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GENERALIZATION OF ABRAMS' LAW

T.S. Nagaraj* and Zahida Banu**
*Professor, **Research Scholar
Department of Civil Engineering
Indian Institute of Science Bangalore, 560 012 INDIA

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ABSTRACT

Considering cement based composites as chemically bonded ceramics (CBC) the consequent strength development with age is essentially a constant volume solidification process, such that the hydrated gel particles fill the space resulting in the compatible gel space ratios. Analysis has been done of the extensively used graphical method of mix design (British method of mix design) i.e., the relation between the compressive strength and the free water - cement ratio. By considering the strength (S) at w/c 0.5 ($S_{0.5}$) as the reference state to reflect the synergetic effects between constituents of concrete a generalized relationship obtained is of the form $\{S/S_{0.5}\} = a + b\ \{1/(w/c)\}$.

Introduction

Continued research efforts and ingenuity have established concrete as a construction material which conforms to new concepts in design with the possibility of realizing the same through advanced processing technology. Cement based composites have been regarded as chemically bonded ceramics in contrast to the thermally bonded ceramics (1). The solidification of cement paste is a constant-volume process. When high density cement particles are mixed with low density water, they react to form a solid hydration product consisting of solids of intermediate density and interspersed residual porosity. According to Idorn (2) the strength of hardened cement paste is influenced by various factors. Before 1970, the potential strength of cement paste at theoretical density, has never been approached, because considerable porosity (20 to 30% or more total porosity) always remained after complete hydration of the cement (3). Since 1970 cement matrix composites have been developed. As such strengths of an order of magnitude higher than those of normal cement pastes were achieved by warm pressing different types of cements, such as portland varieties, calcium aluminate cements, and individual cement compounds; cement paste compressive strengths upto 650 MPa were obtained, compared to the typical 30 MPa (4).

In engineering practice, the strength of a fully compacted concrete processed with a specific type of aggregates at a given age cured at a prescribed temperature, is assumed to depend primarily on the water-cement ratio. But, the compressive strength of a concrete at a water-cement ratio also depends on the fineness and chemical composition of cements. Hence concern has been expressed about the effects of these cements on the strength of concrete at similar water-cement ratios and vice-versa. Generalization of Abrams' law is no exception to this (5-9). In practical concrete technology, it is inevitable that concrete is to be processed in its green or fresh state such that in its in-situ state strength development takes place with age. In proportioning of concrete mixes the first step is to arrive at water cement ratio which primarily dictates the desired strength development.

In this paper, an attempt is made to generalize Abrams' law. The well known logical rational basis for this generalization is to consider water-cement ratio as an inverse function of the compressive strength. It is intended to enhance the applicability of this law by the proposed generalization for practical applications by covering high strength range thus making concrete mix proportioning simpler and faster.

Abrams' Law - Present Generalization

Critical examination of the extensive published data (10) reveals that as the free water cement ratio increases compressive strength decreases such that strength can be regarded as an inverse function of water-cement ratio. Besides the water-cement ratio, the characteristics of the coarse

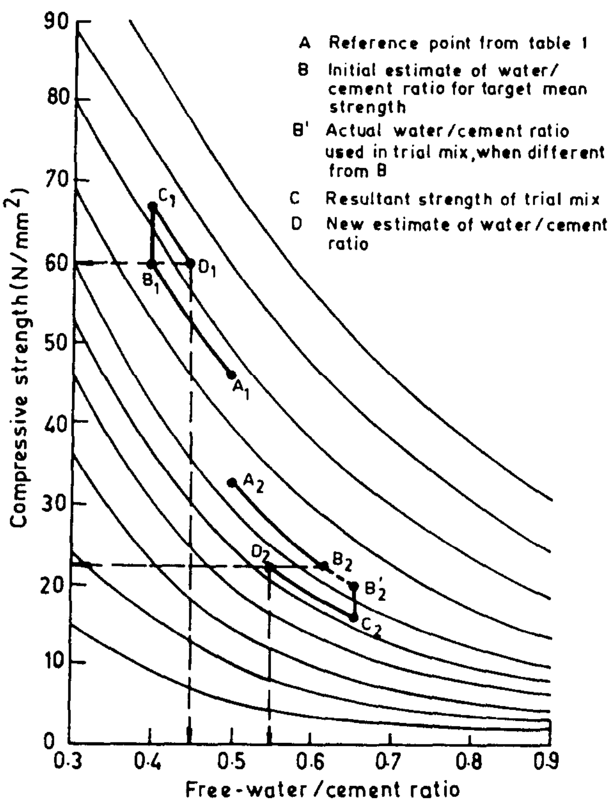

FIG. 1.
Relationship between compressive strength and free water-cement ratio with graphical procedu:e (Ref 10).

