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Improvement of the biaxial flexure test method for concrete

Jihwan Kim [a], Dong Joo Kim [b], Goangseup Zi [a,*]

[a] School of Civil, Environmental and Architectural Engineering, Korea University, 1, 5-ga, Anam-dong, Seongbuk-gu, Seoul 136-713, Republic of Korea

[b] Department of Civil and Environmental Engineering, Sejong University, 98 Gunja-dong, Gwangjin-gu, Seoul 143-747, Republic of Korea

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ABSTRACT

The biaxial flexure test (BFT) method for the biaxial tensile strength of concrete has been improved. To ensure the equi-biaxial stress condition during the test, the geometry of the specimen, proper treatment of the specimen surface, and the support conditions were proposed. The biaxial flexural strength of concrete was measured with the improved biaxial flexure test method. The coefficient of variation (c.o.v) of the biaxial flexure test method is comparable to that of the four-point bending test and ASTM C 1550. The improved biaxial flexure test provides a practical and reliable means of determining the biaxial flexural strength of concrete. The biaxial tensile strength given by the biaxial flexure test method and the ASTM C 1550 method is greater than the uniaxial tensile strength using the four-point bending test method.

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1. Introduction

Thin plate structures such as pavement, slabs, and roofs are subjected to multi-axial stress states due to complex loading configurations and the geometry of plate structures. In estimating the safety of plate structures in the non-uniaxial stress state, it is important to identify the correct multi-axial properties of the materials used. However, studies on the multi-axial behavior of concrete are scarce due to the need for multi-actuator control systems [1], which is very expensive and difficult to control. Thus, the simple biaxial flexure test (BFT) method has been developed recently to investigate the biaxial properties of concrete without using the complex test setups [2]. The biaxial flexure test utilizes a round-shaped specimen supported by an outer support ring, and loaded by an inner loading ring with a diameter smaller than the support ring. The state of equi-biaxial stress is generated within the loading ring; thus, this test method could be successfully applied to investigate the biaxial tensile strength of concrete.

With the aid of the biaxial flexure test, the biaxial fracture characteristics of concrete and the factors influencing its biaxial tensile strength were investigated by Zi et al. [2]. However, the state of equi-biaxial tensile stress was not confirmed in their work. In addition, there was no discussion about the geometry of the concrete test specimen, the proper treatment of the specimen surface, and the support conditions.

The aim of this research is to provide an improved biaxial flexure test to evaluate the biaxial tensile strength of concrete. The geometry of the test specimen for concrete was determined based on both numerical and experimental studies. The proper treatment of the specimen surface and the support conditions were proposed to ensure the equi-biaxial stress condition during the test. With the improved biaxial flexure test, the biaxial tensile strain of concrete was measured to confirm the state of equi-biaxial tensile stress in two different directions. The biaxial tensile strength of concrete obtained from the improved biaxial flexure test and ASTM C 1550 [3], another test method for the biaxial tensile strength, was compared with the uniaxial tensile strength of concrete measured from a four-point bending test.

2. Various test methods for biaxial tensile strength

Various biaxial tensile strength test methods are available to investigate the biaxial properties of ceramics, such as piston-on-three-balls test, ball-on-three-balls test, ring-on-ring test, and ball-on-ring test. In these test methods, a disk-shaped specimen is used, as schematically described in Fig. 1.

The piston-on-three-balls test method was standardized in ASTM F 394 [4] and ISO 6872 [5]. ASTM F 394 was first issued in 1974 and withdrawn in 2001, while ISO 6872, first issued in 1984, is still active [6–9]. The biaxial tensile strength was determined by using a disk-shaped specimen, which was supported on three balls and loaded through a flat piston, as shown in Fig. 1a. The stress distribution under the piston is uncertain [10], even though the piston-on-three-balls test has an advantage in the

* Corresponding author. Tel.: +82 2 3290 3324; fax: +82 2 928 7656.
E-mail address: g-zi@korea.ac.kr (G. Zi).

