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A fuzzy-probabilistic durability concept for strain-hardening cement-based composites (SHCCs) exposed to chlorides
Part 1: Concept development

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A B S T R A C T

Strain-hardening cement-based composites (SHCCs) are high-performance fibre-reinforced composites characterised by their high ductility under tensile load. To utilise their advantageous properties fully, a performance-based durability concept is required. Probabilistic approaches developed for crack-free ordinary concrete provide a rational basis for this. However, the approaches currently available require extension due to a lack of data required to quantify the input variables and the need to adapt the underlying analytical formulas describing chloride ingress. These formulas need to account for material-specific conditions and resultant behaviour, such as multiple cracking in the case of SHCC.

In this first part of a two-part treatise, a fuzzy-probabilistic concept to assess the durability of SHCC exposed to chlorides is presented. The analytical solution for chloride ingress used in the DuraCrete approach has been adapted to allow for a mathematically correct description of the influence of aging and to reflect clearly the contribution of cracks to chloride ingress. Furthermore, the considerable non-stochastic uncertainty associated with most variables in a new material is accounted for with the help of fuzzy-probability theory. In the second part, the durability of a SHCC member exposed to a marine environment will be assessed using this new concept.

1. Introduction

Strain-hardening cement-based composites (SHCCs) are a group of high-performance fibre-reinforced cement-based composites (HPFRCCs) characterised by their pseudo strain-hardening tensile behaviour (cf. Fig. 1). This behaviour arises out of their ability to form multiple, finely spaced cracks of limited width. Details on the superior ductility and promising durability properties of SHCC can be found in Refs. [1–5].

To utilise these properties fully for the efficient design of durable SHCC members which attain the desired service life with the required reliability, a performance-based durability concept is required. For crack-free ordinary concrete a comprehensive framework for performance-based durability design was introduced in the DuraCrete project [7], which has since been developed further into the fib Model Code for Service Life Design [8]. In this framework, the various deleterious processes are described by simple analytical formulas. The respective input variables are stochastically quantified to account for variations observed in laboratory experiments and field investigations. This allows a probabilistic assessment of the durability of the member in question and its expected service life.

A similar concept is required for SHCC, which must account not only for cracks and the influence of a composition that differs from ordinary concrete but also for the limited availability of experimental and field data, precluding a purely stochastic variable quantification. Hence, it has to allow for the transparent inclusion of additional sources of information such as expert knowledge, subjective assessment, and the transfer of experience and knowledge available for ordinary concrete.

This work presents a performance-based durability concept for the initiation stage of chloride-induced steel reinforcement corrosion utilising fuzzy-probability theory. This corrosion process is of significant importance since structural SHCC members are expected to contain steel reinforcement [9].

2. The initiation stage of chloride-induced corrosion

2.1. Chloride ingress

Chloride ingress involves complex physical and chemical processes. This behaviour is due to the time-dependence of the

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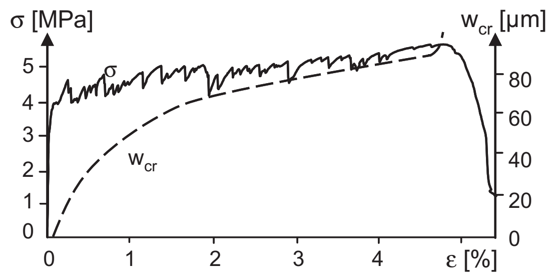

Fig. 1. Typical tensile stress–strain curve and stress-mean crack width curve for SHCC [6].

chloride exposure as well as the matrix, the pore structure, and the composition of the pore solution. Furthermore, the transport itself involves multiple physical and chemical processes, including diffusion, capillary suction and wick action, chemical and physical chloride binding, and chloride interaction with the ions of the pore solution [10].

Numerous models have been developed in recent years to describe chloride ingress into concrete. They can be grouped into (i) empirical models based on analytical or numerical solutions of Fick's Second Law, and (ii) scientific or physical models based on flux equations in which the transport and binding of chloride ions are described with separate expressions. A comprehensive review of the different concepts can be found in some recent state-of-the-art reviews [11–13]. For the concept presented in this paper, an approach was chosen based on Fick's Second Law describing chloride ingress as a diffusion process. This approach has the advantage of being analytically solvable and allowing a fuzzy-probabilistic analysis with acceptable computational effort.

2.2. The DuraCrete approach

In the DuraCrete project [7] a performance-based durability concept was developed, which describes the ingress of chloride ions with a simple analytical expression on the basis of Fick's Second Law of Diffusion. To account for the uncertainty of load and resistance, they are quantified as random variables.

This empirical solution was later modified to account for a near-surface convection zone of depth Δx, e.g. due to cyclic wetting and drying and an initial chloride content C_0 [8,14,15],

$$C(x,t) = C_0 + (C_{S,\Delta x} - C_0)\,erfc\left(\frac{x-\Delta x}{2\sqrt{D_a(t)\cdot t}}\right) \tag{1}$$

where $C(x,t)$: total chloride concentration in the concrete, $C_{S,\Delta x}$: surface chloride concentration, x: distance from the surface exposed to chlorides, *erfc* is the complement to the error function, and $D_a(t)$: the apparent chloride diffusion coefficient. It is assumed that $C_{S,\Delta x}$ is constant in time.

To assess the durability of a member, it is not sufficient merely to forecast the ingress of deleterious materials; indeed a limit state must be defined. In the case of chloride-induced steel reinforcement corrosion, it is most common to define the limit state as having been reached when the chloride concentration at depth $x = c$ of the steel reinforcement $C(x = c, t)$ exceeds the critical chloride concentration C_{crit} and steel corrosion may begin (depassivation). Other limit states are corrosion-induced cracking, spalling and collapse [8]. For the limit state depassivation, it is

$$g(x=c,t) = C_{crit} - C(x=c,t) \leqslant 0. \tag{2}$$

Applying the limit state function according to Eq. (2) as the failure criterion, the failure probability

$$p_f = p\{g(x=c,t) \leqslant 0\} \leqslant p_{f,max} \tag{3}$$

can be computed. For sufficient durability, the failure probability p_f must be smaller than an acceptable probability $p_{f,max}$ over the intended service life time of the member.