TABLE 1
Approximate Compressive Strengths (N/mm^2) of Concrete with a Free Water-Cement Ratio of 0.5 (Ref 10)

Type of cement	Type of coarsec aggregate	Compressive strengths (N/mm^2) Age (days)			
		3	7	28	91
Ordinary Portland (OPC) or Sulphate-Resisting Portland (SRPC)	Uncrushed	18	27	40	48
	Crushed	23	23	47	55
Rapid-Hardening Portland (RHPC)	Uncrushed	25	34	46	53
	Crushed	30	40	53	60

aggregates also affect the properties of concrete. The characteristics of the aggregate particles that affect the properties of concrete are strength, particle shape, size, gradation and surface texture. Taking all the factors into consideration a set of curves were arrived at (page 9 of reference 10). The British method of mix design has specified a graphical procedure to arrive at the appropriate water-cement ratio for the required compressive strength by making minor adjustments. But if a series of strengths are expected from a set of concrete ingredients to avoid the delay, the method suggests to have a number of trial mixes.

In the present investigation, all the ten curves in Fig.1 have been transformed into linear plots by regression analysis with high correlation coefficients 'r' (Fig.2). The possibility of obtaining a single functional relationship between compressive strength and inverse of free water - cement ratio has been examined by considering the compressive strength at water - cement ratio of 0.5 i.e., $s_{0.5}$ as the normalization parameter.

Normally the water- cement ratios in usage are in the range of 0.3-0.7 with 0.5 being the intermediate value. The proposed normalization of concrete strength which is a non-particulate

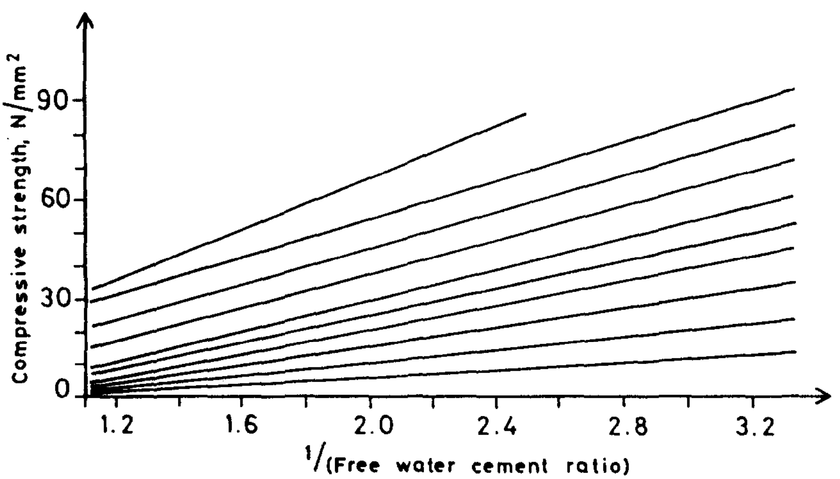

FIG. 2.
Transformed linear plots of the ten curves.

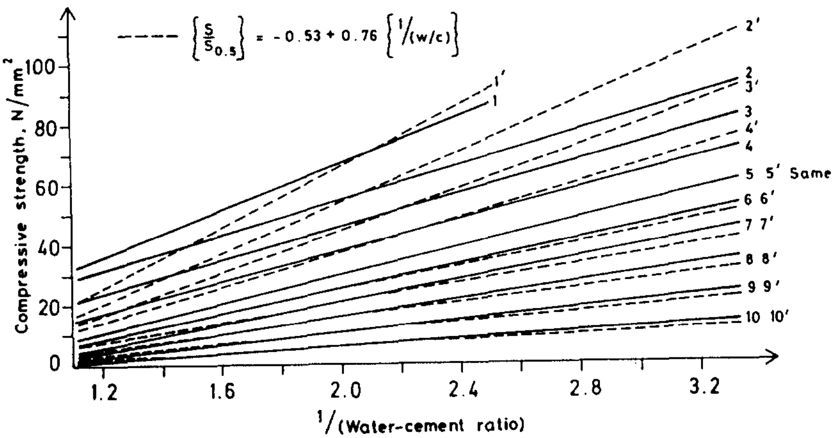

FIG. 3.
Linearized paths with the superimposition of paths predicted from the single generalized relationship.

behaviour is a reflection of synergetic effects between compatible cement mortar strength and its bond with coarse aggregate dictated by its size, shape and surface characteristics. Water-cement ratio in the fresh concrete in the preset condition is a material state parameter. Upon setting, it is directly responsible for the microstructure of hardened mortar and hence for its strength for subsequent synergetic effects.