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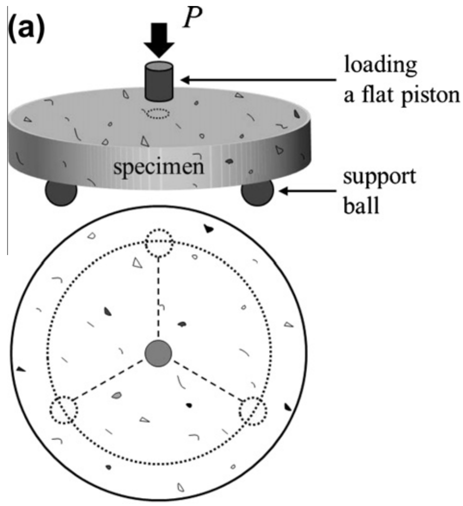

Fig. 1. Schematic drawings of test methods for biaxial tensile strength: (a) piston-on-three-balls test, and (b) ring-on-ring test.

tolerance for warpage during the test. The specimen is usually broken into three parts, and cracks are initiated at the same location because of the non-uniform stress distribution caused by the support balls.

Instead of the flat piston for the piston-on-three-balls test, a ball was used in the ball-on-three-balls test [11]. The test method also has a tolerance for non-uniform smoothness of the specimen surface, and generates lower variation in the test results than other test methods. However, the method has similar weaknesses to the piston-on-three-balls test. Much research has been carried out to investigate the biaxial strength of ceramics by using the ball-on-three-balls test method [12–15], although the test is not a standard method.

The ring-on-ring test (or concentric ring test), which is standardized in ISO 6474 [16] and ASTM C 1499 [17], has been used extensively to measure the strength of glass and ceramic [18–21]. A circular specimen is supported on a ring and loaded by another ring with a smaller diameter, as shown in Fig. 1b. Thus, the ring-on-ring test is able to generate equi-biaxial stress within the specimen, unlike other biaxial test methods using a piston or ball. However, the test method produces considerable variation in the results of the test, since the test results are strongly influenced by the interfacial condition between the specimen and the two rings [22].

A modified version of the ring-on-ring test to reduce the variation is the ball-on-ring test. In this test, a specimen is loaded by a ball instead of the loading ring, though it has support conditions identical to those of the ring-on-ring test [23,24]. The method is still not a standard method.

In estimating the biaxial tensile behavior of concrete, only two test methods are currently available. One method is that given in ASTM C1550 [3], which was developed to determine the flexural toughness of fiber-reinforced shotcrete and concrete, using a round panel with a diameter of 800 mm and a thickness of 75 mm. The round panel is supported by threefold symmetric pivots and subjected to a point load at the center of the specimen. The method in ASTM C1550 is quite similar to the piston-on-three-balls test shown in Fig. 1a, and the ball-on-three-balls test for ceramic. The other method is the biaxial flexure test illustrated in Fig. 2, which has been modified from the ring-on-ring test for glass and ceramics, to estimate the biaxial tensile strength of plain concrete. In the biaxial flexure test, the specimen thickness *h*, the radius of the specimen *R*, and the ratio of the thickness to the radius of the support ring *a* should be suitable for concrete materials, while in the

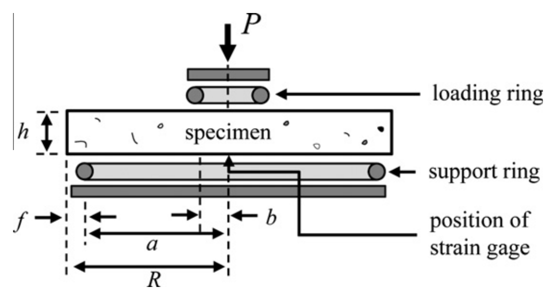

Fig. 2. Schematic diagram of biaxial flexure test (BFT) method.

ring-on-ring test, a very thin and large-aspect-ratio specimen (23–45 mm in diameter, and 0.4–2.2 mm in thickness) is used to measure the tensile strength of ceramic.

The biaxial flexure test has an advantage in that it is capable of generating constant equi-biaxial flexural stress within the area of the loading ring, which is necessary and useful for the investigation of the stochastic nature of the strength of a brittle material [25,26]. In addition, the biaxial flexure test especially has an advantage in investigating the equi-biaxial flexural behavior of fiber-reinforced cementitious composites (FRCCs), because the stress inside the loading ring is identical in all directions, which means cracks would initiate in any direction. If fibers are well distributed without any directional bias, then the directions of the cracks would be completely random inside the loading ring. Otherwise, if fibers are aligned with a certain directional bias, the directions of the cracks would not be random.

