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SOME FACTORS AFFECTING THE DYNAMIC MODULUS OF ELASTICITY OF HIGH STRENGTH CONCRETE

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ABSTRACT

A set of 18 concrete and six mortar mixes is described, with cube strengths at 28 days of 74 to 117 MPa and electrodynamic E values of 47 to 55 GPa (37 to 42 GPa for the mortar). Very limited potential for increased E is seen after 28 days and comparative detrimental effects are noted due to air-drying and/or self desiccation. Increase from 10 to 15 percent silica fume increases cube strength (by about 3 MPa here) but not E. Ultrasonic pulse velocity and cube strength have expected good correlations with E, the former being closely dependent on mix composition and density. Simple empirical prediction models for E are found to be satisfactory for values up to about 50 GPa for the concretes studied.

Introduction

There has been major interest in high strength concrete (HSC) in recent years (1,2); a good deal of the attention has been devoted to achieving high compressive strength using conventional materials and production methods, the incorporation of silica fume (SF) and superplasticisers (SP) having allowed strength of up to about 120 MPa to be achieved at 28 days with no great difficulty. But there is some general unease with other engineering properties viz tensile strength (3) and modulus of elasticity E (4). This paper is concerned with the latter.

Concern arises because it is likely that HSC will be used where deformation is more important (e.g. in lower columns of very tall structures) and yet E is a parameter given comparatively little critical attention in normal concrete except in rather special circumstances. It has generally been approximated in Codes of Practice (5) and in guides to structural behaviour (6). Usually the approach has been to relate it, simply, to compressive strength although in particular applications it has been used with more discrimination (7). Coupled to the simple approach is the knowledge that as strength increases E does not increase correspondingly and uncertainty over E becomes stronger, especially if a suspected 'ceiling' value is imminent.

Different testing techniques also give rise to some ambiguity; whereas most structural engineers use the static E (and interpretations thereof) the dynamic E is much more reliable and reproducible while being very much simpler to measure (8), although giving a higher value.

E, like strength, is related to porosity and in properly compacted concrete porosity is largely a paste-dependent property, for normal aggregates. Hence hydration and ageing are important and thus curing history and age at test are relevant. The aim was to check the development of the dynamic E with age, especially for concretes which might approach their 'ceiling' values, where continued hydration was encouraged and where it could cease due to drying or self-desiccation.

Materials and Mixes Used

Details of the coarse aggregates used are given in Table 1 together with some details of the Class 42.5 N Portland cement (9). Silica fume (SF), with about 97 percent SiO_2, and a superplasticser (SP) of the sulphonated naphthalene formaldehyde type were also incorporated. The sand was marine dredged of generally BS882 'F' grading (10) but with only about 2.0 percent finer than 300 microns.

Details of the concrete mixes used are given in Table 2.

TABLE 1

Some details of aggregates and cement used

Coarse Aggregate	Maximum size (mm)	Relative density		24-hour absorption (% dry mass)	Portland cement calculated Bogue composition (%mass)	
		oven dry	Surface dry			
Crushed limestone	10	2.64	2.69	1.7	C_3S	57
					C_2S	13.5
					C_3A	9.2
Crushed limestone	20	2.67	2.70	1.0	C_4AF	8.1
Uncrushed gravel	10	2.57	2.62	2.0	Finess $=322\ m^2/kg$	

TABLE 2

Details of concrete mixes used with each coarse aggregate

Mix	A/B	W/C	W/B	SF/(C+SF)	SP (%)	C (kg/m^3)	V_a
1a	3.23	0.26	0.23	0.10	1.5	495	0.42
b	3.23	0.27	0.23	0.15	1.7	465	0.42
2a	3.59	0.32	0.29	0.10	1.0	450	0.42
b	3.59	0.34	0.29	0.15	1.2	420	0.42
3a	3.99	0.42	0.38	0.10	0.7	405	0.42
b	3.99	0.45	0.38	0.15	0.9	375	0.42

(All mixes had 37.5% sand by mass of combined aggregate)

A = combined aggregate (kg/m^3)
C = cement (kg/m^3)
W = water (kg/m^3)
B = (C + SF) (kg/m^3)
SP = Superplasticiser
SF = silica fume
V_a = Vol fraction of coarse agg

Two sets of corresponding mixes of mortar were also made (i.e. with 10 and 15 percent SF) having the same mix proportions as the mortars in the concretes. Thus the total number of mixes was 24, all with medium/high workability (100-150mm slump) and compacted easily by vibration.