Due to its relative simplicity, flexibility, and transparency, the framework developed in the DuraCrete project is an appropriate foundation for the development of a performance-based durability concept for SHCC. The analytical formulas can be modified and variables can be requantified to account for the different behaviour of the new material.

2.3. Deterministic solution for chloride ingress in SHCC

2.3.1. The apparent diffusion coefficient $D_a(t)$

With increasing age and time of exposure, the diffusion coefficient decreases. This feature is accounted for by defining a time-dependent apparent diffusion coefficient $D_a(t)$. In accordance with [16], the time-dependence is described as

$$D_a = D_0\frac{1}{1-n}\left[\left(1+\frac{t'_{ex}}{t}\right)^{1-n} - \left(\frac{t'_{ex}}{t}\right)^{1-n}\right]\left(\frac{t'_0}{t}\right)^n \tag{4}$$

where D_0: diffusion coefficient at the reference time t'_0 according to Eq. (5), t: duration of exposure, t'_0: reference time, t'_{ex}: concrete age at first exposure, n: age factor. Following the k-factor approach introduced in the DuraCrete framework, the diffusion coefficient at the reference time is

$$D_0 = k_e k_t k_c D_{ex,0} \tag{5}$$

with k_e: temperature factor, k_t: test method factor, k_c: curing factor, and $D_{ex,0}$: experimentally determined chloride diffusion coefficient at time t'_0. Combining the k-factors of Eq. (5) and the time-dependence term of Eq. (4) in one modification variable

$$T = k_e k_t k_c\frac{1}{1-n}\left[\left(1+\frac{t'_{ex}}{t}\right)^{1-n} - \left(\frac{t'_{ex}}{t}\right)^{1-n}\right]\left(\frac{t'_0}{t}\right)^n \tag{6}$$

it is

$$D_a = TD_{ex,0}. \tag{7}$$

It should be noted that the curing factor k_c was dropped in later approaches based on the DuraCrete framework [8,14,15] without elaboration. Due to the importance of curing to concrete quality and in turn to its resistance to chloride ingress, k_c has been included in Eq. (5).

In contrast to the DuraCrete solution, Eq. (4) differentiates between the actual momentary diffusion coefficients $D(t')$ at a point in time t' and the apparent diffusion coefficients $D_a(t)$ for longer exposure periods t, which are determined by integrating $D(t')$ over the exposure time t. The underlying mathematics are described in detail in [16] with the notable difference that the authors use a different definition for the variable T. If used in the knowledge of the underlying assumptions and definitions, both approaches of considering the time-dependence of the diffusion coefficient yield correct results. Apart from the mathematical rigour, the main advantage of Eq. (4) is that the influence of the age at first exposure t'_{ex}, which can be significant for early-age exposure, is considered.

2.3.2. Consideration of cracks

The formulas above were derived for chloride ingress in crack-free concrete. However, multiple fine cracks will in most cases be encountered under service load when using SHCC. To account for their influence while clearly distinguishing between the contribution of the matrix (index *cf*: crack-free) and that of cracks (index *cr*) to chloride ingress, and in line with the smeared steady-state diffusion coefficient proposed in [17], it is reasonable to model the non-steady-state diffusion coefficient as

$$D = D_{cf} + k_{cr} D_{cr} \tag{8}$$

with k_{cr}: the crack intensity factor. Following [17], D_{cr} may be assumed to be identical to the diffusion coefficient in the bulk fluid D_{bulk}. In this case k_{cr} accounts for the difference between chloride diffusivity in the bulk fluid and in the crack (cf. e.g. [18]).

Alternatively it may be determined experimentally for reference conditions. In their definition, the critical variables which determine crack status must be identified. While it is widely accepted that the average crack width is not critical, there is no general consensus at this stage as to how to describe the crack status. Options put forward include maximum crack width; crack density, defined as the number of cracks per unit length; and a stochastic description of the crack and crack size distribution. If D_{cr} is determined for reference conditions, k_{cr} accounts for the influence of differences between the reference conditions and the actual crack pattern in the member.

Combining Eqs. (7) and (8) it is

$$D_a = T_{cf} D_{ex,0,cf} + T_{cr} D_{ex,0,cr} \tag{9}$$

with $D_{ex,0,cf} + D_{ex,0,cr} = D_{ex,0}$ and T_{cf} and T_{cr} according to Eqs. (10) and (11)

$$T_{cf} = k_e k_t k_c \frac{1}{1-n_{cf}} \left[\left(1 + \frac{t'_{ex}}{t}\right)^{1-n_{cf}} - \left(\frac{t'_{ex}}{t}\right)^{1-n_{cf}} \right] \left(\frac{t'_0}{t}\right)^{n_{cf}} \tag{10}$$

$$T_{cr} = k_e k_{t,cr} k_{cr} \frac{1}{1-n_{cr}} \left[\left(1 + \frac{t'_{ex}}{t}\right)^{1-n_{cr}} - \left(\frac{t'_{ex}}{t}\right)^{1-n_{cr}} \right] \left(\frac{t'_0}{t}\right)^{n_{cr}}. \tag{11}$$

2.4. Quantification of input variables

Eq. (1) is an empirical solution for the description of chloride ingress. Thus, many variables do not have a clear physical or chemical meaning and require a large database for their specification using probability theory. However, only few experimental results are available for chloride ingress and chloride-induced rebar corrosion in SHCC, either in the literature or from the authors' own investigations. Furthermore, due to an information deficit, the interpretation of the results published in the literature is often not straightforward. As examples, the curing conditions and the exact experimental setup are often not known. But even if they were reported, their influence on the variable in question cannot always be quantified based on the information available in the literature.

