of 0.5, volume ratio of recycled coarse aggregate of 40.4%, 100% replacement of river sand, 0% crushed brick, and water-washed aggregate.

Although the mixture of A1B2C2D2E2 is the best of the sixteen mixtures identified by grey relational analysis, it is not guaranteed to be the optimal one of all possible mixtures. The overall optimal mixture has to be determined by main effects table, main effects plot, and ANOVA. When adopting DOE or the Taguchi methods, every main effects plot on each response has to be produced. Figs. 2 and 3 are the main effects plots for average value (Table 5) on each response and S/N ratio (Table 6) on each response, respectively. The effects of each control factor at different levels on every response can be observed in these main effects plots. As shown in Figs. 2 and 3, different performances of the mixtures are obtained depending on the response of interest. Thus, the comprehensive main effects plot can be further obtained by the weighted Grey-Taguchi method.

**Table 11**
Grey relational grades for mixtures on full responses.

| No. | Designation | Grey relational grade |
|---|---|---|
| 1 | A1B1C1D1E1 | 0.69 |
| 2 | A1B1C1D2E2 | 0.71 |
| 3 | A1B1C2D1E2 | 0.76 |
| 4 | A1B1C2D2E1 | 0.79 |
| 5 | A1B2C1D1E2 | 0.69 |
| 6 | A1B2C1D2E1 | 0.61 |
| 7 | A1B2C2D1E1 | 0.71 |
| 8 | A1B2C2D2E2 | 0.80 |
| 9 | A2B1C1D1E2 | 0.59 |
| 10 | A2B1C1D2E1 | 0.62 |
| 11 | A2B1C2D1E1 | 0.65 |
| 12 | A2B1C2D2E2 | 0.71 |
| 13 | A2B2C1D1E1 | 0.55 |
| 14 | A2B2C1D2E2 | 0.60 |
| 15 | A2B2C2D1E2 | 0.58 |
| 16 | A2B2C2D2E1 | 0.61 |

Table 12 shows the effects of each control factor on grey relational grade at different levels, and can be further drawn as the main effects plot, as shown in Fig. 4. The dashed line in Fig. 4 is the total mean value of the grey relational grades and is equal to 0.668. According to the computed effects in Table 12 and Fig. 4, the priority of control factors for affecting the multiple performance responses is sequentially A (water/cement ratio), C (replacement of river sand), B (volume ratio of recycled coarse aggregate), D (content of crushed brick), and E (cleanliness of aggregate).

Further confirmation can be conducted by ANOVA of the control factors on the grey relational grades, as shown in Table 13. An *F*-test is also adopted to determine which control factors have significant effects on the performance responses [24]. Results of ANOVA indicate that A (water/cement ratio), B (volume ratio of recycled coarse aggregate), and C (replacement of river sand) are indeed the significant control factors for affecting the multiple responses, and that A (water/cement ratio) is the most significant control factor due to its highest contribution ratio (50.591%) among the control factors.

Finally, according to this analysis, the optimal mixture with multiple responses for recycled aggregate concrete assessed in this paper is A1B1C2D2E2, meaning water cement ratio of 0.5, volume ratio of recycled coarse aggregate of 42.0%, 100% replacement of river sand, 0% crushed brick, and water-washed aggregate.

### 4.2. Reduced responses design

The proposed weighting technique can not only determine the weights of responses but also detect inefficient responses to reduce time/cost of experiments. As shown in their computed weights, the weights of ultrasonic pulse velocity (at 7, 14, and 28 days) are rather small relative to other responses, meaning ultrasonic pulse velocity cannot distinguish the differences among mixtures of recycled aggregate concrete and can be ignored in the experiment. Hence, a set of reduced responses including slump, slump flow, resistivity (7-day, 14-day, 28-day) and compressive strength (7-day, 14-day, 28-day) can obtain the same results as the set of full responses.