Test Specimens and Curing Regimes

All test specimens were kept in their moulds, covered, for about 24 hours, in the laboratory and then de-moulded. Concrete cubes, 100mm size, were stored in water at 20°C and tested as standard control specimens (BS1881) after 7 and 28 days.

Prisms, nominally 76 x 76 x 254mm size, were used for checking E. (This prism size is not usually recommended for use with aggregates of 20mm maximum size but being so convenient - and in practice was easy to use - it was felt to be worth while to check potential detrimental effects. See comments later.)

Curing history is known to affect E (11,12) so three sets of pairs of prisms from each mix were cured as follows:

A - water cured at 20°C,
B - air-dried in the laboratory at about 20° ± 5°C with no positive control of relative humidity and
C - nominally sealed (in two layers of plastic film, as used for food wrapping) and stored as B.

Tests on Prisms

The electrodynamic E was found from the longitudinal resonant frequency (8) and had the great virtue of being non-destructive, which allowed the same specimen to be re-tested many times. Tests were done at 7, 28 and 56 days and, typically, after 21 to 27 weeks when the rate of change was extremely slow. Further tests will be carried out at later ages.

Ultrasonic pulse transit times were also measured, using standard equipment. These provided a very useful check on E values as well as an indication of internal changes in the structure of the concrete due to ageing and curing, or lack of it. Each prism was weighed at test.

Results and Discussion

Because the interactions between Portland cement, SF and SP are very complex and specific to the chemistry of the materials used (13) observations on properties of concretes made with ingredients from single sources cannot be strictly definitive; nevertheless observed trends should be indicative of the behaviour of mixes of reasonably similar compositions but containing materials which can only be nominally similar.

Compressive Strength

From the results given in Table 3 there are no concrete 'ceiling' strengths evident although the rate of increase in strength with decrease in water/binder ratio is less with the gravel aggregate mixes. Similar results were given by the 15% SF mixes.As the rates of hardening of UK Portland cements have been changing over some decades (14) so the ratios of 28 to 7 day strengths have reflected that. The latter, here, vary from about 1.43 to 1.25, 1.41 being a value for typical structural concrete quoted in 1993 (15).

The mortar is close to an apparent ceiling strength, which should be reflected in E values. Consequently, improvement in the composition of the mortar would be necessary to effect further increase in the E of the concrete.

Excellent correlation was seen between cube strengths f_{c10} and f_{c15} of mixes with 10 and 15 percent SF respectively:

TABLE 3

Standard, control, cube strengths (MPa) with 10% SF

Mix	Crushed limestone				Gravel		Mortar	
	10mm		20mm		10mm			
	7*	28*	7	28	7	28	7	28
1	86	116	84	114	78	99	74	93
2	68	91	72	96	68	88	70	91
3	53	76	58	81	48	72	52	75

*7 and 28 days

$f_{c15} = 0.98\, f_{c10} + 4.14$ (r = 0.997, n = 18 pairs of concrete cubes)

$f_{c15} = 0.95\, f_{c10} + 6.73$ (r = 0.991, n = 24 pairs of concrete and mortar cubes)

Thus cubes with 15 percent SF are likely to be about 2 to 3 MPa stronger.

Modulus of Elasticity

Results of dynamic E tests are given in Table 4, for water cured prisms, with 10% SF; similar values were found with 15% SF.