3. Improvement of the biaxial flexure test method

3.1. Specimen geometry

In determining the geometry of the biaxial flexure test specimen for concrete, the following aspects are carefully considered: (1) the concrete specimen containing coarse aggregates needs to have larger size than the ceramic specimen used in ASTM C 1499; and (2) the field of the equi-biaxial tensile stress field should be obtained and maximized during the test.

In the current ASTM C 1499 method for the biaxial strength of ceramics, it is recommended that the support diameter be significantly larger than the thickness of the specimen, while the length margin f, the difference between R and a, should be 0.5–3.0 times the thickness. However, these recommendations cannot be applied for concrete specimens, because the minimum dimensions should be at least three times larger than the maximum size of the coarse aggregate. If concrete specimens are designed according to ASTM C 1499, they are significantly larger and heavier; thus, it would be more difficult to deal with the concrete specimen during the test. Based on these considerations, the geometry of the concrete specimen, e.g., h/a (0.24) and f/a (0.05), was chosen to ensure the equi-biaxial tensile behavior described by simple plate theory [27]. The ratios were referenced from the literature [14,28], and the results of previous work [2].

According to ASTM C 1499, the equi-biaxial tensile strength of a disk specimen in the ring-on-ring test can be calculated by Eq. (1) [27,29]:

$$\sigma_b = \frac{3P}{2\pi h^2}\left\{(1-\upsilon)\frac{(a^2-b^2)}{2R^2} + (1+\upsilon)\ln\frac{a}{b}\right\} \quad (1)$$

where P is the applied load, h is the specimen thickness, a is the radius of the support ring, b is the radius of the loading ring, and υ is Poisson's ratio of the material. Here, the radius of the specimen R is greater than the radius of the support ring a as illustrated in Fig. 2.

Since the weight of margin in the specimen possibly generates the effect of end confinement on the equi-biaxial tensile strength, by assuming no margin f to minimize the effect, Eq. (2) was proposed for maximum equi-biaxial tensile strength by Zi et al. [2], based on plate bending theory [27].

$$\sigma_{\mathrm{BFT}} = \frac{3P}{4\pi h^2}\left\{(1-\upsilon)\left[1-\left(\frac{b}{a}\right)^2\right] - 2(1+\upsilon)\ln\left(\frac{b}{a}\right)\right\} \quad (2)$$

The biaxial tensile strength σ_{BFT} of the specimen, as described in Eq. (2), is a nonlinear function of b/a. Thus, the ratio b/a should be carefully determined. A numerical study has been performed to find an appropriate ratio and investigate its sensitivity.

In the numerical study to determine the ratio b/a and to validate Eq. (2), 3D linear elastic finite element models were developed using commercial software (ABAQUS/CAE Version 6.7-1) [30]. In the model, the friction between the specimen and the loading and support rings was assumed to be negligible, since a Teflon sheet was applied at the interface between the specimen and the loading and support rings during the test. The boundary conditions of the support ring constraint are fixed in the vertical direction only; uniform normal line loads were applied at the location of the loading ring. Only half of the specimen is modeled due to symmetry. The size of the element in the model used within the loading ring was refined until there was no further change in the stress value observed according to the size of elements.

The biaxial tensile stresses according to different b/a ratio are obtained from the theoretical and numerical investigation and are compared in Fig. 3. When the b/a ratio is larger than 0.25, an identical result is obtained from both the analytical and numerical investigations. However, the biaxial tensile stress at the location of the loading ring is slightly higher than that at the center of specimen, and the difference between these stresses is even higher when the b/a ratio increases. Thus, the b/a ratio is determined as 0.25, while the diameter of the loading ring is determined as more than twice the thickness of the specimens. The ratio between the two analytical solutions in Eqs. (1) and (2) is 1.006 when the ratio b/a is 0.25; thus the difference is clearly negligible. The dimensions of the improved biaxial flexure test specimen are summarized in Table 1.