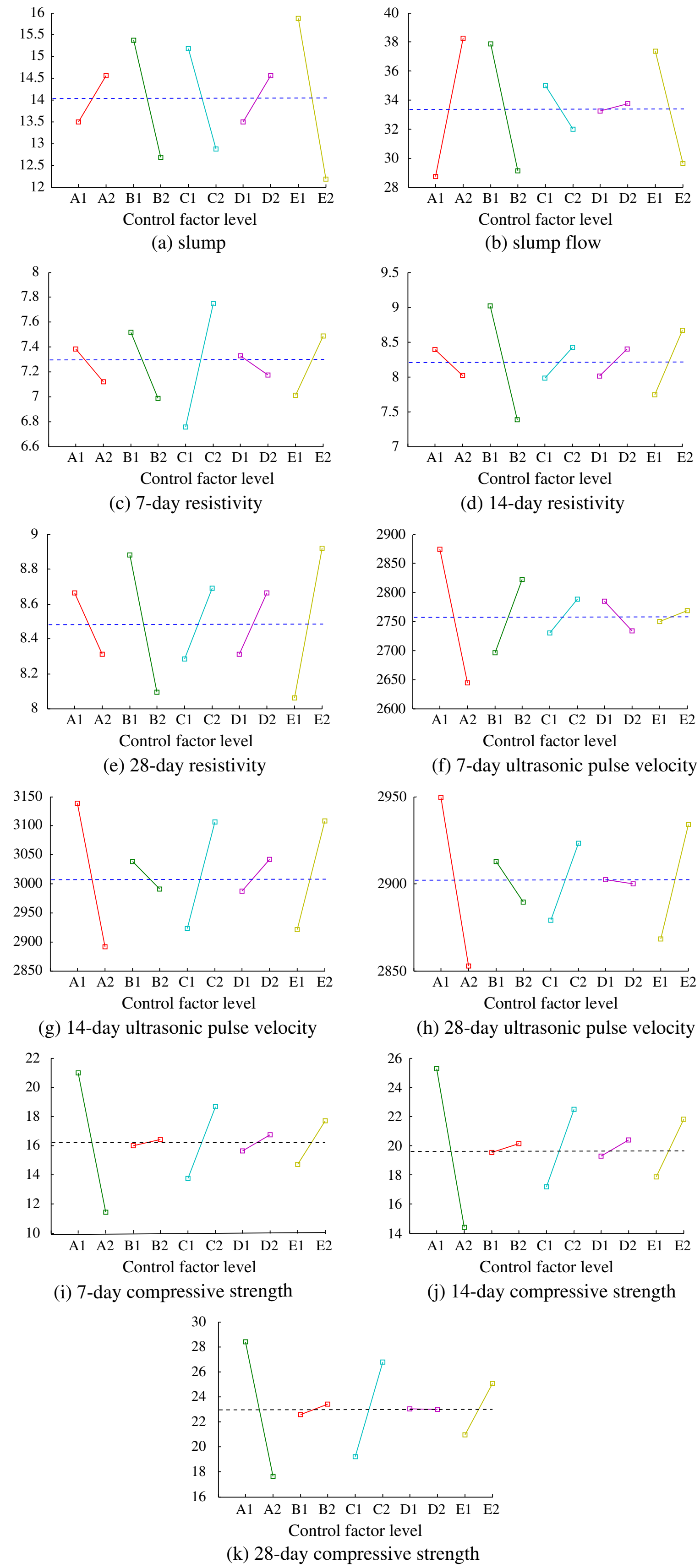


**Fig. 2.** Main effects plot for average value for each response.

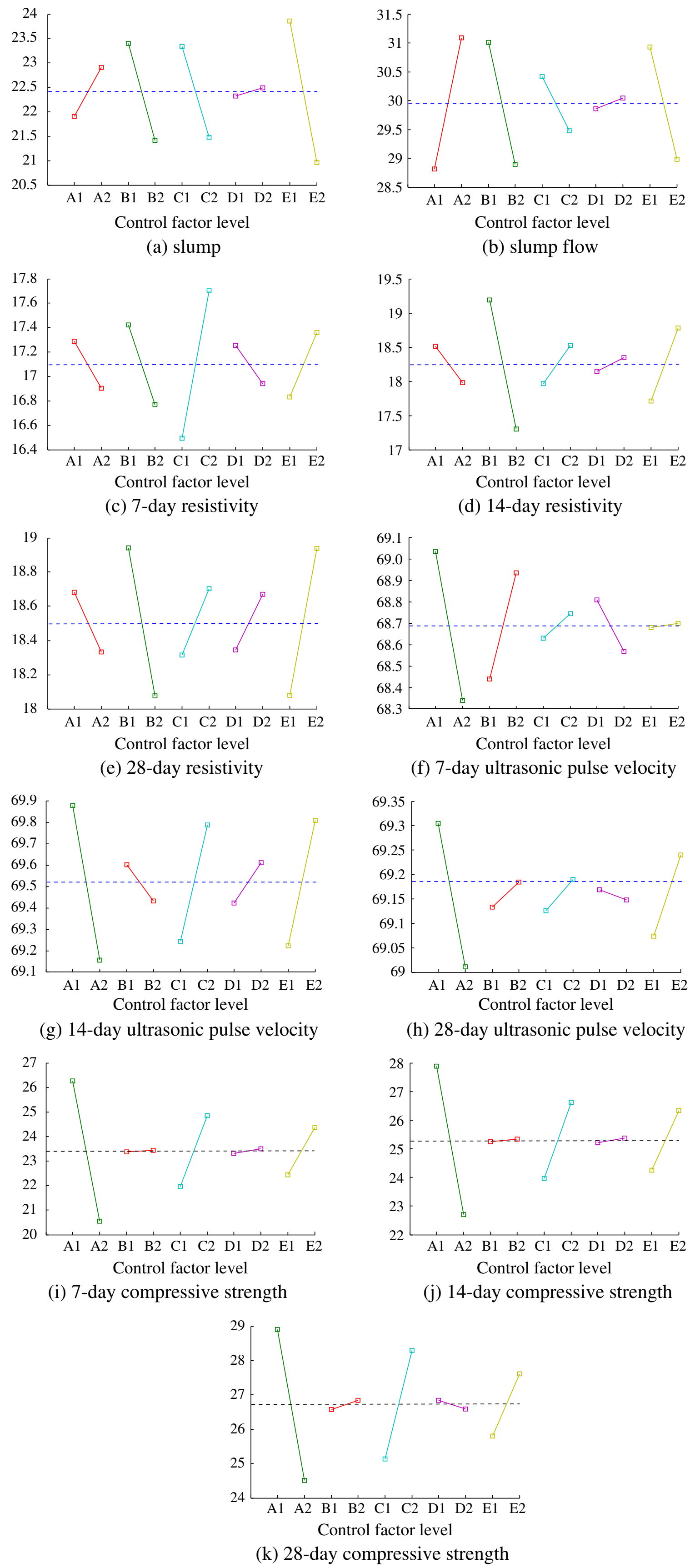


**Fig. 3.** Main effects plot for S/N ratio for each response.

**Table 12**
Main effects table for the grey relational grade on full responses.

| Designation | Control factors | Grey relational grade | | |
|---|---|---|---|---|
| | | Level 1 | Level 2 | Effect |
| A | Water/cement ratio | 0.72 | 0.61 | 0.11 |
| B | Volume ratio of recycled coarse aggregate | 0.69 | 0.64 | 0.05 |
| C | Replacement of river sand | 0.63 | 0.70 | 0.07 |
| D | Content of crushed brick | 0.65 | 0.68 | 0.03 |
| E | Cleanliness of aggregate | 0.65 | 0.68 | 0.03 |