For the pairs of prisms tested on each occasion, standard deviations were calculated; there was no observable trend between these and mean values of E and they were taken to be independent. No

TABLE 4

Moduli of elasticity (E) of water cured prisms

Material	Mix	E (GPa)			
		10% SF			
		7*	28*	56*	L**
10mm CL	1	51.0	53.1	53.4	53.8
	2	46.6	49.5	50.0	51.0
	3	44.5	47.4	48.3	48.5
20mm CL	1	53.3	55.6	55.9	57.6
	2	51.0	53.5	53.6	54.3
	3	48.3	50.7	51.2	51.6
10mm G	1	49.6	52.1	52.7	53.0
	2	48.8	51.3	51.6	51.7
	3	44.0	46.8	47.6	48.0
Mortar	1	39.9	42.2	42.7	43.7
	2	38.0	40.8	41.3	41.8
	3	34.4	37.2	37.6	37.6

*Age in days; **Later ages, from 21 to 27 weeks.

significant effects on variability, due to the different SF quantities, was found so the overall values, S_T, of standard deviations were obtained by pooling the results of the two sets. The following S_T (GPa) values were calculated:

Curing A - $S_T = 0.367$ (47 pairs)
Curing B - $S_T = 0.669$ (48 pairs)
Curing C - $S_T = 0.589$ (48 pairs)

Each of these represents the component of variability of these results due to the overall testing procedure and is a good measure of their reliability. Thus, for water cured prisms, individual results could reasonably be expected to be within the range $\overline{E} \pm (1.96/\sqrt{2})0.367 = \overline{E} \pm 0.51$ (where $\overline{E}$ is the mean value of a pair).

The S_T values for water cured prisms are significantly less variable than for sealed and air dried ones, no significant difference being apparent between the latter two sets. Coefficients of variation range from about 0.7 to about 1.7 percent, overall. Clearly this confirms the expected low variability of the technique and validates the methodology used here.

Comparison between E values (E_{10} and E_{15}) with 10 and 15 percent SF respectively shows near identity:

$E_{15} = 0.94\ E_{10} + 2.59$ (r = 0.96, n = 35 pairs of concrete prisms)

$E_{15} = 0.91\ E_{10} + 3.89$ (r = 0.98, n = 47 pairs of concrete and mortar prisms)

The difference in the amount of SF used therefore appears to have no significant effect on E and the values can be pooled.

Examination of Table 4 shows E increasing with age, for most mixes, but with many approaching apparent ceiling values. Despite occasional, apparently coincidental, similarities between results from different sources (see, for example ref. 16 which has data quite similar to those for mix 2 in the present work) the general position regarding relative increases in E from 28 days onward can be observed from published work from the early 1990s (17, 18) to be specific to the materials and tests used. Of course this is not a new phenomenon as shown from data of about 40 years ago (19,20), where material - specific results are clearly evident. There is the added complication in modern concretes of the presence of SF and SP.

The older data show E values of 48 to 52 GPa (19) even though strengths were only 58 to 65 MPa. It has been much easier to increase strengths (roughly doubled) than to enhance E values, a point very worthy of attention. Present generation concretes do not have the same reserves of strength or E (for example, see ref. 21, working from the same source as ref. 20).