Fig. 4 shows the contour plot of the maximum principal stress on the bottom surface of the specimen. It is clear that the stress is maximum and uniform with the position of the loading ring,

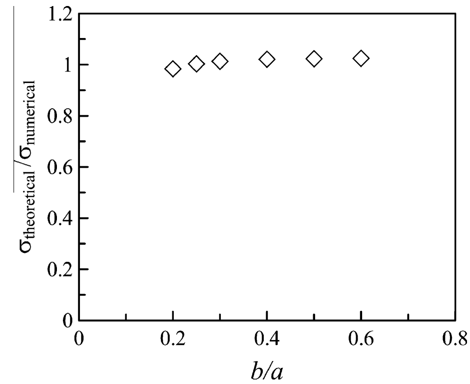

Fig. 3. Comparison between theoretical and numerical solutions for various b/a ratios.

Table 1
Geometry of the improved biaxial flexure test specimens from the numerical study.

h/a	f/a	b/a
0.24	0.05	0.25

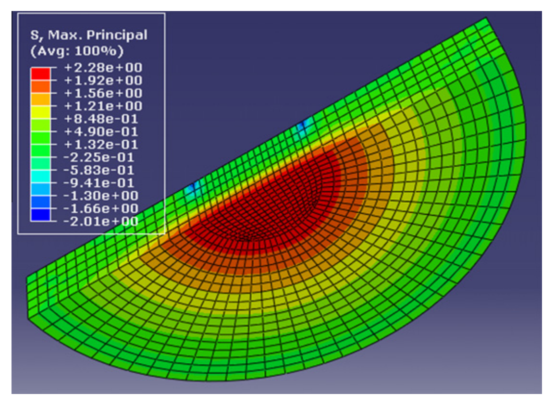

Fig. 4. Contour plot of the maximum principal stress on the bottom surface of a biaxial flexure test specimen.

where the first crack might be initiated. In Fig. 5, the radial and circumferential stresses in the specimen were normalized by the maximum stress $\sigma_{\max}$, while the distance from the center of the

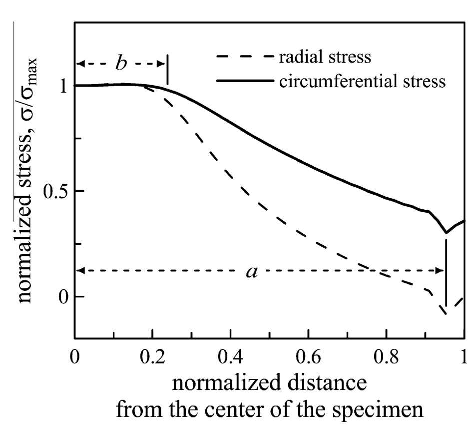

Fig. 5. Radial and circumferential stress distribution on the bottom surface of the biaxial flexure test specimen by the finite element analysis.

specimen was normalized by the radius of the specimen. As mentioned in the previous section, the normalized stresses are uniform and equal to each other, as the normalized distance is less than 0.2.

3.2. Loading and support condition

Maintaining the state of equi-biaxial tensile stress during the test is one of the most important conditions in the investigation of the biaxial flexural behavior of concrete by using the biaxial flexure test method. To satisfy the condition, the load should be uniformly applied on the specimen, and the surface of the specimen should be smooth. Even though the centers of the loading device and the specimen are carefully aligned to the same position so that there is no apparent eccentricity, the surface roughness may cause uneven distribution of the applied load along the perimeter of the loading device, which clearly changes the center of the resultant load [23,31]. To smooth the surface of the specimen, an annular soft rubber layer and four Teflon sheets of 0.3 mm thickness were placed between the rings and the specimen, as shown in Fig. 6.

In addition, as illustrated in Fig. 6, the surface of the specimen in contact with the rings was treated by applying high-strength gypsum, similar to the capping of concrete cylinders for the compression test [32,33]. The capping of the high-strength gypsum was performed according to the following procedures: (1) water and laitance was removed from the top of the specimen before capping; (2) the high-strength gypsum was prepared and stirred prior to pouring each cap; (3) the high-strength gypsum was poured into the specimens, and the high-strength gypsum was pressed to achieve flatness of the cap by using a ring-shaped capping plate; (4) the capping plate was covered with a sheet of plastic to prevent the capping material from adhering to the surface of the plate; and (5) the capping plate was removed as the capping material hardened.