Since water-cement ratio is generic parameter for concrete strength development with age it is an appropriate parameter for normalization directly or indirectly through the strength it can develop upon hardening. Further by normalization of the entire data by their corresponding strength values at water-cement ratio of 0.5 (i.e. $S_{0.5}$), the following relationship was obtained.

$$S/S_{0.5} = -0.53 + 0.76\,(1/w/c) \quad r = 0.98$$

On checking for accuracy, when the lines obtained using this equation, were superimposed on the published set of lines i.e.,broken lines in Fig.3, it was noticed that there was a shift in the top four lines indicating the possible deviations in the level of predictions when the generalized relation for the entire set is used for practical application. This predicament is attributed to the fact, that as mentioned earlier, the ten set of curves have been arrived at by extensive experimental investigations involving many variable parameters and hence lack an unique equation for the entire range.

Though in principle, there should be an unique equation, but for sake of usage with maximum accuracy, the ten lines were divided into two sets of the top four and the bottom six. Adopting the same earlier procedure, the following two equations were obtained.

$$S/S_{0.5} = -0.2 + 0.6\,(1/w/c) \quad r = 0.995 \quad \text{For } S_{0.5} > 30 \text{ MPa}$$

$$S/S_{0.5} = -0.73 + 0.865(1/w/c) \quad r = 0.997 \quad \text{For } S_{0.5} \leq 30 \text{ MPa}$$

Similarly the lines superimposed using the two equations match closely with the linearized published plots (Fig.4).

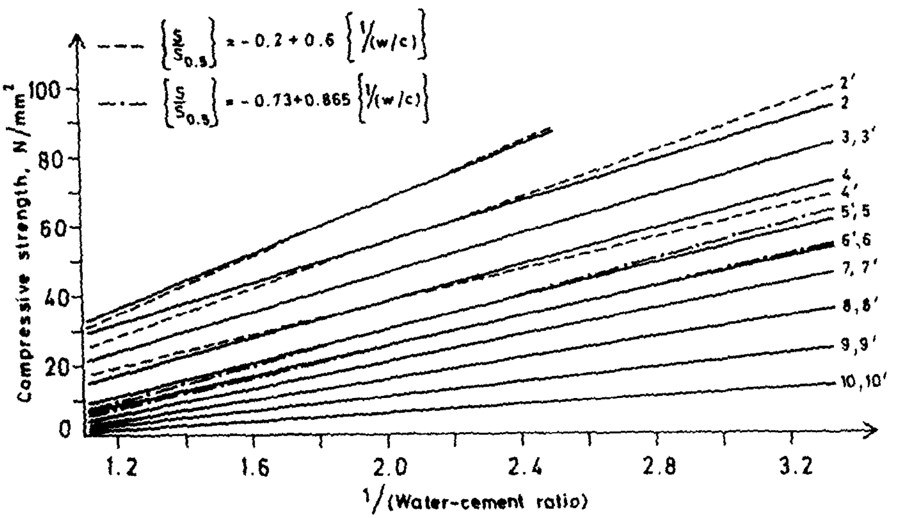

FIG. 4.
Linearized paths with the superimposition of predicted paths from the two generalized relationships.

It is interesting to observe that the estimated values of water-cement ratios cited by graphical procedures as examples in Fig.1 are exactly the same as those computed by the two equations. Finally, the same graphical procedure was tried out for lines, as a substitute for curves and the cited values are similar (Fig.5). This generalized approach is only applicable for proportioning concrete mixes, where the strength of the aggregate is higher than the matrix strength i.e., concrete failure without aggregate crushing.