Effect of Maximum Aggregate Size on E

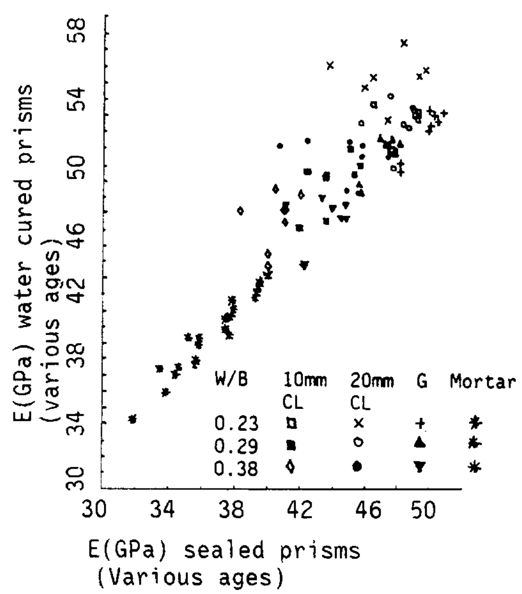
Comparing E of the 10mm crushed limestone concretes E_{10} with that of the 20mm ones E_{20} (for both 10 and 15 percent SF) shows $E_{10} = 1.17\ E_{20} - 11.97$, with r = 0.97, n = 23 pairs i.e. the latter is typically 5 to 10 percent greater than the former. This may be the result of the change in maximum particle size; a crude calculation indicates the likelihood of about four times as many 10mm particles as 20mm ones in the cross-section of the prism, which probably influence the passage of the sound wave (and hence its resonant frequency) through the prism. It should also influence ultrasonic pulse velocity. (See later)

The correlation between the results of the 10mm crushed limestone E_{10CL} and 10mm gravel E_{10G} mixes is exemplified by $E_{10CL} = 0.94\ E_{10G} + 2.70$, $r = 0.90$, $n = 23$ pairs, which shows no significant difference between them for E of 45 to 55 GPa. It is not surprising that the correlation here is not quite so good since the two crushed limestone materials are derived from the same parent rock and should have very similar E values of the particles, whereas that of the gravel particles is likely to be different. It seems that the maximum particle size, rather than type, of aggregate - relative to the prism size - has a significant effect on E for these water cured prisms.

A further check is of the air-dry prisms since drying is likely to accentuate cracking and reflect differences between concretes of different maximum particle size. The correlation between E_{20} (for the 20mm crushed limestone concrete) and E_{10} (for the 10mm one) is $E_{20} = 0.53\ E_{10} + 22.22$, with $r = 0.65$ and $n = 24$ pairs, showing considerable scatter; the 95 percent confidence limits are ± 4.4 GPa. It may be that the much greater variability is related to the inclusion of the 20mm particles.

Sealed Prisms

With such high cement contents and low water contents, together with the presence of SF, self-desiccation is to be expected. Figure 1(a) shows that even at 7 days (i.e. with the lowest E values) there seems to be a significant reduction in the sealed prism E, followed by a trend with an increasing difference between E of the water cured and sealed prisms, but with a good deal of scatter. Figure 1(b) shows a good correlation between water cured and sealed E values at 7 days, but with the former being about 2 to 3 GPa greater. But in all cases, for the latter, there is a significant increase in E from 7 to 28 days and therefore it seems that the reduction at 7 days (compared to the water cured values) is due to (a) some loss of moisture immediately after demoulding, before prisms were sealed and/or (b) some moisture loss due to inefficient sealing (and some increased hydration in the water-cured prisms). Insufficient data do not allow confirmation of the cause but these results emphasise the need for considerable care in handling and testing such specimens.



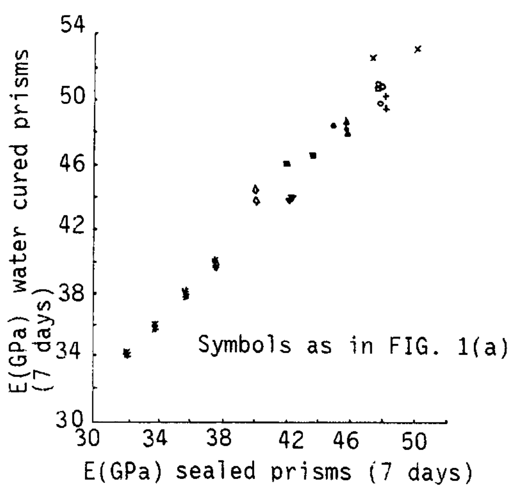


FIG. 1(a) FIG. 1(b)

Comparison between E of water cured and sealed prisms.