3.3. Experimental verification of the improvement

By applying the improvement on the surface condition of the specimen as mentioned above, the equi-biaxial tensile strain was measured with two strain gages, attached to the center of the bottom surface of the specimen (Fig. 2), to discover whether or not the state of equi-biaxial tensile stress is obtainable during the test. The geometry of the biaxial flexure test specimen was predetermined according to the information given in Table 1. The panel specimen used has a 75 mm thickness and a 657 mm diameter. The radiuses of the load and support ring are 78 mm and 312.5 mm, respectively. The mix proportion of the concrete is provided in Table 2 and the compressive strength of the concrete at 28 days is 30 MPa. Type I Portland cement, natural fine aggregates, and crushed coarse aggregates are used. The compressive test was performed according to ASTM C 39 [32].

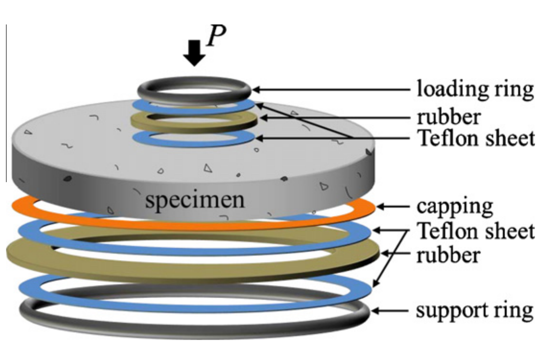

Fig. 6. Schematic diagram of experimental test setup of the biaxial flexure test.

Table 2
Mix proportion of concrete.

W/C (%)	S/a (%)	Unit weight (kg/m^3)			
		W	C	S	G
42	49	170	405	866	934

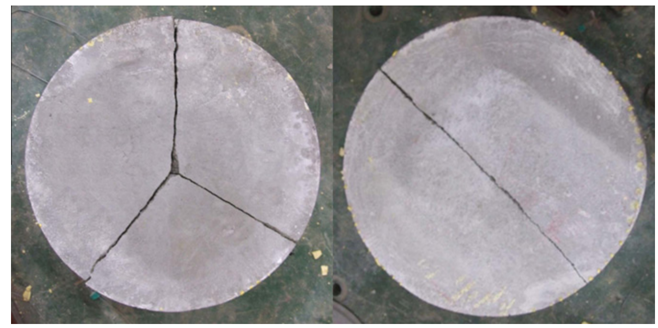

Fig. 7. Typical biaxial fracture patterns by improved biaxial flexure test.

The specimens cast were stored in a laboratory at room temperature for 2 days prior to demolding, followed by curing in a water chamber for 28 days. A hydraulic machine was used to perform the test, and the speed of the stroke during the test was 1 mm/min.

In order to minimize the effect of load eccentricity during the test, a plumb was used to align the center of the specimen and the loading and support ring. As mentioned, an annular soft rubber layer and four Teflon sheets were placed between the rings and the specimen. The annular surface of the test specimens was capped with high-strength gypsum. The thickness of the capping is 2 mm, which is about 4% of the thickness of the specimen. The load signal was measured from the load cell inside the hydraulic machine, while the biaxial strains at the bottom surface of the specimens were measured from the attached two strain gages.

Fig. 7 shows the fracture patterns of the biaxial flexure test specimens. Most of the specimens are broken into two or three pieces. The direction and location of the first crack were completely arbitrary. Both the longitudinal and transversal strain histories were measured, as well as the history of the applied load, as provided in Fig. 8. The surface improvement of the specimen generated almost identical responses from the two strain gages, as shown in Fig. 8a and b, while the specimen with no surface treatment showed a considerably different response, as shown in Fig. 8c. These results show that the surface treatment of the specimen successfully produced uniform equi-biaxial tensile strain at the bottom of the specimens, regardless of the number of broken pieces, and that the use of the gypsum and rubber was very helpful to align the center of the load resultant to the center of the specimen.