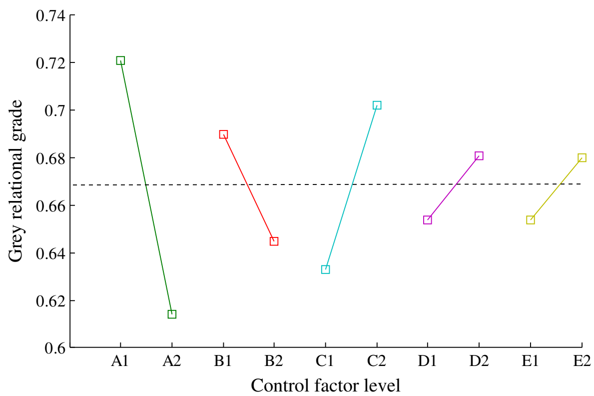


**Fig. 4.** Main effects plot for grey relational grade for full responses.

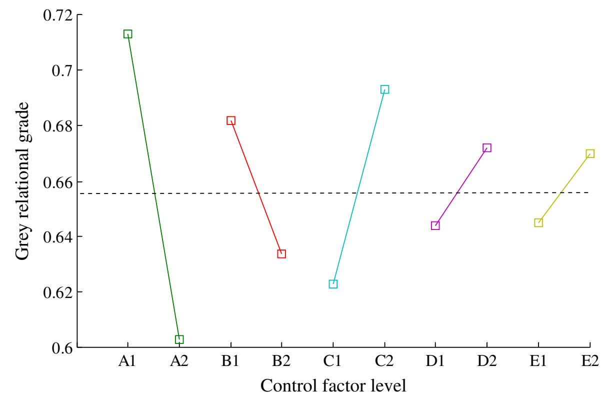


**Fig. 5.** Main effects plot for grey relational grade for reduced responses.

The experimental data of reduced responses were also subjected to the same steps of the weighted Grey-Taguchi method including pre-processing, calculation of weights, computation of grey relational coefficients, and determination of grey relational grades. After calculating the deviations of mixtures using reduced responses, the deviation sums of reduced responses are obtained as $D(j)$ = (33.03, 24.50, 15.74, 16.88, 11.83, 37.39, 32.55, 28.30) and the total deviation sum is also calculated as $\sum_{j=1}^{11} D(j) = 200.23$. Thus, the weights of reduced responses can be calculated as $w'(j)$ = (0.16, 0.12, 0.08, 0.08, 0.06, 0.19, 0.16, 0.14). The weights of these reduced responses are more similar to each other.

Differences between $r_0(j)$ and $r_i(j)$ for reduced responses can be also calculated. Then, $\Delta max$ and $\Delta min$ are assessed as 0.48 and 0.00, respectively. Grey relational coefficients can be further calculated to obtain the grey relational grades of the mixtures. The best of the sixteen mixtures is identified as A1B2C2D2E2 (GRG of 0.80).

A main effects plot (or table) and ANOVA are also conducted to confirm the optimal mixture. Fig. 5 (dashed line of 0.658) illustrates the main effects plot and shows the priority of control factors for affecting the reduced responses is sequentially A (water/cement ratio), C (replacement of river sand), B (volume ratio of recycled coarse aggregate), D (content of crushed brick), and E (cleanliness of aggregate), which is the same as the full response design.

The results of ANOVA also indicate that A (water/cement ratio), B (volume ratio of recycled coarse aggregate), and C (replacement of river sand) are indeed the significant control factors for affecting the reduced responses. A (water/cement ratio) is also the most significant control factor due to its highest contribution ratio (50.208%) among the control factors.

Finally, the optimal mixture based on reduced responses is also identified as A1B1C2D2E2 by the proposed weighted Grey-Taguchi

**Table 13**
ANOVA of control factors on grey relational grades on full responses.

| Designation | Degrees of freedom | Sum of squares | Mean square | $F$ | Contribution ratio |
|---|---|---|---|---|---|
| A | 1 | 0.0461 | 0.046 | 140.64 | 50.591% |
| B | 1 | 0.0083 | 0.008 | 25.388 | 8.836% |
| A × B | 1 | 0.0005 | 0.000 | 1.502 | – |
| C | 1 | 0.0187 | 0.019 | 56.976 | 20.280% |
| A × C | 1 | 0.0017 | 0.002 | 5.258 | 1.543% |
| B × C | 1 | 0.0001 | 0.000 | 0.377 | – |
| D × E | 1 | 0.0024 | 0.002 | 7.393 | 2.316% |
| D | 1 | 0.0031 | 0.003 | 9.435 | 3.056% |
| A × D | 1 | 0.0007 | 0.001 | 2.214 | – |
| B × D | 1 | 0.0001 | 0.000 | 0.337 | – |
| C × E | 1 | 0.000 | 0.000 | 0.005 | – |
| C × D | 1 | 0.002 | 0.002 | 6.023 | 1.820% |
| B × E | 1 | 0.0014 | 0.001 | 4.249 | 1.177% |
| A × E | 1 | 0.0005 | 0.001 | 1.565 | – |
| E | 1 | 0.0028 | 0.003 | 8.656 | 2.774% |
| Pooled | 6 | 0.0020 | 0.0003 | 1.000 | – |