The general ageing trend of the sealed concrete prism E values is to show a maximum at, roughly, 28 days, with little increase to 56 days and a significant reduction at later ages. The mortars appear to approach about 22 weeks before reduction occurs, but with little increase during the intermediate period. Microcracking, from desiccation, may be greater as particle size increases and could explain the delayed reduction in the mortar prisms.

Air-dry Prisms

Some effects of air-drying on E from prisms with 10% SF are given in Table 5 which shows E at 7 days and at 150 to 180 days as percentages of the corresponding water cured prisms. The values for 28 days and 56 days are not included nor are those with 15% SF as they are not qualitatively different.

TABLE 5

Relative decrease in E of air-dried prisms, with 10% SF, compared to water cured ones : $(E_{air\text{-}dry}/E_{wet})\%$

Mix	Crushed limestone				Gravel		Mortar	
	10mm		20mm		10mm			
	7 days	21-27 weeks	7 days	21-27 weeks	7 days	21-27 weeks	7 days	21-27 weeks
1	90.2	82.9	92.1	82.3	94.2	92.5	90.7	85.1
2	90.3	85.5	89.6	81.8	92.2	90.7	90.8	86.8
3	85.4	79.2	90.1	78.6	90.9	85.2	87.2	79.0

In all cases there is a significant reduction in E due to air-drying. There is, generally, a noticeably bigger reduction at later ages and, overall, the prisms made with the 10mm gravel seem to be least affected. It is not clear why this is so, especially compared to the mortar prisms, but perhaps it is related to water loss on drying, which is likely to occur at a greater rate with the latter.

There is not a significant effect on E values due to changing from 10 to 15 percent SF.

E and Compressive Strength

The data relating E to cube strength, in Figure 2, could be approximated by a linear correlation, but this is hardly realistic because of the limiting value of the former, which, for most materials appears to be about 60 GPa (1). It is apparent that there is a separate relation for each type of concrete, that for the mortar, as expected, being much below the others. Even if, for a given strength, E_{20} is higher than E of the other concretes (as noted before) the simple expression $E = 15.85f_c^{0.26}$ (where f_c is the cube strength), a form often used, gives a good empirical overall approximation for the present data, the measured values being closer than about 4% to those predicted.

Ultrasonic Pulse Velocity

Ultrasonic pulse velocity (UPV) values are, of course, useful for checking homogeneity of concrete; here they are primarily used in connexion with E values. There seems to be little effect on UPV due to the different amounts of SF incorporated and both sets have been pooled for subsequent discussion.

As expected, there are clear correlations between UPV and E for each set of results, as seen in Figure 3. For a given E, UPV is highest when 20mm aggregate is used; it is higher for the 10mm crushed

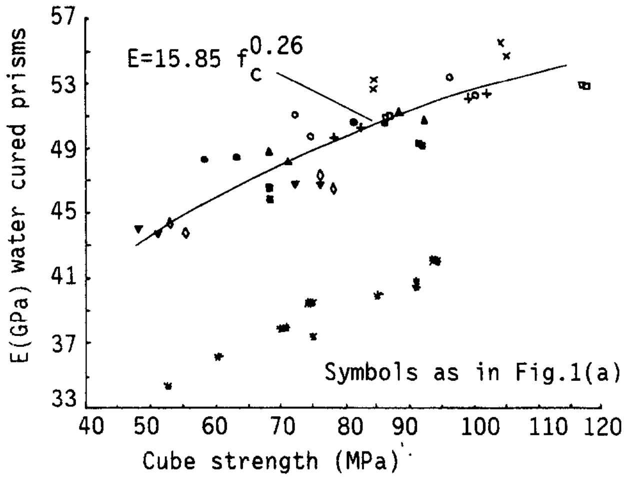


FIG. 2 Relation between E of water cured prisms and standard cube strength.