The chloride diffusion coefficient, for instance, may be determined using various direct and indirect test methods. These include diffusion, migration, and resistivity as well as pressure penetration, permeability, and sorptivity tests. Some of these methods have a "nebulous relationship with what actually occurs in the concrete" [19], which makes the quantification of the diffusion coefficient based on such results challenging. But even the assessment of results from different diffusion tests such as the AASHTO T259-80 salt ponding test [20] and the NT BUILD 443 bulk diffusion test [21] is not without difficulty. While in the former method additional transport mechanisms such as sorption, vapour transport, and wick action influence the result, their relative influence on the test result does not necessarily reflect their impact in the field. In contrast to that, these additional transport processes observed in the field play no role in the latter, the bulk diffusion test [19].

This situation is further complicated by the variables' not always being mutually independent of each other. In Eq. (4), the time-dependence of the diffusion coefficient is expressed with the help of the age factor n. Without knowing n, it is not possible to adjust experimentally determined diffusion coefficients $D(t' \neq t'_0)$ for the influence of time to determine $D_{ex,0}$ (cf. Eq. (5), see also Section 2.4 in Part 2).

Further information for use in the quantification of variables includes expert knowledge and subjective assessment, for example, based on analogies to chloride-induced corrosion in ordinary concrete. In the absence of any investigations, for example, into the critical chloride concentration C_{crit} at which steel corrosion may begin in SHCC, these are the only available sources to quantify this particular variable (cf. Section 2.5 in Part 2 of this paper).

3. Coping with uncertainty in durability design

3.1. Modelling uncertainty in durability design

3.1.1. Data uncertainty

The uncertainty associated with input variables must be considered in the design to quantify the durability of SHCC members. This uncertainty stems from the inherently non-deterministic nature of SHCC and environmental loads as well as from a lack of information. The former is referred to as *aleatory uncertainty* or *randomness*. It is irreducible and objectively assessable. In contrast, information deficit results in *epistemic uncertainty* or *imprecision* [22].

Epistemic uncertainty may be reduced, for instance, by collecting more data, through an improved understanding of the physical and chemical processes at hand and by strict quality control, but it cannot be obviated. This is particularly true for properties relevant to the durability of new materials like SHCC, where field data for long-term behaviour is unavailable and long-term behaviour can at best be assessed based on a typically small number of rapidly conceived and executed laboratory experiments.

Thus, all the variables used in the deterministic solution to describe chloride ingress in SHCC introduced in Section 2 exhibit both aleatory and epistemic uncertainty. This uncertainty may be described using stochastics, fuzzy set theory, and a combination of the two models – fuzzy–stochastics.

Traditionally, both randomness and information deficit are described using *stochastics*. Thereby, epistemic uncertainty is accounted for through conservative assumptions for the probabilistic parameters of the variables in question, such as mean value or standard deviation. However, while very well suited to describe randomness, stochastics have some shortcomings limiting their usefulness in describing the information deficit. Firstly, some restrictions apply to probability density functions $f(\cdot)$. For instance, the sum of the probability of an event and its complementary event must always be unity. This means that if for the random variable x holds that the probability $p\{x \leqslant 2\}$ is equal to 0.30, the probability of exceedance $p\{x > 2\}$ must be 0.70. Secondly, if both types of uncertainty are described using the same model, it is no longer possible to differentiate between randomness and information deficit. It is not obvious from a high probability of failure p_f, calculated based on a random input variable quantification, if this result is due to the unsuitability of the material or to a lack of information about the material's performance. Thus, a design solution might be discarded even though it would have yielded a superior result if efforts had been made to reduce the information deficit.

For instance, in the DuraCrete Report R9 [23], the age factor n is tabulated for different binder types. For concrete containing fly ash or slag, the tabulated stochastic quantification is based on expert judgement rather than field data. Thus, imprecision is quantified using stochastics. This is clearly shown in the table. However, in the Final Technical Report R17 [24], the mean values tabulated in

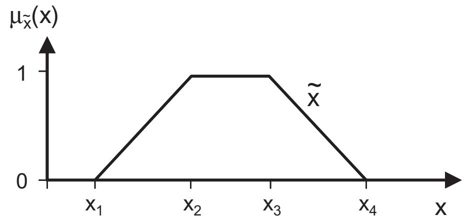

Fig. 2. Trapezoidal membership function $\mu_{\tilde{x}}(x)$ of the variable $\tilde{x} = \langle x_1, x_2, x_3, x_4 \rangle$, with the tilde denoting the fuzziness of the variable.

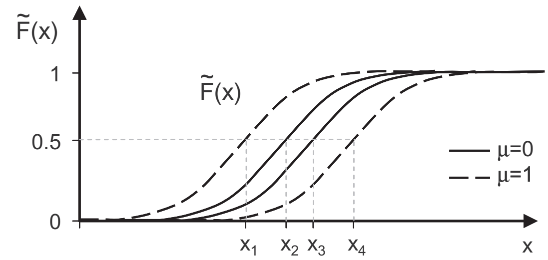

Fig. 4. Fuzzy probability distribution function $\tilde{F}(x)$ for a normally distributed variable $\tilde{x}$ with a fuzzy mean value $\tilde{m}$ as defined by the fuzzy variable $\tilde{x}$ in Fig. 2.

[23] are given as characteristic values for a semi-probabilistic design approach without any clarification as to which values are based solely on expert opinion. Thus, information is lost, and most users will assume that all stochastic quantifications are derived from extensive databases.

Fuzzy set theory, on the other hand, is particularly well-suited to describe epistemic uncertainty. It allows a gradual assessment of the possibility that a variable attains a certain value. This possibility is quantified using a membership function $\mu(\cdot)$, with values between zero if there is no possibility that the variable attains a value and one in the case of maximum possibility. No further constraints exist for the shape of membership functions. It must be noted that the sum of the possibility of an event and its complementary event does not need to be unity. Since these functions represent a subjective assessment, there is no generally applicable method specifying their derivation. However, the limited availability of data usually does not justify elaborate membership functions. In these cases, simple trapezoidal or triangular functions (cf. Fig. 2) offer enough flexibility and precision to reflect the information available.

