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A MATHEMATICAL MODEL FOR THE PREDICTION OF CEMENT STRENGTH

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ABSTRACT

In this paper a mathematical model for the prediction of Portland cement compressive strength after 2, 7 and 28 days is developed and the importance of chemical-mineralogical and fineness factors is pointed out. It is shown that the fineness is the predominant factor affecting the early strength while the chemical and mineralogical synthesis of the cement contribute to the strength development after 7 and 28 days. The 28 day strength of the cement is significantly affected by the distribution of the material in the size fractions <3μm, 3-16μm, 16-24μm and 24-32 μm.

Introduction

The standard 28 day compressive strength test is widely used for the characterisation of the cement properties. Since, in industrial practice, 28 days is a very long period the faster determination of the cement strength is a favourite object of recent research.

There are two methods for the faster determination of cement strength: the accelerated strength test methods (1) and the use of suitable mathematical models (2,3,4,5). Besides other researchers have studied the effect of the clinker composition (6,7), the clinker microstructure (8,9), the porosity and pore structure (10,11) and the cement fineness (12,13,14) on the cement strength.

In the present paper a mathematical model for the prediction of cement strength is developed based on three series of data concerning the chemical-mineralogical synthesis of the cement, the parameters of the particle size distribution of the cement and the material distribution in characteristic size fractions.

Experimental

The data used for the development of the strength model are given in Tables 1 and 2 which present the chemical-mineralogical data and the data concerning the fineness of the cements respectively.

TABLE 1
Chemical and Mineralogical Characteristics
of the Cements used.

No	C_3S (%)	C_2S (%)	C_3A (%)	C_4AF (%)	Na_2O_{eq} (%)	SO_3 (%)	LSF	C_3S/C_2S	C_3A/C_4AF
1	60.5	11.1	7.2	11.9	0.57	2.60	96.2	5.45	0.61
2	57.7	14.0	6.0	12.5	0.62	2.50	94.5	4.14	0.48
3	53.0	18.5	6.5	12.8	0.59	2.45	92.4	2.86	0.51
4	59.3	12.5	6.4	12.4	0.54	2.50	95.3	4.73	0.52
5	54.4	16.7	6.8	12.2	0.61	2.70	93.4	3.26	0.56
6	56.6	14.9	6.7	12.4	0.60	2.72	94.2	3.80	0.54
7	63.0	9.4	6.3	12.2	0.52	2.40	96.8	6.70	0.51
8	56.5	15.1	6.9	12.0	0.60	2.60	94.2	3.74	0.57
9	54.2	17.0	7.2	11.9	0.59	2.75	93.3	3.18	0.61
10	56.2	16.4	7.3	11.1	0.65	2.60	93.7	3.42	0.65
11	59.6	12.2	7.0	11.9	0.62	2.60	95.6	4.88	0.59
12	62.5	8.8	7.8	11.6	0.54	2.55	97.5	7.12	0.67
13	57.0	13.9	6.7	12.8	0.59	2.55	94.7	4.10	0.53
14	54.1	16.6	7.1	12.8	0.61	2.40	93.5	3.26	0.56
15	59.1	11.9	7.3	12.2	0.54	2.50	95.8	4.97	0.60
16	59.8	12.8	6.3	12.2	0.60	2.60	95.1	4.65	0.51
17	58.8	13.6	6.9	11.0	0.64	2.66	94.9	4.33	0.63
18	49.9	20.8	6.6	13.1	0.65	2.65	91.4	2.40	0.50
19	60.0	12.5	6.9	11.3	0.59	2.55	95.5	4.78	0.61
20	57.2	15.6	6.9	11.3	0.59	2.60	94.0	3.66	0.61
Min	49.9	8.8	6.0	11.0	0.52	2.40	91.4	2.40	0.48
Max	63.0	20.8	7.8	13.1	0.65	2.75	97.5	7.12	0.67

Results And Discussion

The following series of variables have been tested in order to certify their effect to the cement strength.

a. chemical - mineralogical variables: % content of C_3S, C_2S, C_3A, C_4AF, Na_2O_{eq}, SO_3, fCaO, values of C_3A/C_4AF, C_3S/C_2S, lime saturation factor LSF (%), alumina ratio AR.

b. particle size distribution variables: specific surface S_b, position parameter pp, 80% passing size P_{80}, uniformity factor n.

c. size fractions variables : % content in 3-32μm, ‹3μm, 3-16μm, 16-24μm, ›24μm, ›32μm.