limestone concrete than for the 10mm gravel material and there seems to be a single relation for mortar and gravel prisms. As the E_a (the E of the coarse aggregate) is greater for the limestone than for the gravel (see later) it is not surprising that E of the concrete reflects that. But there is a difference between the crushed limestone concretes despite E_a being nominally the same for both. It may well be the case that, despite the presence of the SF, the structure of the aggregate/mortar interface is not as dense as that of the bulk mortar and thus the greater the number of interfaces that lie in the path of the ultrasonic wave the more it is slowed. This may account for the reduced UPV in the 10mm aggregate prisms. The similarity of E_a of the sand particles to that of the gravel would contribute to the similarity of correlations between UPV and E for the gravel and mortar mixes.

It is postulated later (See 'Prediction Models') that E of concrete is related to $E_a^{V_a}$ (V_a being the coarse aggregate volume fraction which in this case is equal to 0.425) and therefore it may be reasonable to modify E values of the 20mm and 10mm crushed limestone mixes by the ratio $E_{aG}^{0.425} / E_{aCL}^{0.425} \approx 0.97$, where the suffixes refer to gravel and crushed limestone respectively. This

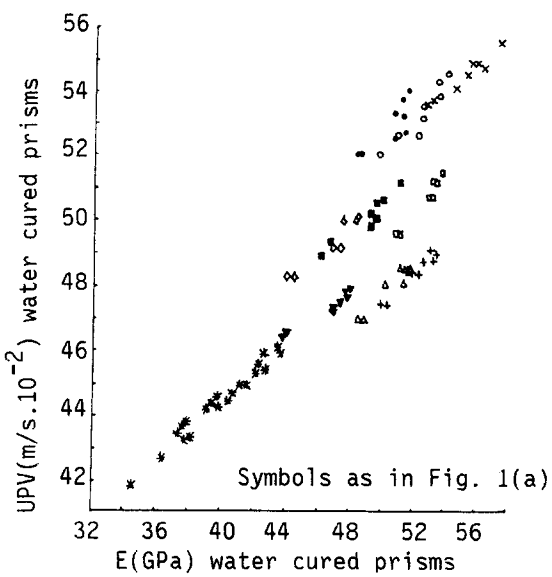


FIG. 3 Relation between UPV and E of water cured prisms.

naturally reduces the range of UPV values for a given E of concrete and lessens the dependency of the UPV/E relation in Figure 3 on each individual set of mixes.

The density of the concrete is, of course, significantly higher than that of the mortar, largely because of the higher water content of the latter. Since the UPV, denoted by V, is related to E of concrete and ρ_c its density, by a relation of the form $V = k(E/\rho_c)^{1/2}$ - where k is a constant dependent upon Poisson's ratio of the concrete - a plot of $V^2\rho_c$ against E, shown in Figure 4, (where V of the limestone mixes is modified by multiplying by 0.97), shows some degree of unification. Improvement could be effected by more accurate measurement of UPV and ρ_c. Lack of fit of the 20mm aggregate data, still evident, is presumably related to the interfacial effects previously discussed.

In passing, it may be noted that the correlations between UPV and E, for sealed and for air-dry prisms, were very similar to those for the water cured ones, except for the 20mm aggregate mixes where, for a given UPV, there was a significantly reduced E, compared to that of the latter.

Prediction Models for E

It is always best to measure E of concrete, where it is important, but several well known models have been suggested for predicting E, as noted, for example, in Ref. (12). They are helpful for estimating approximate, but often realistic, values and also for assessing the likely magnitude of changes due to changes in mix composition. On the whole, complicated models are not much better than simple ones which can be very effective. One of the simplest is based on a model proposed for strength (22) which has been shown to be useful and easy to apply (12). It is referred to here as the BNC model (after its original proposers). It assumes that E of the concrete is dependent on E_m, E_a and V_a, where the suffixes refer to mortar and coarse aggregate respectively; it implies adequate operative bond between coarse aggregate and mortar. Thus $E = E_a^{V_a} E_m^{1-V_a}$.

