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ESTIMATION OF THE ELASTIC MODULI OF LIGHTWEIGHT AGGREGATE

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ABSTRACT

This research presents a method for the estimation of the elastic modulus of lightweight aggregate. An experimental study was carried out to measure the elastic modulus of lightweight concrete in function of the amount of lightweight aggregate. The elastic modulus of lightweight aggregate was obtained using composite theory models applied to the experimental data. Based on the Mori-Tanaka model, the elastic modulus of three different types of lightweight aggregates ranged from 12.5 to 19.6 GPa. These values are in agreement with experimental results reported in the literature.

Introduction

The elastic properties of rock based aggregate can roughly be estimated from general information about the rock or directly measured on the bedrock itself, however it is not as easy to obtain information about the elastic properties of artificially produced small particles such as lightweight aggregate (LWA). Müller-Rochholz (1) used ultrasonic pulse velocity to determine the dynamic elastic properties of a variety of different lightweight aggregates, including two of those tested in the present study. These tests were performed on single aggregate particles polished into prisms. In this special sample preparation, however, the stiffer shell of the particle is removed during the polishing, and may lead to lower values of the elastic modulus. Also, since the properties may vary from one particle to another, a large number of tests must be carried out to obtain representative results.

A recent study has demonstrated the advantage of using models based on composite materials theory for the prediction of the elastic properties of concrete (2). The calculated elastic modulus is based on information on the elastic properties of the aggregate and the matrix, and their respective volume fractions. Such models can also be used to solve the inverse problem: determination of the elastic modulus of the aggregate once the elastic modulus of the concrete and the matrix are determined experimentally, and the volume fractions of the aggregate and the matrix are known. However, the inverse problem tends to be more complex than the forward problem, and identification of the parameters requires greater care. Mor and Monteiro (3) successfully used the method developed by Kuster and Toksöz (4) to determine the elastic modulus of a weak lightweight aggregate. Zimmerman et al. (5) also used the same method to study the effect of porosity on the elastic behavior of cement paste and mortar. In the present work both the Kuster and Toksöz and the Mori-Tanaka methods (6) are used.

Theoretical Background

Kuster-Toksöz Model

Kuster and Toksöz (4) proposed a homogenization technique based on a compressional (or shear) wave impinging on an assemblage of inclusions, and calculated the sum of the waves which were scattered back from each inclusion. This wave was then equated to that which would be scattered back from a single equivalent spherical inclusion, thus leading to an expression for the effective elastic modulus. By assuming the aggregate as spheres, this theory predicts that

$$\frac{K}{K_1} = \frac{1+[\,4G_1\,(K_2 - K_1)\,/\,(3K_2 + 4G_1)\,K_1]\,c}{1 - [\,3\,(\,K_2 - K_1)\,/\,(3K_2 + 4G_1)]\,c} \tag{1}$$

and

$$\frac{G}{G_1} = \frac{(6K_1 + 12G_1)G_2+(9K_1+8G_1)[(1\text{-}c)G_1+cG_2]}{(9K_1+8G_1)G_1+(6K_1+12G_1)[(1\text{-}c)G_2+cG_1]} \tag{2}$$

where K and G are the bulk and shear moduli, respectively, c is the volume fraction of aggregate, and the subscripts 1 and 2 refer to the matrix and to the aggregate, respectively.

Mori-Tanaka method

The Mori-Tanaka method involves rather complex manipulations of the field variable along with special concepts of eigenstrain and backstress. The method does not have any physical description and is essentially mathematical rather than physical. By assuming spherical aggregate particles, the Mori-Tanaka method yields the following equations for the effective bulk modulus ($K_{M\text{-}T}$) and shear modulus ($G_{M\text{-}T}$) when the aggregate particle is softer than the matrix (7), typically the case for lightweight concrete.

$$K_{M\text{-}T} = K_2 + \frac{c_1}{\dfrac{1}{K_1 - K_2} + \dfrac{3c_2}{3K_2 + 4G_2}} \tag{3}$$

$$G_{M\text{-}T} = G_2 + \frac{c_1}{\dfrac{1}{G_1 - G_2} + \dfrac{6(K_2 + 2G_2)c_2}{5G_2(3K_2 + 4\,G_2)}} \tag{4}$$

when $(G_2\text{-}G_1)(K_2\text{-}K_1)\geq 0$, with $(G_2\text{-}G_1)\leq 0$, $(K_2\text{-}K_1)\leq 0$

c_1 and c_2 are the volume fractions of the matrix and the aggregate, respectively.