Furthermore, fuzzy set theory allows the use of linguistic parameters such as *very low*, *low*, *medium*, *high*, and *very high* to describe a variable. This approach is particularly useful for variables which are not fully quantified, e.g., due to equipment not being readily available. One such variable is the crack status. In most cases, only one or more of the indicators maximum crack width, average crack width, and number of cracks over the measurement length are given. However, a full quantification would also include information about crack width distribution and crack spacing distribution. All of these indicators may be quantified, albeit with some uncertainty, using photogrammetry (see, e.g., [25]). Where this technology is not available, a linguistic assessment as shown in Fig. 3 offers a suitable alternative.

The apparent vagueness of fuzzy quantifications must not be misunderstood as a deficiency; instead, it is a strength of fuzzy set theory, which affords the quantification of imprecision the required flexibility and transparency. While the sources of and thoughts behind stochastic quantifications based on expert assessment are usually unclear, each characteristic point defining a membership function (e.g., x_1 to x_4 in Fig. 2) can be transparently related to sources of information, for example, an experimental result or a deduction based on first principles.

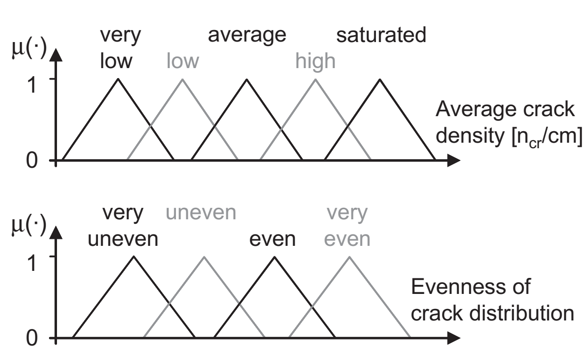

Fig. 3. Linguistic assessment of the crack status to supplement measured values such as the maximum crack width $w_{cr,max}$.

As noted above, aleatory and epistemic uncertainties exist simultaneously. Hence, it is advantageous to describe the two phenomena in a *fuzzy-random* quantification of input variables [22,26]. Data and expert assessment can be accounted for transparently through a fuzzy quantification of the parameters of a variable's probability function. This yields fuzzy probability density functions $\tilde{f}(x)$ and fuzzy probability distribution functions $\tilde{F}(x)$ (cf. Fig. 4). The fuzzy parameters of the probability density function are called fuzzy bunch parameters $\tilde{s}_i$ and form the vector $\underline{\tilde{s}} = [\tilde{s}_1, \tilde{s}_2, \ldots]^T$. The fuzzy probability density function $\tilde{f}(x)$ can thus be expressed as $f(\underline{\tilde{s}}, x)$.

This approach combines the usefulness of stochastics for quantifying randomness with the flexibility of fuzzy set theory to transparently describe input variables. It also allows the clear separation of the influence of imprecision on the results of durability design calculations from the impact of randomness.

If, for instance, only two experimentally determined values for the chloride diffusion coefficient D of a concrete mix are available, the associated uncertainty may be quantified transparently using fuzzy randomness. Based on the fib Model Code [8], the diffusion coefficient may be modelled as being normally distributed with a coefficient of variation that is not influenced by the mix design. The mean value of the distribution may be quantified as a fuzzy parameter with a trapezoidal membership function (cf. Fig. 2). It is known that D increases with an increasing water-to-binder ratio. Thus, if the mean value of D for a similar mix with a lower water-to-binder ratio is known, this value may be attributed to x_1. The actual mean value most possibly lies somewhere between the two experimental values, which means, they define x_2 and x_3. Since the diffusion coefficient of chloride in concrete is less than in the bulk pore fluid, x_4 may be defined by the bulk fluid diffusion coefficient. The corresponding fuzzy probability distribution function is shown in Fig. 4 [6].

Still further, corrosion processes and associated variables such as the chloride diffusion coefficient D are time-dependent. This time-dependence may be modelled by describing $\tilde{x}(t)$ as a fuzzy-random process [27]. This process is simply a family of fuzzy random variables $\tilde{x}(t_i)$ for each point in time t_i, which can be written in vector-form as $\underline{\tilde{x}} = [\tilde{x}(t_1), \tilde{x}(t_2), \ldots]^T$. Thus, the time-dependent fuzzy probability density function of $\tilde{x}(t)$ can be expressed as the joint probability density function of the family of variables $\tilde{x}(t_i)$ denoted as $\tilde{f}_t(\underline{x}) = f_t(\underline{\tilde{s}}, \underline{x})$.

3.1.2. Model uncertainty

Empirical analytical solutions, such as the one used in the DuraCrete project for chloride ingress, are mathematical fits to observations and have no clear physical meaning. Due to this so-called model uncertainty, predictions about future behaviour beyond