**Table 14**
Results of verification experiment.

| Designation | S/N ratios | | | | | | | | | | | GRG |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $y_1$ | $y_2$ | $y_{31}$ | $y_{32}$ | $y_{33}$ | $y_{41}$ | $y_{42}$ | $y_{43}$ | $y_{51}$ | $y_{52}$ | $y_{53}$ | |
| A1B1C1D1E1 | 23.747 | 30.999 | 17.612 | 17.832 | 18.004 | 68.952 | 69.212 | 68.711 | 24.897 | 27.937 | 28.088 | 0.753 |
| A1B1C1D2E1 | 24.823 | 31.922 | 16.244 | 19.498 | 19.019 | 68.761 | 70.197 | 69.536 | 25.042 | 27.104 | 27.160 | 0.816 |
| A1B2C2D2E2 | 14.114 | 27.602 | 19.121 | 18.612 | 20.072 | 68.927 | 70.356 | 69.540 | 29.114 | 30.591 | 31.173 | 0.795 |
| A1B1C2D2E2 | 23.591 | 30.874 | 17.490 | 19.256 | 19.321 | 69.307 | 70.101 | 69.245 | 29.071 | 30.515 | 31.169 | 0.899 |

method. These results demonstrate the effectiveness of the proposed weighting technique.

### 4.3. Verification experiment

A confirmation experiment has also been conducted to compare the performance among mixtures of A1B1C1D1E1, A1B1C1D2E1, A1B2C2D2E2, and A1B1C2D2E2. A1B1C1D1E1 is considered as the initial mixture. A1B1C1D2E1 and A1B2C2D2E2 are the optimal mixtures based on slump and 28-day compressive strength, respectively, proposed in a previous study [25]. Besides, A1B2C2D2E2 is also the mixture based on the largest grey relational grade, and A1B1C2D2E2 is the optimal mixture proposed in this paper. Table 14 shows the results of the verification experiment and demonstrates that A1B1C2D2E2 (GRG of 0.899) is still the best among the compared mixtures.

## 5. Conclusion

Based on the results of this study, the following five conclusions can be drawn:

1. This paper has proposed the weighted Grey-Taguchi method to assess the optimal mixture with multiple responses for recycled aggregate concrete. Water/cement ratio, volume ratio of recycled coarse aggregate, replacement by river sand, content of crushed brick, and cleanliness of aggregate were selected as control factors with responses of slump, slump-flow, resistivity (at 7, 14, and 28 days), ultrasonic pulse velocity (at 7, 14, and 28 days), and compressive strength (at 7, 14, and 28 days). Results show that the optimal mixture of recycled aggregate concrete is water cement ratio of 0.5, volume ratio of recycled coarse aggregate of 42.0%, 100% replacement of river sand, 0% crushed brick, and water-washed aggregate.
2. The algorithm of the weighted Grey-Taguchi method is described by a step-by-step procedure in this paper. The weighted Grey-Taguchi method takes the Taguchi method as its basic structure and adopts grey relational analysis to deal with multiple responses as well as using the proposed weighting technique to enhance the distinguishing ability of grey relational analysis. The proposed weighting technique can not only provide reasonable weights on responses but also detect inefficient responses to reduce time/cost of experiments.
3. In the full responses design, the priority of control factors for affecting the full eleven responses has been identified by the weighted Grey-Taguchi method as water/cement ratio (effect of 0.11), replacement of river sand (effect of 0.07), volume ratio of recycled coarse aggregate (effect of 0.05), content of crushed brick (effect of 0.03), and cleanliness of aggregate (effect of 0.03), sequentially. Besides, based on the results of ANOVA, water/cement ratio ($F = 140.639$), volume ratio of recycled coarse aggregate ($F = 25.388$), and replacement of river sand ($F = 56.976$) are further identified as the significant control factors for affecting the multiple responses. The water/cement ratio is considered as the most significant control factor due to its highest contribution ratio (50.591%) among the control factors.
4. Owing to ultrasonic pulse velocities (7-day, 14-day, 28-day) being identified as inefficient responses by the proposed weighting technique, reduced responses of slump, slump flow, resistivity (7-day, 14-day, 28-day), and compressive strength (7-day, 14-day, 28-day) are discussed in this paper and compared with the results of the full responses design. Results show that the mixture assessed from the reduced responses design is the same as that of the full responses design, and also demonstrate the effectiveness of the proposed weighting technique.
5. The practicability of the weighted Grey-Taguchi method has been demonstrated in the application of recycled aggregate concrete. This method can be easily applied to related mixture problems.

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