The Young's modulus E and Poisson's ratio ν can be found by

$$E = \frac{9KG}{3K + G} \tag{5}$$

$$\nu = \frac{3K\text{-}2G}{6K+2G} \tag{6}$$

Experimental

Concrete samples were made from normal weight sand, portland cement, silica fume and three different expanded shale types of lightweight aggregates (LWA), Type 1, 2 and 3. The physical properties of the aggregates are reported elsewhere (8). Aggregate Type 1 was more porous and weaker than Type 2 and 3, while Type 3 was the strongest of the three types of LWA used. The water-cementitious ratio was kept at 0.30 throughout the test program. A high-range naftalen based water reducer was used to obtain the workability required. In order to keep the effective water-cementitious ratio as constant as possible the LWA was immersed in water for 10 minutes before mixing and then surface dried with a towel. In addition to the pure matrix (reference), mixes with LWA volume fractions of 15, 30 and 50% were tested (see Table 1).

TABLE 1

Mix Proportions (kg/m^3).

Materials	Matrix			
	Ref	15%	30%	50%
Water	251	213	176	125.6
Cement	753	640	527	376.7
Silica Fume	83.7	71.1	58.6	41.9
Sand	1272	1081	890.6	636.2
Superplasticizer	10.1	8.5	8.2	6.0
Aggregate Content	**Aggregate Type**			
	Type 1	Type 2	Type 3	
Coarse Aggregate (4-8 mm) 15%	74	91	110	
30%	147	181	219	
50%	246	303	365	
Coarse Aggregate (8-16 mm) 15%	74	91	111	
30%	147	181	222	
50%	246	303	370	

Concrete cylinders (100 by 200 mm) were cast and cured in water (20±2° C) until the time of testing. At 28 days the static modulus of elasticity and the compressive strength were measured according to ASTM Test Method for Static Modulus of Elasticity and Poisson's Ratio of Concrete in Compression (C-469) (9) and ASTM Test Method for Compressive Strength of Cylindrical Concrete Specimen (C-39) (10), respectively. All cylinders tested were ground and polished before testing to achieve smooth end surfaces, and all the test results are the average of two specimens. A testing machine with 3000 kN load capacity was used.

Results and Discussion

Figures 1 and 2 show the elastic modulus of concrete and the compressive strength of concrete as a function of the volume fraction of LWA.

The Poisson's ratio of the concrete samples was assumed to be 0.2. This allows the computation of the bulk and shear moduli for the matrix and the concrete from the experimental values for the modulus of elasticity. Equations (1) and (2) and eqs. (3) and (4) were used to calculate the bulk and shear moduli of the LWA according to the Kuster-Toksöz and the Mori-Tanaka method, respectively. Since the equations for both the Kuster-Toksöz and the Mori-Tanaka method are non-linear in K_2 and G_2, a non-linear regression was performed to identify these parameters.

Table 2 shows the elastic modulus of the different lightweight aggregates obtained from the Kuster-Toksöz and the Mori-Tanaka methods. Figure 3 shows the quality of the fit both for these methods. As expected, the elastic moduli of all the lightweight aggregates were lower than that of the mortar matrix. For comparison, the experimental values obtained by Müller-Rochholz are also shown. It is interesting to note that Müller-Rochholz results are higher for Type 1 aggregate and lower for Type 3. One possible explanation is that Müller and Rochholz measured the dynamic elastic properties which are higher than the static values. Their