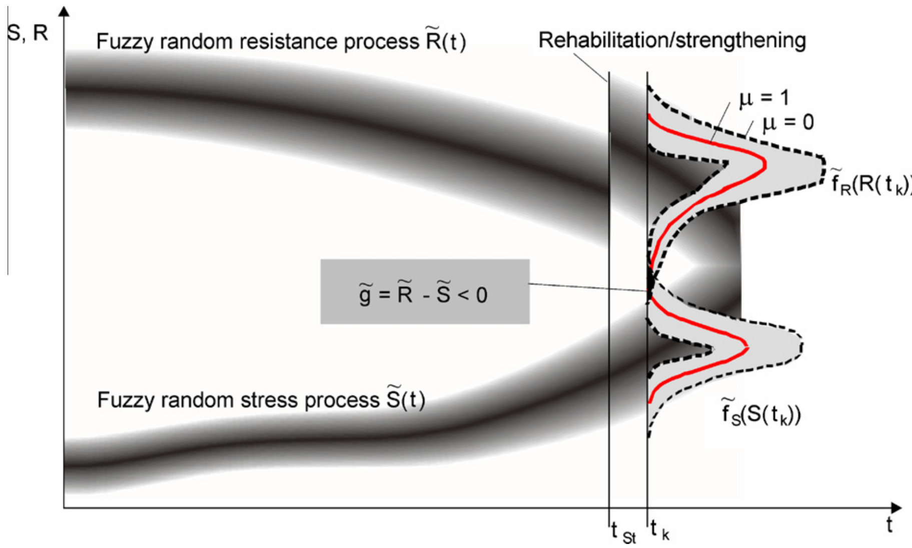

Fig. 5. Time-dependent fuzzy-probabilistic environmental loads $\widetilde{S}(t)$, resistance $\widetilde{R}(t)$ and limit state $\tilde{g}(t) < 0$ [27].

the timeframe for which data are available must be interpreted with caution [12]. Here the uncertain input variables of the model are determined by parameter identification using the inverse of the same analytical solution. Consequently the model uncertainty is included in the uncertainty of the input variables. If uncertain variables are appropriately defined, the model uncertainty is reflected in the results of a fuzzy random analysis. Thus, the approach outlined below can predict valuable information, albeit with significant non-stochastic uncertainty, on chloride ingress in SHCC for future situations.

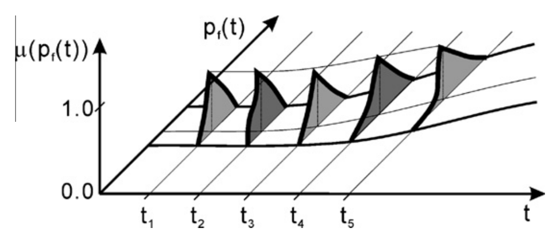

Fig. 6. Time-dependent fuzzy probability of failure $\tilde{p}_f(t)$.

3.2. Uncertainty in the performance-based design concept

Limit-state based failure occurs if the load $\widetilde{S}$ is equal to or larger than the resistance $\widetilde{R}$. The failure event is assessed by probability. Due to imprecision, this results in *fuzzy failure probability*

$$\tilde{p}_f = p\{\tilde{g} = \widetilde{R} - \widetilde{S} \leqslant 0\}. \tag{12}$$

For time-dependent processes such as chloride ingress, the fuzzy input variables $\tilde{x}_i$ of the performance-based design concept and load, resistance, and failure probability are in turn time-dependent (cf. Fig. 5).

Modelling time-dependence as a fuzzy random process as introduced in Section 3.1.1, the uncertainty of the input variables may be denoted as $\tilde{f}_t(\underline{x}) = f_t(\underline{\tilde{s}}, \underline{x})$ with $\underline{x} = [x_1(t_1), x_1(t_2), \ldots, x_2(t_1), x_2(t_2), \ldots]^T$ and $\underline{\tilde{s}} = [\tilde{s}_{x1,1}, \tilde{s}_{x1,2}, \ldots, \tilde{s}_{x2,1}, \tilde{s}_{x2,2}, \ldots]^T$. The fuzzy bunch parameters $\underline{\tilde{s}}$ describe the imprecision of the input variables $\tilde{x}_i$. The probability distribution function of $\tilde{g}(\underline{x}) = g(\underline{\tilde{s}}_g, \underline{x})$ with the fuzzy bunch parameters $\underline{\tilde{s}}_g = [\tilde{s}_{g,1}, \tilde{s}_{g,2}, \ldots]^T$ is unknown. However, the failure probability $\tilde{p}_f(t_i)$ at a given time t_i

$$\tilde{p}_f(t_i) = \left\{ p_f(t_i) = \int_{x|g(\underline{s}_g)\leqslant 0} f_t(\underline{s}, \underline{x}) dx \forall \underline{s} \in \underline{\tilde{s}} \wedge \underline{s}_g \in \underline{\tilde{s}}_g \right\} \tag{13}$$

may be determined based on the joint probability density function $f_t(\underline{\tilde{s}}, \underline{x})$ of the input variables (cf. Fig. 6).

3.3. Numerical solution

3.3.1. Fuzzy stochastic sampling

To determine the fuzzy failure probability according to Eq. (13) efficiently, fuzzy stochastic sampling [27] is used. This three-loop algorithm consists of a fuzzy, a stochastic, and a deterministic analysis. It is independent of the applied deterministic analysis. Here, the time-dependent chloride concentration *C(x,t)* is computed and related to the critical chloride content according to Eq. (2). To account for the time-dependence of chloride-induced corrosion and allow the calculation of $\tilde{p}_f(t)$, a time-loop is added (cf. Fig. 7).

Before the failure probability can be calculated, the uncertain input variables have to be quantified. Depending on the source of uncertainty and the available information, fuzzy, random, and fuzzy-random variables are specified by membership functions $\mu(x)$, distribution density functions $f(x)$, and fuzzy distribution density

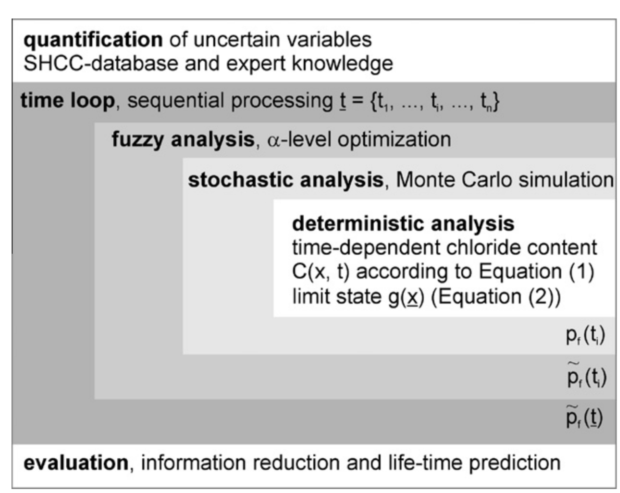

Fig. 7. Schematic representation of the numerical algorithm of the performance-based durability design concept.