The selection of the variables that contribute to the prediction of the cement strength is based on stepwise regression analysis. In figure 1 the variables that are inserted in mathematical models by this statistical procedure are illustrated. It must be noticed that, in case of strongly correlated parameters, the effect of each one on the development of cement strength cannot be drawn from this figure.

The stepwise regression analysis of the data presented in Tables 1 and 2 leads to the relation 1.

TABLE 2
Fineness Characteristics of the Cements used

No	Sb (cm^2/g)	n	pp (µm)	P_{80} (µm)	‹3µm (%)	3-32µm (%)	›32µm (%)	3-16µm (%)	16-24µm (%)	›24µm (%)
1	4220	0.99	16.1	26.0	16.7	70.4	12.9	41.9	16.3	25.1
2	3940	1.02	16.3	26.1	16.5	70.2	13.3	40.2	17.7	25.6
3	3820	0.97	17.8	29.1	16.0	70.9	13.1	38.6	18.8	26.6
4	3940	1.01	15.2	24.4	17.5	70.9	11.6	42.4	16.8	23.3
5	3900	0.99	18.2	29.4	15.7	68.4	15.9	37.9	17.3	29.1
6	3870	1.03	16.1	25.6	15.5	72.8	11.5	43.2	17.9	23.2
7	4100	1.05	16.0	25.1	15.7	70.5	12.0	39.9	19.4	23.2
8	3910	1.00	15.6	25.1	17.5	70.0	12.5	41.2	16.6	24.7
9	3750	1.00	16.0	25.7	17.1	70.7	12.7	39.7	17.8	25.9
10	3230	0.92	23.6	39.8	14.0	60.2	25.6	32.8	13.0	40.0
11	4000	0.97	17.4	28.4	16.2	71.3	12.5	40.2	18.4	25.2
12	4050	1.01	16.4	26.3	16.6	70.4	13.0	39.0	18.3	26.1
13	3980	1.02	16.4	26.1	15.6	71.9	12.5	41.8	18.0	24.6
14	3900	0.99	16.2	26.1	17.0	72.0	13.4	41.0	16.8	27.6
15	4000	1.03	17.5	27.8	14.6	71.7	13.7	37.9	19.8	27.7
16	3500	0.93	22.5	37.6	14.1	62.7	23.2	33.7	15.8	36.4
17	3320	0.94	23.2	38.5	13.6	62.1	24.3	32.8	16.1	37.5
18	3720	0.99	17.6	28.4	15.5	69.3	15.2	39.6	17.3	27.6
19	3260	0.97	22.7	37.1	13.0	60.5	26.5	32.7	15.5	38.8
20	3540	0.97	19.5	31.9	14.9	66.7	18.4	38.1	16.6	30.4
Min	3230	0.92	15.2	24.4	13.0	60.2	11.5	32.7	13.0	23.2
Max	4220	1.05	23.6	39.8	17.5	72.8	26.5	43.2	19.8	40.0

Variables	2 d. Strength	7 d. Strength	28 d. Strength
C_3S/C_2S		■	■
C_3A/C_4AF			■
LSF (%)		■	■
Sb (cm^2/g)	■		
pp (µm)		■	■
P_{80} (µm)	■	■	■
‹3µm (%)		■	■
3-32 µm (%)	■		■
3-16 µm (%)	■		
16-24 µm (%)	■		■

FIG. 1
Selected variables by stepwise regression analysis for the prediction of cement strength (shading=selection)

$$\begin{vmatrix} S_2 \\ S_7 \\ S_{28} \end{vmatrix} = \begin{vmatrix} 0.00 & 0.00 & 0.00 & 0.0066 & 0.00 & 0.31 & 0.00 & -0.90 & 0.98 & 0.93 \\ -1.34 & 0.00 & 1.64 & 0.0000 & -13.93 & 6.77 & -3.58 & 0.00 & 0.00 & 0.00 \\ -11.75 & 35.37 & 8.46 & 0.0000 & -74.05 & 35.47 & -9.87 & -5.39 & 0.00 & 6.93 \end{vmatrix} \cdot \begin{vmatrix} C_3S/C_2S \\ C_3A/C_4AF \\ LSF(\%) \\ S_b\ (cm^2/g) \\ pp(\mu m) \\ P_{80}\ (\mu m) \\ <3\mu m\ (\%) \\ 3\text{-}32\mu m\ (\%) \\ 3\text{-}16\mu m\ (\%) \\ 16\text{-}24\mu m(\%) \end{vmatrix} \quad (1)$$

where S_2, S_7, S_{28} are the compressive strength after 2, 7 and 28 days (N/mm^2) respectively