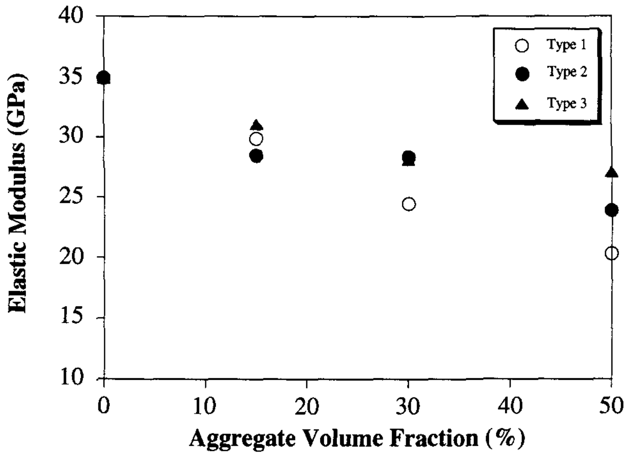

FIG. 1
Elastic Modulus of Concrete as a Function of the Volume Fraction of Aggregate.

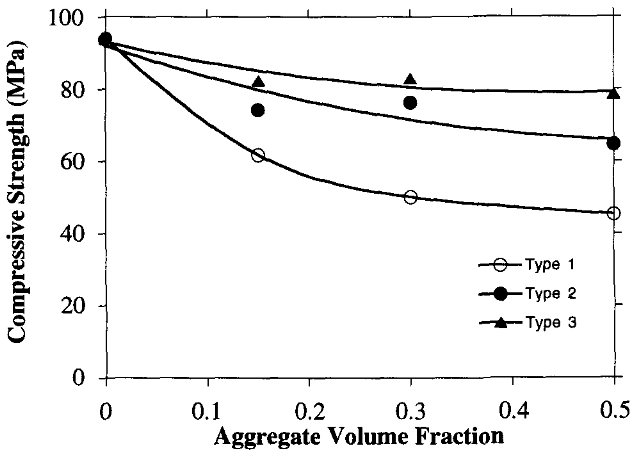

FIG. 2
Strength of Concrete as a Function of the Volume Fraction of Aggregate.

experimental technique removed the stiffer shell of the particle during the polishing which should decrease the elastic moduli, however since the porous Type 1 aggregate does not have a significantly stiffer shell, the dynamic measurements overshadow the removal of the shell where for the dense Type 3 aggregate the reverse happens.

TABLE 2 - Elastic Modulus of Lightweight Aggregates (GPa).

Aggregate	Kuster-Toksöz	Mori-Tanaka	Müller-Rochholz(1)
Type 1	10.3	12.5	13.0
Type 2	15.3	16.4	*
Type 3	19.1	19.6	17.5

*not measured

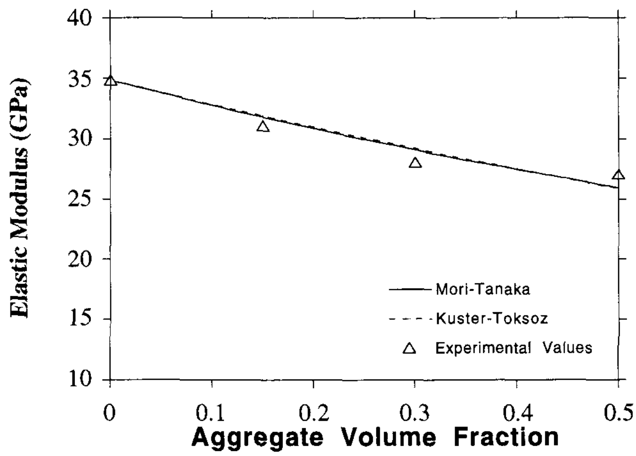

FIG. 3
Comparison of the Kuster-Toksöz and the Mori-Tanaka methods

Conclusions

Models based on composite materials theory appear to provide a good basis for determination of the elastic properties of lightweight aggregate. Based on the Mori-Tanaka method, the elastic modulus of the three different lightweight aggregates tested ranged from 12.5 to 19.6 GPa, which corresponds to experimentally observed values reported in the literature.

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