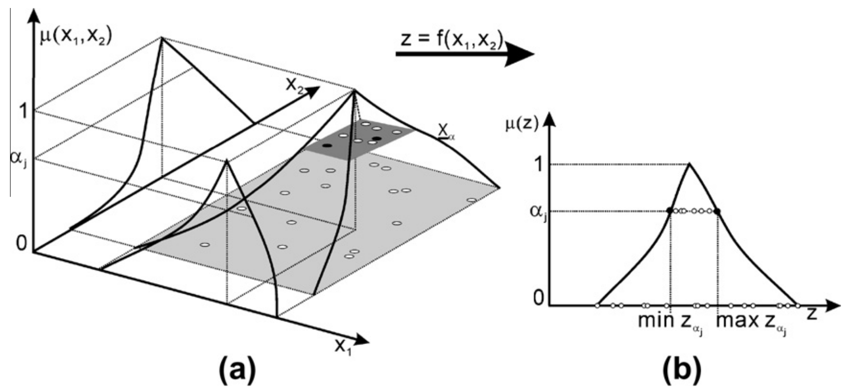

Fig. 8. (a) Schematic representation of the combined space $\underline{\mathbf{X}}$ (the x_1x_2-plane) for the fuzzy input variables $\tilde{\underline{x}} = [\tilde{x}_1, \tilde{x}_2]^T$. Also shown are the membership function $\mu(\underline{x})$ for each combination of $\tilde{x}_1$ and $\tilde{x}_2$ and the subsets $\underline{X}$ of $\underline{\mathbf{X}}$ for the α-levels $\alpha = 0$ (light grey) and α_j (dark grey). (b) Fuzzy result variable $\tilde{z}$ obtained with the aid of the deterministic analysis $z = f(x_1, x_2)$.

function $\tilde{f}(x)$, respectively. In a next step, the time scale is discretised in a set $\{t_1,\ldots,t_i,\ldots,t_n\}$. For every time t_i, the failure probability $\tilde{p}_f(t_i)$ is then computed by a fuzzy analysis. The principle of this analysis is described below. More detailed information can be found in [22,27,28].

3.3.2. Fuzzy analysis

The analysis with certain (or uncertain) algorithms and with fuzzy quantities as input and model parameters is referred to as fuzzy analysis. Its first step is the formation of a combined space $\underline{\mathbf{X}}$ comprising all input variables $\tilde{x}_i$; in Fig. 8 the combined space $\underline{\mathbf{X}}$ for the two variables $\tilde{x}_1$ and $\tilde{x}_2$ is formed by the x_1x_2-plane. The membership value $\mu(\underline{x})$ for each combination of realisations of fuzzy input variables $\underline{x} = [x_1,\ldots,x_i,\ldots,x_n]^T$, denoted by white dots in Fig. 8a, is determined using the minimum operator. This means that for each $\underline{x}$ the membership value is equivalent to the smallest membership value of all input variables $\tilde{x}_i$: $\mu(\underline{x}) = min\{\mu(x_i)|i = 1,\ldots,n\}$.

The results of fuzzy analyses are also fuzzy quantities $\tilde{z}_j$ (cf. Fig. 8b) forming the vector $\tilde{\underline{z}} = [\tilde{z}_1, \tilde{z}_2, \ldots]^T$. In the design concept presented in this work, these results are the fuzzy chloride concentration $\widetilde{C}(x,t_i)$ and the fuzzy failure probability $\tilde{p}_f(t_i)$.

Therefore, the analysis is a two-step process. Firstly, all input variables $\tilde{\underline{x}} = [\tilde{x}_1(t_i), \tilde{x}_2(t_i), \ldots]^T$ must be transformed into the space of fuzzy result variables $\underline{z} = p_f(t_i)$. This transformation from $\underline{x}$ to $\underline{z}$ is realised with the aid of a certain analysis $\underline{z} = f(\underline{x})$. Secondly, the membership functions $\mu(\underline{z})$ for all elements of $\tilde{\underline{z}}$ have to be determined. For each element of $\tilde{\underline{z}}$, the value of the membership function is determined by the value of the membership function of the corresponding input variables. If more than one combination of realisations of input variables $\underline{x}$ yields the same result z_i, its membership value $\mu(z_i)$ is defined by the maximum membership value $\mu(\underline{x})$ of these input variable combinations.

Due to the infinite number of possible combinations defined by the combined space $\underline{\mathbf{X}}$, the fuzzy analysis must be solved numerically. In the approach presented in this paper, the analysis is performed with the help of the so-called *α-level-optimisation*. Using this strategy, which is explained in detail in [29], $\mu(\underline{z})$ is not determined for all values between 0 and 1, but only for a number of discrete α-levels $\mu(\underline{z}) = \alpha_j$. This is called α-discretisation. As can be seen in Fig. 8, each *α-level* α_j is associated with a subset $\underline{X}_{\alpha_j}$ of the input variables $\underline{x}$. For each α_j, the minimum and the maximum result variables $min\ z_{\alpha_j}$ and $max\ z_{\alpha_j}$, which define $\mu(\underline{z})$ for this α-level, must be computed. Especially for non-linear functions, the input variable combinations yielding $min\ z_{\alpha_j}$ and $max\ z_{\alpha_j}$ are not necessarily on the contour line or in a corner of $\underline{X}_{\alpha_j}$ (cf. Fig. 8a). Thus, the whole subset of possible input variable combinations $\underline{X}_{\alpha_j}$ for the α-level in question must be investigated to determine $min\ z_{\alpha_j}$ and $max\ z_{\alpha_j}$.

