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ON THE RELATIONSHIP BETWEEN POROSITY AND ELECTRICAL RESISTIVITY IN CEMENTITIOUS SYSTEMS

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ABSTRACT

The applicability of porosity-electrical resistivity relationships for hardened cementitious systems is addressed. The following equation

$$\ln F = \ln \mu + m \cdot \ln \varepsilon = \ln \left(\frac{\tau^2}{\delta} \right) + m \cdot \ln \varepsilon$$

relating the electrical resistivity formation factor (F) to the porosity (ε), tortuosity (τ) and constrictivity (δ) was tested for Portland cement pastes hydrated up to 29 years and mortar hydrated for 28 days. Results for the well-hydrated pastes and, to a lesser degree, 28 day hydrated mortars followed the relationship.

Introduction

The formation factor, F, for micro-porous Portland cement paste systems with solid phases having negligible electrical conductivity is defined as,

$$F = \frac{\rho_{system}}{\rho_{porewater}} = \frac{\sigma_{porewater}}{\sigma_{system}} = \frac{D_{porewater}}{D_{system}} \tag{1}$$

where, ρ is electrical resistivity, σ is electrical conductivity, and D is diffusivity. F is uniquely determined by both the porosity, ε, and pore structure of the cementitious solid when surface conduction is insignificant. Pore structure includes the convergent/divergent nature or constrictivity and the orientation/topography or tortuosity of the interconnecting capillary network. The pore structure variables can be combined into a matrix factor, μ, defined as (1),

$$\mu = \frac{\tau^2}{\delta} \tag{2}$$

where, τ is tortuosity, and δ is constrictivity. F is proportional to both μ and ε (2),

$$F + \mu \cdot \varepsilon^m = \left(\frac{\tau^2}{\delta}\right) \cdot \varepsilon^m \tag{3a}$$

or,

$$\ln F = \ln \mu + m \cdot \ln \varepsilon = \ln\left(\frac{\tau^2}{\delta}\right) + m \cdot \ln \varepsilon \tag{3b}$$

where m is a constant. Eq.[3] represents an expanded version of Archie's law (3, 4),

$$F = A \cdot \varepsilon^{-m} \tag{4}$$

where A usually equals one. The appropriateness of relationships based on Archie's law has been challenged for sedimentary rocks (5) and concrete (6) as total porosity is considered an inadequate descriptor of the dependence of conductivity on pore structure. However, a recent fractal interpretation, in which the conducting pore system is a subset of the total pore space, confirmed the validity of Archie's law for a variety of rocks (7). There are few reported experimental data which confirm or disprove Archie's law for cementitious systems. Therefore, the purpose of this investigation was to determine F and ϵ and test the validity of Eq. [3] for cement pastes and mortars.

Experimental

Paste Systems. Paste specimens were prepared with Type 10 cement as described previously (8). The w/c ratios and hydration times are indicated in Table 1. Specimens were hydrated in

TABLE 1
Hydration Times of Cement Pastes.

w/c Ratio	Hydration (*Years*)
0.25	17.5
0.30	17.5
0.40	27.8
0.50	29.0
0.80	27.1
1.00	27.1

TABLE 2
Mortar Compositions.

Mix	Mass % Sand
A	0
B	0.005
C	0.05
D	0.5
E	1.0
F	2.5
F	5.0
H	10
I	15
J	25
K	35
L	45

TABLE 3

Experimental Results for Paste

w/c	ρ_{paste} ($\Omega \cdot cm$)	$\rho_{porewater}$ ($\Omega \cdot cm$)	F	ε (%) of Paste
0.25	18429	19.1	965	10.4
0.30	6922.3	25.7	269	11.9
0.40	1532.2	31.2	49.1	26.1
0.50	1316.2	31.8	41.4	31.3
0.80	441.7	46.3	9.5	57.5
1.00	369.0	55.5	6.7	57.5

sealed rubber membranes containing a small amount of saturated lime water for periods up to 29 years. Specimens used for resistivity measurements were 3.2 *cm* (OD) x 2.5 *cm* thick paste cylinders. Stainless steel electrodes were pressed against the faces. Paste resistivity was determined using the procedure and apparatus described previously (9). Then, the specimens were crushed. A portion of the crushed specimens was used to determine the total porosity by mercury intrusion porosimetry (10). The porewater was extruded from the remaining portion of the crushed specimens. The resistivity of the porewater was measured with a calibrated resistivity probe.