In all of the concretes used here V_a is 0.425 and $1-V_a$ is 0.575. Because of the problems associated with measurement of E_a a simple alternative is to derive it from the relation established between E_a and ρ_a, the particle relative density (23), $E_a = 8.1\rho_a^2$ (GPa). This gives $E_{aCL} \approx 58$ and $E_{aG} \approx 54.5$ GPa. Thus E can be readily calculated from $E = 58^{0.425} E_m^{0.575}$ for each mix and similarly for the gravel mixes.

The correlation between predicted (E_{pr}) and measured (E_{me}) E values is

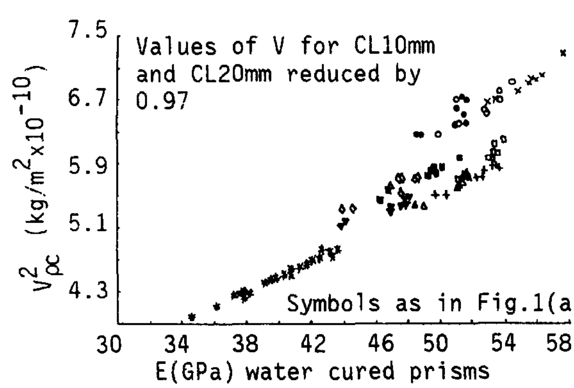


FIG. 4 Relation between $V^2\rho_c$ and E of water cured prisms.

$E_{pr} = 0.45\ E_{me} - 23.53$, r = 0.80, n = 71 pairs

with 95 percent confidence limits of ± 2.1 GPa.

The Hashin model (24) adapted for concrete, often seen as among the best, gives predicted values very similar to those from the BNC model . Predictions, here, are generally less than measured values, above about 45 GPa. Other predictions (10,26) relate concrete density, ρ_c and strength f_c to E; the expression $E = \rho_c^2\, f_c^{1/2} x 10^{-6}$ GPa predicts reasonably here ($E_{pr} = 2.0\ E_{me} - 45.57$, r = 0.91, n = 36 pairs) but overestimates above E of about 50 GPa.

Conclusions

1. Individual (electrodynamic) E values of HSC, from pairs of nominally identical water cured prisms should lie within about ± 0.5 GPa of the mean value. Typical coefficients of variation should be about 0.7 to 1.0%; for sealed prisms and for air-dried ones (with no water curing) they are likely to be greater, being 1.1 to 1.6% in this study.

2. With standard 28-day cube strengths of about 75 to 115 MPa, E values of concrete ranged from about 47 to 55 kN/mm^2 at the same age. Increase in E from 28 days to about 6 months depended on mix composition but varied from about 0.5 to 3.5 percent, the largest increase being with limestone aggregate concretes.

3. Maximum E values of sealed prisms were reached by about 56 days; after about 6 months they tended to be similar to or less than those at 7 days. This may be due to partial drying.

4. E values of air-dried prisms compared to those of the companion water-cured ones ranged from about 0.87 to 0.94 at 7 days and 0.76 to 0.92 by about six months, depending on aggregate type and mix composition.

5. There were no practical problems in making 76 x 76 x 254 mm prisms with 20mm maximum size aggregate but E and UPV were not the same as those measured in corresponding prisms made with 10mm maximum size from the same source. (In this work the former resulted in values of E about 5 to 10 percent greater than with the latter, for example).

6. The increase of SF from 10 to 15 percent caused a 2 to 3 MPa rise in cube strength but had no significant effect on E. E could be reasonably estimated from an empirical expression of the form $E = af_c^b$ where f_c was the cube strength and a, b were constants which, for these materials, were 15.85 and 0.26 respectively.

7. For E values up to 50 GPa the BNC model gave reasonable predictions but then became increasingly conservative. Likewise the relation $E = \rho_c^2\, f_c^{1/2} x 10^{-6}$ worked reasonably up to 50 GPa but overestimated, increasingly, above that.

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