In the approach presented here, the number of combinations which require investigation to find $min\ z_{\alpha_j}$ and $max\ z_{\alpha_j}$ is reduced with the help of a modified evolution strategy [27]. In a two-dimensional space, the computation of $min\ z_{\alpha_j}$ may be described as follows: Beginning with an arbitrary combination $\{x_1, x_2\}^1$ belonging to the subset $\underline{X}_{\alpha_j}$, the result $z = f(x_1,x_2)$ is calculated. Then, the evolution strategy is used to search for a new realisation $\{x_1, x_2\}^2$ within a specified maximum distance from $\{x_1, x_2\}^1$, which results in a smaller result z_{α_j}. This process is repeated until no further improvement can be reached for a specified number of iterations. Realisations $\{x_1, x_2\}^q$ and corresponding results z^q for $\alpha = 0$ and α_j are denoted in Fig. 8 by white dots. Subsequently, the process is repeated to determine $max\ z_{\alpha_j}$. In Fig. 8, the realisations resulting in $min\ z_{\alpha_j}$ and $max\ z_{\alpha_j}$ for α_j are denoted by black dots.

In the case of fuzzy random input variables, a stochastic analysis according to Section 3.3.3 must be carried out for each realisation $\{x_1, x_2\}^q$ investigated during the α-level-optimisation. With the assessment of the limit state of chloride-induced corrosion of the reinforcement according to Eq. (2), it is thus possible to determine the failure probability $p_{f,\alpha_j}(t_i) = z_{\alpha_j}$.

3.3.3. Stochastic analysis

The stochastic analysis can be performed with a state-of-the-art algorithm suitable for the computation of failure probability. In the approach presented in this paper, a Monte-Carlo simulation is used to obtain an estimate for the failure probability $p_{f,\alpha_j}(t_i)$ for the random variables obtained for each combination of realisations of fuzzy input variables of the α-level-optimisation. To achieve this, an indicator function

$$I(g(c,t_i)) = \begin{cases} 1 & \text{if } g(c,t_i) \leqslant 0 \\ 0 & \text{if } g(c,t_i) > 0 \end{cases} \tag{14}$$

is introduced. According to Eq. (12), the failure probability is equal to the probability that $g(c,t_i) \leqslant 0$. Thus, $p_f(t_i)$ can be estimated based on N simulations as

$$\hat{p}_f(t_i) = \frac{1}{N}\sum_{i=1}^{N} I(g(c,t_i)). \tag{15}$$

The mean value and quantiles of interest of the fuzzy chloride profile $\widetilde{C}(x,t_i)$ may be estimated analogously using the Monte-Carlo method.

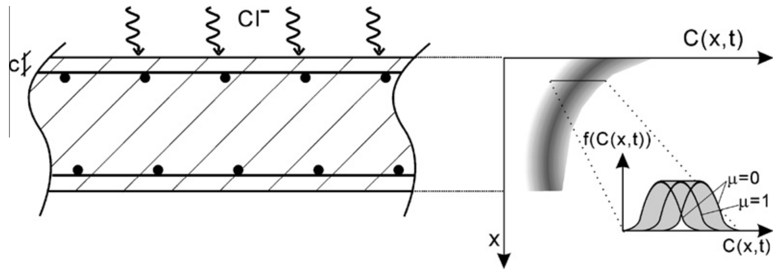

Fig. 9. Fuzzy-random profile of chloride concentration $\widetilde{C}(x,t)$ for a steel-reinforced SHCC member exposed to chlorides.

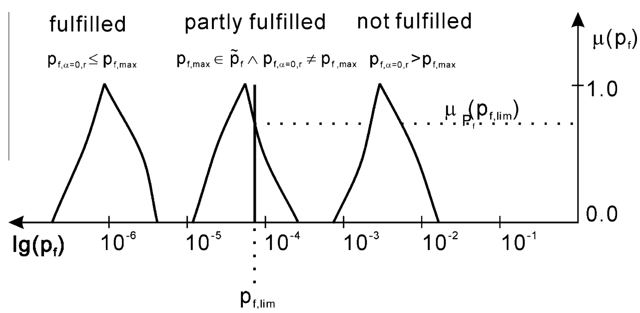

Fig. 10. Possible cases for reliability evaluation with a fuzzy failure probability.

3.4. Interpretation of results

With the use of fuzzy stochastic analysis, the fuzzy-random chloride concentration $\widetilde{C}(x,t)$ according to Eq. (1) may be computed at any given point in time t_i. The resulting chloride profile is described with a fuzzy probability density function $\tilde{f}(C(x,t))$ (cf. Fig. 9).

At the depth of steel reinforcement $x = c$, the uncertain chloride concentration can be used to assess the limit state function according to Eq. (2), resulting in a time-dependent fuzzy failure probability $\tilde{p}_f(t)$ (cf. Eq. (3) and Fig. 6).

To determine the durability and service life of the member in question, the failure probability $\tilde{p}_f(t)$ must be compared with a specified maximum failure probability $p_{f,max}$. Values for $p_{f,max}$ may be obtained from codified requirements for ordinary concrete structures or determined specifically for SHCC members. Traditionally, these values are not fuzzy but certain.

The fuzzy result variables may be assessed directly against the limit value $p_{f,max}$ or they can first be defuzzified. If assessed directly, sufficient member durability is proven if it is

$$\tilde{p}_f(t_i) \leqslant p_{f,\max} \tag{16}$$

for all points in time $t_i \leqslant t_{life}$, with t_{life}: the intended service life of the member. Conversely, the service life is reached for the first point in time for which the Inequality (13) does not hold.

Inequality (13) can lead to the three different cases shown in Fig. 10. It is certainly fulfilled, if the largest value of $\tilde{p}_f(t_i)$ is smaller than $p_{f,max}$, and it is certainly not fulfilled, if the smallest value of $\tilde{p}_f(t_i)$ is greater than $p_{f,max}$. However, it is also possible that the condition is only partially fulfilled. In this case, a subjective decision is required, which may, for example, be based on the membership value $\mu_{\tilde{p}_f}(p_f)$ for $p_f = p_{f,max}$. Assessed conservatively, partial fulfilment leads to a rejection of the proof of durability.