The relation (1) results to:

$$S_2 = 0.0066S_b + 0.31P_{80} - 0.90(\%\ 3\text{-}32\mu m) + 0.98(\%\ 3\text{-}16\mu m) + 0.93(\%\ 16\text{-}24\mu m) \quad (2)$$

$$S_7 = -1.34C_3S/C_2S + 1.64LSF - 13.93pp + 6.77P_{80} - 3.58(\% < 3\mu m) \quad (3)$$

$$S_{28} = -11.75C_3S/C_2S + 35.37C_3A/C_4AF + 8.35LSF - 74.05pp + 35.48P_{80} - 9.87(\% < 3\mu m) - 5.39(\%\ 3\text{-}32\mu m) + 6.93(\%\ 16\text{-}24\mu m) \quad (4)$$

The measured and predicted strength values are presented in Table 3.

In order to investigate the fitting of the multiple regression models 2,3 and 4 to the set of data, the statistic multiple coefficient of determination R^2 (R square) is determined. The R^2 is 0.9979, 0.9985 and 0.9978 for the models 2, 3 and 4 correspondingly. That means that the 99.8% of the sum of squares of deviations of the measured strength values about their mean is attributable to the least-squares prediction equations therefore the model fits the data best.

The F-test statistics are 1786, 541 and 1073 for the models 2, 3 and 4 correspondingly which greatly exceed the tabled values 2.90, 2.90 and 2.85. That means that at least one of the parameters of the independent variables does not equal zero for a confidence interval of 95%.

The adequateness of the obtained mathematical models has been examined by Student's t-test and all calculated values of t-test are less than tabulated at a=0.05. Therefore an exact functional dependence can be established between the studied

TABLE 3
Measured and Predicted Strength Values of the Cements used.

No	Measured Strength (N/mm^2)			Predicted Strength (N/mm^2)			Difference (%)		
	2 d.	7 d.	28 d.	2 d.	7 d.	28 d.	2 d.	7 d.	28 d.
1	27.8	41.6	56.2	29.0	42.7	59.6	4.2	2.6	6.0
2	27.7	39.5	52.0	27.0	39.7	54.6	2.5	0.4	5.0
3	24.2	36.6	56.1	26.0	39.1	57.1	7.2	6.8	1.7
4	27.6	41.4	56.0	27.2	40.3	56.6	1.6	2.6	1.0
5	28.5	41.3	54.1	26.7	38.8	55.2	6.2	6.1	2.0
6	27.3	44.5	57.6	27.2	43.1	55.1	0.4	3.2	4.4
7	29.7	41.4	59.6	28.8	41.3	55.0	3.1	0.4	7.8
8	26.6	40.2	60.6	26.6	39.2	60.3	0.2	2.6	0.4
9	25.9	39.6	65.6	24.8	39.0	64.4	4.3	1.5	1.9
10	24.8	39.8	57.3	23.9	39.7	56.4	3.8	0.1	1.5
11	27.7	40.9	63.8	27.8	42.0	62.0	0.2	2.7	2.9
12	26.2	40.3	55.7	27.0	40.8	56.7	3.0	1.3	1.9
13	27.7	43.2	56.4	27.6	42.7	57.3	0.4	1.1	1.7
14	24.6	39.0	52.1	25.1	39.8	51.0	1.9	2.0	2.1
15	27.3	42.4	57.5	26.3	43.1	61.0	3.8	1.6	6.0
16	27.9	41.0	58.2	26.2	40.7	58.2	6.0	0.8	0.1
17	23.9	39.8	54.6	25.3	39.2	56.0	5.7	1.6	2.6
18	25.5	35.8	50.7	26.1	38.7	51.5	2.4	8.2	1.6
19	23.6	37.7	50.9	25.2	38.9	51.3	6.7	3.1	0.8
20	28.1	42.9	62.8	26.2	40.3	58.5	6.8	6.2	6.9

variables and the cement strength. The average differences between measured and predicted strength are 4.73 %, 3.98 % and 4.83% or more specifically 1.26, 1.61 and 4.83 N/mm^2 for the models 2, 3 and 4 correspondingly.