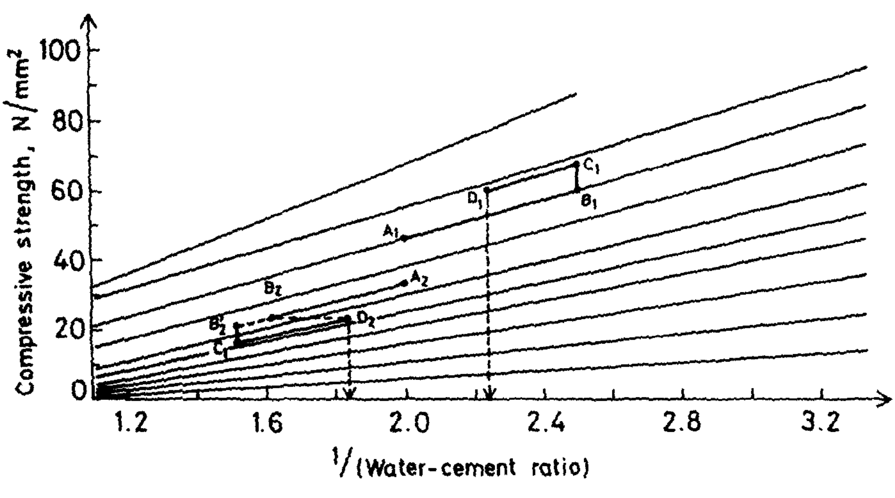

FIG. 5.
Graphical procedure for the linearized paths.

Abrams' Law

Proof for the two examples (Fig. 1)

1. Computation from Equation $S/S_{0.5} = -0.2 + 0.6\ (1/(w/c)$ For $S_{0.5} > 30$
   $S_{trial} = 67\ N/mm^2$ $(w/c)_{trial} = 0.4$
   $S_{0.5} = 51.54\ N/mm^2$
   so for S = 60 w/c = 0.44 as against 0.45
2. Computation from Equation $S/S_{0.5} = -0.73 + 0.865\ (1/(w/c))$ For $S_{0.5} \leq 30$
   $S_{trial} = 16\ N/mm^2$ $(w/c)_{trial} = 0.655$
   $S_{0.5} = 27.09\ N/mm^2$
   So for S = 22.5 w/c = 0.55 as against 0.55

Experimental Investigations and Discussions

Extensive experimental compressive strength data at different workability levels have been generated using the developed generalized Abrams' law. Table 2 gives the details of the materials used. Accordingly trial mix details at 7 and 28 days were generated for free water-cement ratio of 0.5 i.e., $S_{0.5}$ (Table 3). Based on these data, using the appropriate equations, concrete mixes were reproportioned for different strengths and workability levels (Tables 4 and 5). It is interesting to observe the close agreement between the targeted and obtained values for all these levels of workability.

TABLE 2
Details of the Materials Used

Coarse aggregate : Crushed (25 mm and down)

Bulk density : 1605 kg/m^3
(S.S.d)
Specific gravity : 2.4
(S.S.d)

Fine aggregate : Natural sand

Fineness Modulus : 2.65
Specific gravity : 2.6

Ordinary portland cement

Specific gravity : 3.15
Normal consistency : 30%
Specific surface (cm^2/g) : 3320
Chemical composition

SiO_2	Al_2O_3	Fe_2O_3	CaO	MgO	K_2O	SO_3	Chloride
20.83	5.156	4.297	62.8	1.013	0.323	2.119	-

TABLE 3
Trial Mix Details

Water-Cement Ratio = 0.5 and Medium Workability

Water content	= 195 Kg/m^3
Cement	= 390 Kg/m^3
Coarse aggregate	= 1108 Kg/m^3
Fine aggregate	= 533 Kg/m^3
Compaction factor obtained	= 0.93

Compressive Strength (N/mm^2)

7 days	28.38
28 days	43.93

Equations used for computation

7 days: $S/S_{0.5} = -0.73 + 0.865\,(1/W/C)$

28 days: $S/S_{0.5} = -0.2 + 0.6\,(1/W/C)$

Note: All the Compressive Strength results are the mean of five cubes of size 15x15x15 cms.

It is very well known that, concrete being a composite material, the strength realized upon hardening depends upon the microstructure of hardened mortar in the interfacial zone, the surface characteristics of the aggregates, total surface area of the interfacial zone depending upon the size, gradation and quantity of coarse aggregate.

Further investigations were carried out to find out as to how pronounced is the increase in interfacial zone due to marked reductions in the size of the coarse aggregate. For a particular

TABLE 4
Reproportioning of Concrete Mixes for Compressive Strength.
Predicted and Obtained Compressive Strengths at
Different Water-Cement Ratios and Levels of Workability

Water-cement ratio	7 days strength (N/mm^2) predicted strength	obtained strength	percent variation	28 days strength (N/mm^2) predicted strength	obtained strength	percent variation	Remarks on workability
		16	- 5.8		31	- 3.12	low
0.65	17	17	nil	32	30	6.25	medium
		18	+ 5.8		32	nil	high
		24	- 7.6		38	- 7.3	low
0.53	26	26	nil	41	38	- 7.3	medium
		24	- 7.6		39	- 4.8	high

TABLE 5
Reproportioning of Concrete mixes for Compressive Strength. Predicted and Obtained Compressive Strengths at Different Water-Cement Ratios and Levels of Workability