If result variables are first defuzzified, information reduction methods are used to obtain a certain value for the failure probability $\tilde{p}_f(t_i)$. Typical methods include distribution methods where, for instance, the centre of gravity of the area under the membership function $\mu(z)$ may be used, and maxima methods for which a result z with the highest membership value $\mu(z)$ is used. If more than one maximum exists, for example, the smallest, the largest, or a random maximum value is chosen, depending on the method. Further details on these and other methods can be found in [30,31]. The advantage of this approach is that Inequality (13) is transformed into $p_f(t_i) \leqslant p_{f,max}$, one with certain instead of fuzzy parameters. However, the visibility of the epistemic uncertainty in the results, which is the main benefit of the fuzzy stochastic approach, is lost.

4. Sensitivity analysis

The relative influence of a specific uncertain input variable on the result of a fuzzy–stochastic analysis (FSA) according to Section 3.3 depends on three factors. Firstly, it depends on the value of the other input variables. Secondly, it depends on how much the result changes if the input variable changes, the so-called significance. Finally, its influence depends on the uncertainty of the input variable.

For a given combination of input variables, a variable's significance and contribution to the uncertainty of the result may be determined with the help of a sensitivity analysis (SA).

The results of a SA allow the targeting of future research efforts to reduce the imprecision of FSA results efficiently. Furthermore, the computational effort required for the FSA is influenced decidedly by the number of uncertain input variables. Therefore, the concentration on significant uncertain variables identified with the SA may reduce this effort. Finally, information obtained from a SA may be used to optimise the design of R/SHCC members efficiently.

While different approaches have been developed to analyse sensitivity (see, for example, [32]), sampling-based methods have the advantage that they may be carried out in conjunction with the fuzzy–stochastic uncertainty analysis (FSA) according to Section 3.3. However, they are usually more demanding than the FSA itself, which is why in a first step the influence of individual input variables on the uncertain result is estimated by determining the coefficients of correlation between the input variables $\tilde{x}_i$ and the chloride concentration at the depth of the reinforcement $\widetilde{C}(c,t)$. This coefficient describes on a scale from −1 to 1 the degree of linear correlation between the variables in question. For complete linear correlation, it attains an absolute value of 1, while it becomes zero for no linear correlation.

For N results of the Monte Carlo analysis, the empirical coefficient of correlation for the variables x_1 and x_2 is defined as

$$r_{x_1 x_2} = \frac{\sum_{i=1}^{N}(x_{1,i} - \bar{x}_1)(x_{2,i} - \bar{x}_2)}{(N-1)s_{x_1}s_{x_2}} \tag{17}$$

with $\bar{x}_j$: the mean value of the samples for x_j and s_{xj}: the sample standard deviation for x_j.

The failure probability $\tilde{p}_f$ reduces the stochastic uncertainty to one certain value. In contrast, this uncertainty is expressed with the help of a probability density function for the chloride concentration $\widetilde{C}(c,t)$. Thus, the influence of both stochastic and non-stochastic uncertainty can be determined with the coefficients of correlation above. However, the influence of the critical chloride content cannot be investigated in this manner since it is only introduced in the limit state function $\tilde{g}(x,t)$ (cf. Eq. (2)).

For fuzzy-random variables, the coefficient of correlation is fuzzy and its interpretation may not be straightforward. In such cases it may be helpful to assume the fuzzy variables to be equally distributed over (parts of) the membership function, yielding certain coefficients of correlation. However, in this way the information on the influence of non-stochastic uncertainty is lost. Furthermore, in case of an equal parameter distribution over the whole membership function $\mu(\cdot)$, the extreme values of $\mu(\cdot)$ are given undue weight.

If these limitations are considered in the interpretations of the coefficients of correlation, the results yield a good first result for the sensitivity of chloride ingress to individual input variables and their uncertainty.

5. Summary

A performance-based durability concept for SHCC exposed to chlorides has been presented here. Based on the DuraCrete approach, this concept may also serve as a template for durability concepts for other degradation phenomena and other new materials such as ultra-high performance concrete or textile-reinforced concrete. Similar to SHCC, the latter is characterised by pronounced strain-hardening and multiple cracking with relatively small crack widths under tensile load, and has been shown to have promising durability properties [33].

Using fuzzy probability theory, the concept allows a transparent quantification of variables and both their random and fuzzy uncertainties. In contrast to other performance-based concepts, no extensive database is required to quantify input variables. Instead, the limited available data is supplemented with expert knowledge and subjective assessment.

The calculations are carried out using fuzzy–stochastic sampling, a three-loop algorithm consisting of fuzzy, stochastic, and deterministic analyses. To this, a time-loop is added to account for the time-dependence of chloride corrosion.

Apart from uncertain chloride profiles, it is possible to determine the development of the chloride concentration over time at the reinforcement depth. With the definition of a limit state – in this case, the onset of steel reinforcement corrosion – and a maximum probability of exceeding that limit state, the fuzzy time-dependent failure probability, and hence the durability of a member can also be determined.

Using fuzzy set theory, the computed results transparently reflect the impact of imprecision due to a lack of available information. Thus, it is possible to differentiate between, for instance, a high probability of failure due to imprecision and one due to the unsuitability of material or geometry for the exposure conditions in question.

In the second part of this work, the concept is applied to an SHCC member exposed to seawater. It is shown that at present the non-stochastic uncertainty of the input variables and in turn that of the results are considerable. Nonetheless, the potential of the approach as presented is evident: With the help of sensitivity analyses, it is possible to identify those variables crucial to the imprecision of results. This approach will allow targeted research to improve the database and reduce the imprecision of these variables.

It is envisaged that further research efforts to improve the database in light of the results of a more rigorous sensitivity analysis will allow the development of the concept into a design tool for the durability of SHCC members. This goal will also require improvements of the deterministic analysis, for example, to describe more precisely the influence of cracks.

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