Mortar Systems. Type 10 Portland cement mortars were prepared with ASTM C109 Ottawa sand which passed #50 mesh and was retained on #100 mesh. All mortars were prepared at w/c=0.35 . The compositions are given in Table 2. Mortars were cast into Plexiglas molds (2.54 *cm* x 2.54 *cm* x 2.54 *cm*) with stainless steel electrodes on opposite faces for the mortar resistivity measurements. The mortars were cured for 28 days at 100% *rh*. Subsequently, the mortar resistivities, mortar porosities and porewater resistivities were measured in a manner similar to that for the paste specimens.

TABLE 4

Estimated Results for Mortar

Wt. % Sand	ρ_{mortar} ($\Omega \cdot cm$)	$\rho_{porewater}$ ($\Omega \cdot cm$)	F	ε (%) of Mortar
0	1428.5	20.7	69.0	15.3
0.005	1465.8	21.8	67.2	14.2
0.05	1465.8	22.3	65.7	15.0
0.5	1473.2	20.5	71.9	14.4
1.0	1388.6	21.0	66.2	14.8
2.5	1296.4	25.1	51.7	14.4
5.0	1283.5	23.6	54.4	15.0
10	1390.1	24.4	56.9	14.4
15	1486.4	22.6	65.8	13.8
25	1595.6	22.2	71.9	13.2
35	2186.7	22.2	98.5	11.7
45	2499.4	21.4	117	11.7

Results and Discussion

Values of the resistivities, formation factors and porosities are summarized in Tables 3 and 4 for the paste and mortar systems, respectively. Porosity, ε, of the pastes increases as the w/c ratio increases. Conversely, F decreases towards its theoretical lower limit of one. Porosity, ε, decreases and F increases for the mortars as the cement content is increasingly replaced by sand. The experimental data have been plotted according to Eq. [3b] in Figures 1 and 2, for the pastes and mortars, respectively. It can be seen in Figure 1 that for the extremely well-hydrated pastes, there is good confirmation of Eq. [3b]. The agreement with Eq. [3b] for the mortars of Figure 2 which have been hydrated for only 28 days is less definitive but it is within the uncertainty encountered in cementitious systems. Values of μ calculated from the indicated least squares lines in Figures 1 and 2 were 1.89 and 1.02 for pastes and mortars, respectively. When μ=1, there is an exact adherence to the classic Archie's law given in Eq.[4] for A=1. Eq.[4] is the limiting case of the more generalized form of Archie's law given by Eq.[3b] when $\mu \rightarrow 1$. The slopes in Figures 1 and 2 are 2.55 and 2.14 for paste and mortar, respectively. The values are approximately mid-range of the known values for rocks (5, 7).

Christensen and co-workers reported normalized conductivity-porosity data for white cement paste, w/c = 0.40, hydrated for relatively short periods of time (11). The data are presented in Figure 3. The agreement of Eq.[4] was good at high porosities for an exponent of $m \approx 1.5$. Adjustment of the exponent value to $m \approx 2.5$ would provide reasonable conformance with Archie's law at lower porosity values. A more precise fit of the data over the whole porosity spectrum is provided by Eq.[3b] with μ = 0.14 and m = 4.8 . Therefore, Christensen's data is compatible with Eq.[3b] provided μ and m are suitably selected.