Water-cement ratio	7 days strength (N/mm^2) predicted strength	obtained strength	percent variation	28 days strength (N/mm^2) predicted strength	obtained strength	percent variation	Remarks on workability
		34	- 5.5		51	- 3.7	low
0.43	36	39	+ 8	53	55	+ 3.7	medium
		38	- 5.5		55	+ 3.7	high
		47	- 4.0		69	+ 2.9	low
0.35	49	48	- 2.0	67	69	+ 2.9	medium
		47	- 4.0		71	+ 5.9	high

cement and water cement ratio of 0.5, three trial mix details were generated for the three different sizes of the same coarse aggregate independently (Tables 6 and 7). All three mixes exhibited medium workability, as against the low workability aimed at. It has been realized that, as the size reduces with a marked increase in the surface area, a higher cement content is required to arrive at the same workability. Similarly the effect of the increase in the interfacial zone is reflected in the reduction of the compressive strength. Finally, all the three mixes were

TABLE 6
Characteristics of the Materials Used

Aggregate type	Cube strength MPa	Specific gravity (saturated surfacedry)	Bulk density Kg/m^3 40-25 mm	25mm & down	12.5mm & down
			A	B	C
Crushed coarse aggregate	112	2.4	1633	1605	1505
Fine aggregate					
Natural sand Fineness modulus = 2.65	---	2.6			
Cement					
Ordinary portland cement		3.15			

TABLE 7
Proportioning of Trial Mixes for Constant Workability and Strength

Aggregate size	Water content Kg/m^3	Cement Kg/m^3	C.A Kg/m^3	F.A Kg/m^3	A/C	C.f	Compressive Strength (N/mm^2) 7 days	28 days
A	160	320	1209	584	5.60	0.92	23	39
B	185	370	1107	577	4.55	0.91	20	33
C	200	400	858	757	4.04	0.93	12	21

reproportioned for a common matrix strength (i.e. w/c = 0.35) and low workability (Table 8). The applicability of the two equations for different aggregate sizes has also been highlighted. It is interesting to observe the close agreement between the predicted and the observed values, based on the three independent $S_{0.5}$ values.

It has been stated recently (11) that 10 to 12 mm is the maximum size of aggregates preferable for making high strength concrete. A discussion of this statement is presented by Francois and Albert (11) supported by experimental data (for normal strength and high strength mixes) and is concluded that in the range of mixes that can be made with the used components - crushed limestone aggregates, portland cement, silica fume and superplasticiser, the classical theory still seems applicable: 20 to 25 mm maximum size aggregates lead to better performances and economy than smaller size aggregates. The present experimental investigations also contribute to the above concept.

Figures in brackets are the predicted strengths from the relations as indicated below:

$$S/S_{0.5} = -0.73 + 0.865\ (1/W/C) \quad \text{for } S_{0.5} \leq 30 \text{ Mpa}$$

$$S/S_{0.5} = -0.2 + 0.6\ (1/W/C) \quad \text{for } S_{0.5} > 30 \text{ MPa}$$

TABLE 8
Comparison of Compressive Strengths of Concrete
Proportioned for the Same Matrix Strength
(i.e. w/c = 0.35) and Same Low Workability (i.e. c.f. = 0.85)

Aggregate size	Water content Kg/m^3	Cement Kg/m^3	C.A Kg/m^3	F.A Kg/m^3	A/C	C.f	Compressive Strength (N/mm^2) 7 days	28 days
A	145	414	1209	545	4.23	0.84	41 (39)	56 (59)
B	170	486	1107	520	3.35	0.86	34 (35)	46 (50)
C	185	529	858	690	2.92	0.84	29 (22)	38 (37)

Concluding Remarks

The generalized approach suggested in this paper would enable one to take into account the synergetics of various ingredients of concrete in its hardened state. But for a given batch of cement and aggregates and placement conditions, further reproportioning concrete mixes reinforces the well known observation that the compressive strength of concrete is dominantly influenced by water - cement ratio. Using the appropriate relations:

$$S/S_{0.5} = -0.2 + 0.6\,(1/w/c) \quad \text{For } S_{0.5} > 30 \text{ MPa}$$

$$S/S_{0.5} = -0.73 + 0.865\,(1/w/c) \quad \text{For } S_{0.5} \leq 30 \text{ MPa}$$

for the compressive strength data at free water-cement ratio 0.5.e., $S_{0.5}$, obtained by the trial mix; it is possible to compute free water-cement ratio for any desired concrete strength.

Acknowledgment

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