4. Uniaxial tensile strength vs. biaxial tensile strength of concrete

4.1. Specimen preparation and test procedure

To compare the uniaxial and biaxial tensile strengths of concrete, three different test methods were carried out, including the four-point bending test (4PBT) for uniaxial tensile strength, ASTM C 1550, and the biaxial flexure test for biaxial tensile strength. In all three experiments, the thickness of specimens was maintained at 48 mm. The geometry of the specimen for the four-point bending test was 48 × 48 × 244 mm, while the geome-

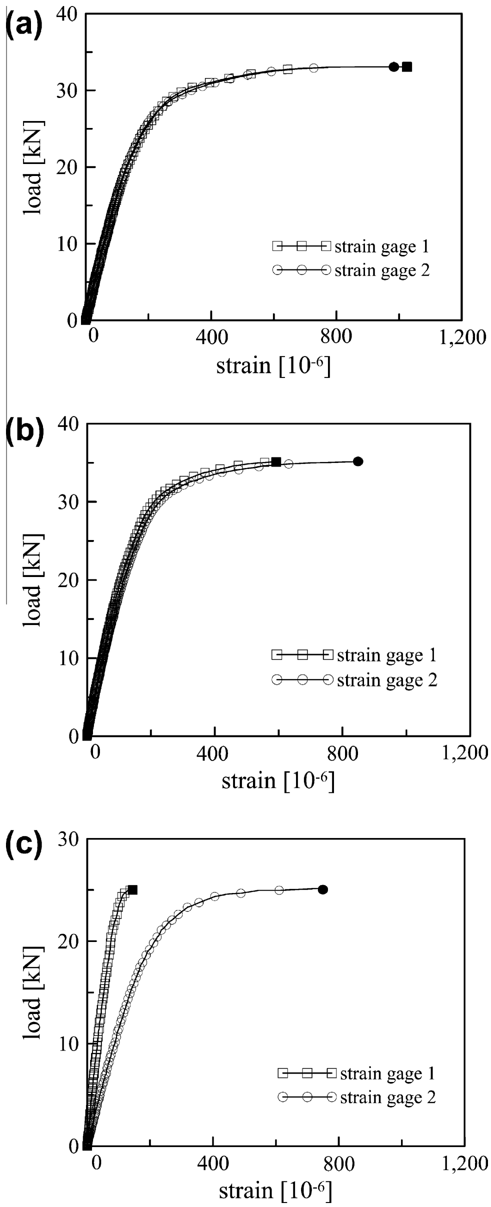

Fig. 8. Experimental load–strain curves of specimens broken into (a) three pieces, (b) two pieces by improved biaxial flexure test and (c) the specimen by improper biaxial flexure test in which solid symbols represent the termination of the measurement.

try of the specimen for the ASTM C 1550 and biaxial flexure tests are provided in Table 3. Thirteen concrete beams and 26 concrete panel specimens were prepared to investigate the stochastic nature of strength according to the different test methods. The same materials and mix proportions were used in both series of test specimens. The size of the maximum coarse aggregate in the matrix composition was 6.5 mm, and the compressive strength was 33 MPa at 28 days.

For the uniaxial tensile strength of concrete, a four-point bending test was carried out according to ASTM C 78 [34]. The equivalent flexural strength σ_u from four-point bending test is calculated by:

$$\sigma_u = \frac{PL}{b'h^2} \tag{3}$$

where P is the maximum applied load, L is the span length, and b' and h are the width and thickness of the specimen, respectively.

The equi-biaxial tensile strength of concrete was measured using ASTM C 1550 and the improved biaxial flexure test method. While the failure strength of concrete by the biaxial flexure test was obtained using Eq. (1), that by ASTM C 1550 was determined by the equations for the ball-on-three-balls test in ceramics [10,35] due to the lack of an analytical solution to provide the biaxial strength of the concrete specimens subjected to ASTM C 1550. The biaxial tensile strength for ASTM C 1550, σ_{B3B} was calculated as follows:

$$\sigma_{\mathrm{B3B}} = \frac{3P(1+\upsilon)}{4\pi h^2}\left[1 + 2\ln\left(\frac{a}{b_e}\right) + \frac{(1-\upsilon)}{(1+\upsilon)}\left\{\frac{2a^2 - b_e^2}{2R^2}\right\}\right] \tag{4}$$

$$b_e = \sqrt{1.6\,b_0^2 + h^2} - 0.675\,h, \tag{5}$$

where P is the applied load, υ is the Poisson's ratio of the material, a is the radius of the support circle, and R and h are the radius and thickness of the specimen, respectively. b_e is the equivalent radius, and b_0 is the radius of the loaded area measured from tests. Eq. (5) is applicable if $b_0 < 0.5\,h$. The equivalent radius (b_e) of the loaded area in this test is 17.58 mm. Poisson's ratio used in estimating σ_b was obtained from compressive tests; $\upsilon = 0.184$. The speed of loading in all tests was 1 mm/min.