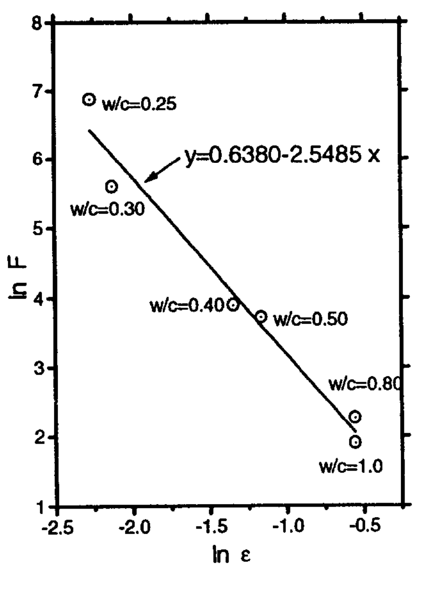

FIG. 1.
Eq. (3b) for paste.

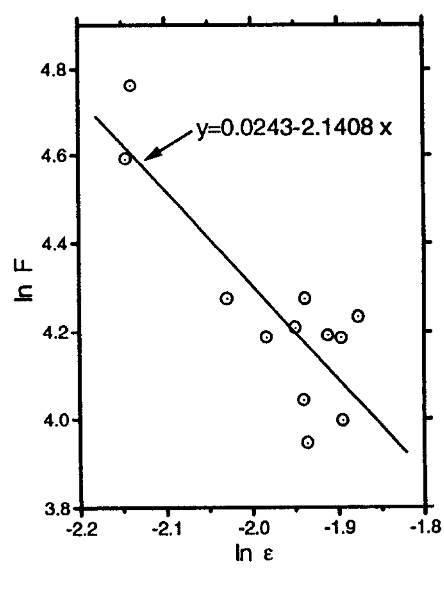

FIG. 2.
Eq.(3b) for mortar.

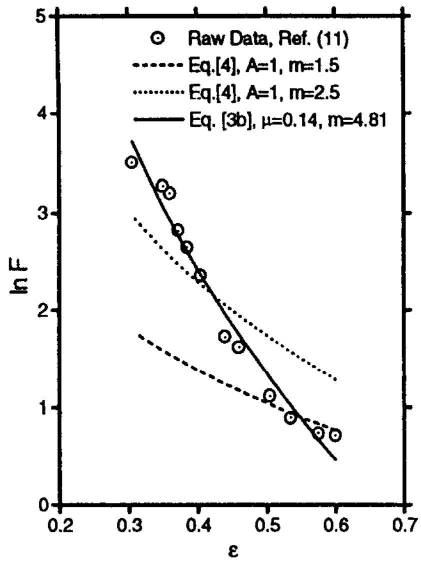

FIG. 3.
Eq. (4) with m = 1.5/2.5 and eq. (3b) with μ = 0.14 and m = 4.81 for Ref. (11) data.

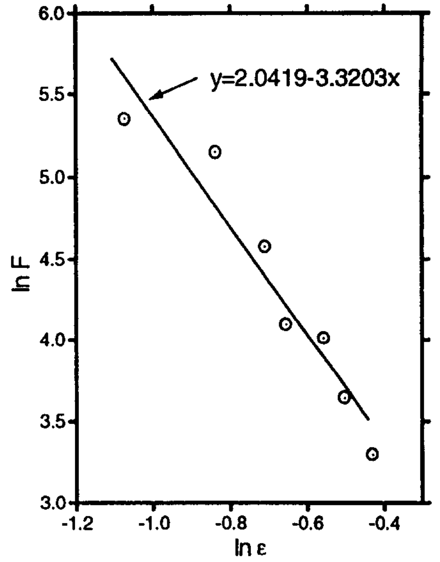

FIG. 4.
Eq. (3b) plotted for Ref. (12) data.

The experimental data of Taffinder and Batchelor (12) for neat pastes hydrated for 90 days are presented in Figure 4. The experimental data are well represented by the indicated least squares lines for Eq.[3b] with $\mu = 7.71$ and m = 3.32 .

Conclusions

Archie's law in the form of Eq. [3b] appears to be valid for extremely well-hydrated paste systems. Archie's law also appears to be followed for mortars hydrated for 28 days. The relationship is less certain than for the paste systems.

The combination of Eq.[1] and [3b] allows the quick determination of diffusivities which are increasingly important for service-life models based on the ingress of aggressive ions.

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