In Figure 2 the values of cement strength after 2, 7 and 28 days vs. predicted values are presented. Therefore the simulation of the strength development is very satisfactory.

From the relations 2,3 and 4 it is obvious that the fineness of the cement is the significant factor for the strength after 2 days. More specifically the particle fractions ‹3, 3-16 and 16-24μm have a positive effect on the strength while the fraction 24-32μm has a negative one. In addition the increase of the specific surface and P_{80} of the cement leads to higher strength values.

The cement strength after 7 days is affected by the ratios C_3S/C_2S and the LSF value as well as by the characteristics of the particle size distribution pp and P_{80}. The fraction with size less than 3μm lowers the strength value as it was expected.

The 28 day strength is affected by the LSF value and the ratios C_3S/C_2S and C_3A/C_4AF. Besides it is observed that the fraction 16-24μm has a positive effect while the fractions ‹3, 3-16 and 24-32 have a negative one.

It must be noticed that it is not possible to extract conclusions concerning the individual contribution of the C_3S/C_2S, C_3A/C_4AF and LSF values on the strength as these variables are strongly correlated.

The models 2, 3 and 4 have been tested for the prediction of the strength of cements produced by the Greek companies and the results were very satisfactory.

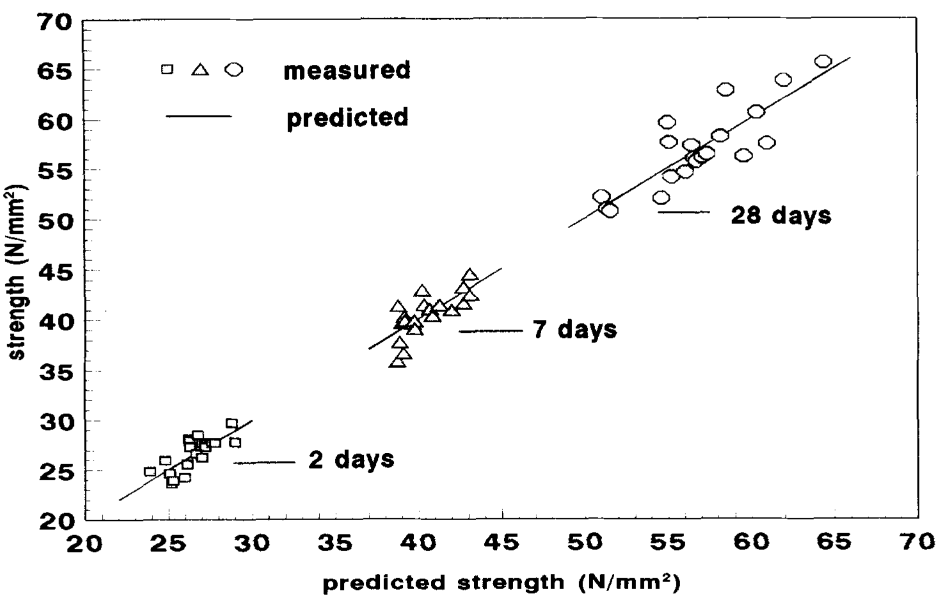

FIG. 2.
Compressive cement strength vs predicted cement strength after 2, 7 and 28 days.

Conclusions

The following conclusions can be drawn from the present study:

- A mathematical model for the prediction of the 2, 7 and 28 day compressive strength of the cement is developed based on stepwise regression analysis.
- The proposed model predicts the cement strength with a satisfactory accuracy.
- At early ages the strength is affected mainly by the fineness parameters.
- At later ages the chemical - mineralogical synthesis of the cement influences the strength growth.
- The 28 days strength is strongly affected by the distribution of the cement particles in the size fractions <3μm, 3-16μm, 16-24μm and 24-32 μm. Specifically the 16-24μm fraction is the only one which has a positive effect on the strength development.

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