5. Results and discussions

The fracture patterns of the round panel specimens from the two biaxial tests are comparable, and the fracture initiated from the maximum tensile surface of the specimens. The results of the four-point bending test, ASTM C 1550, and the biaxial flexure test methods are summarized in Table 4. The uniaxial tensile strength from the four-point bending test is about 43% of the biaxial tensile strength measured from the ASTM C 1550, and about 64% of that obtained from the biaxial flexure test. The same tendency has been reported elsewhere [8,9,36]. It is well known that the biaxial strength measured from the ball-on-three-balls method is generally higher than that by the ring-on-ring method, and greater than the modulus-of-rupture.

In ceramic research, the difference between uniaxial and biaxial tensile strength under flexural load is thought to originate from the different effective areas (or volume) of the material subjected to maximum flexural tensile stress based on the Weibull theory [25,37–39]. However, the strength of quasi-brittle materials such as concrete, rock, ceramics, and composite materials is affected not only by the shape of structures, but also by the size. The size effect on the biaxial tensile strength of concrete is beyond the scope of this paper, but a discussion is available elsewhere [40].

Table 3
Dimensions of the ASTM C 1550 and biaxial flexure test specimens.

	Thickness of specimen h	Diameter of specimen $2R$	Radius of			
			Support circle a	Support ring a	Loaded area b_0	Loading ring b
ASTM C 1550	48 mm	420 mm	200 mm	–	11 mm	–
Biaxial flexure test	48 mm	420 mm	–	200 mm	–	50 mm

Table 4
Test results obtained from the four-point bending test, ASTM C 1550, and biaxial flexure test methods.

No.	Tensile strength (MPa)		
	Four-point bending test	ASTM C 1550	Biaxial flexure test
1	5.23	11.62	6.35
2	4.45	10.29	7.28
3	4.79	11.15	6.67
4	4.22	10.58	6.38
5	4.83	8.75	5.57
6	4.08	9.14	6.37
7	3.78	10.28	6.92
8	4.62	11.30	7.63
9	4.90	10.68	7.76
10	4.58	9.59	6.22
11	3.87	8.75	7.43
12	4.51		7.71
13	3.41		7.14
Mean strength	4.41	10.19	6.88
Standard deviation	0.49	0.96	0.65
c.o.v	0.11	0.09	0.10

The ring-on-ring test for ceramics has shown higher variation in the test results due to the imperfect surface conditions of specimens. Thus, the number of specimens needed per series for the ring-on-ring test is generally more than that needed for other test methods. However, in this study, the coefficient of variation (c.o.v) for the flexural strength of the four-point bending test, ASTM C 1550, and the biaxial flexure test were about 11%, 9%, and 10%, respectively. These values are in good agreement with those obtained by Kim et al. [41], although different sizes were used for their experiment. Our results indicate that the biaxial flexure test method is a practical and reliable means of determining the biaxial tensile strength of concretes.

6. Conclusions

To improve the biaxial flexure test method for concrete, the geometry of the biaxial flexure test specimen (h/a = 0.24, f/a = 0.05 and b/a = 0.25) was carefully determined by performing numerical and experimental studies. In addition, proper treatments for the specimen surface and the support conditions are proposed and verified. In order to provide a smooth surface for the biaxial flexure test specimens, the specimens are capped on the perimeter of the bottom face where the specimen rests on the support ring, and soft rubber pads are added at the interface between the specimen and the loading and support rings.

The equi-biaxial tensile strength of concrete was successfully determined by using the improved biaxial flexure test method. The cracks were consistently initiated in the maximum tensile surface within the loading ring of the biaxial flexure test method, and uniform biaxial strain histories were measured on the middle of the bottom tensile surface of the biaxial flexure test specimens. Furthermore, the coefficient of variation (c.o.v) in the biaxial tensile strengths measured by using the biaxial flexure test method is comparable to that from the four-point bending test and ASTM C 1550. Thus, the biaxial flexure test method is a practical and reliable means of determining the biaxial strength of concrete. The biaxial tensile strengths given by the biaxial flexure test and the ASTM C 1550 method are higher than the uniaxial tensile strength using the four-point bending test method.

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