



PII S0008-8846(96)00183-4

## RESIDUAL FATIGUE STRENGTH AND STIFFNESS OF ORDINARY CONCRETE UNDER BENDING

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(Received July 25, 1996; in final form October 22, 1996)

### ABSTRACT

On the basis of the fatigue equation that had been previously proposed by the authors, a new formula for predicting the residual fatigue strength of ordinary concrete was deduced. This formula was further verified and modified by the flexural tests on the concrete beam specimens and compared with other previous test data. Also, some residual fatigue stiffness properties of concrete such as Young's modulus, Poisson's ratio and shear modulus were investigated using non-destructive testing techniques. There was a good correlation between the stiffness properties and the residual fatigue strength. *Copyright ©*

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### Introduction

The failure of many concrete structures, such as concrete pavements, airport runways, bridges and offshore structures, etc., is mainly caused by the fatigue rupture of concrete. Some of these concrete structures were built several decades ago, so it becomes a very urgent task to predict the residual strength or the residual life of these existing structures.

The research on the fatigue properties of concrete started at the end of the last century. A lot of research has been done since then to investigate the effects of various factors including material composition, loading forms and environmental conditions on the fatigue of concrete [1-4]. Some efforts are also focused on the fatigue of high-performance concrete under more complex stress states [5-7]. However, very few papers on the prediction of the residual fatigue strength and other residual properties of concrete have been published.

In the past forty years, some experimental work has been done to investigate the effect of preloading history (static, sustained and cyclic) on the residual compressive strength of concrete [8-12]. Under pre-sustained compression up to 80%  $f_c'$  for up to 500 days, both the residual strength and the Young's modulus increase by as much as 15%. Even higher pre-sustained compression (90%  $f_c'$ ) only increases the residual strength in the early stage (<30% T, T is the failure time) but the strength reduces thereafter. Under pre-cyclic compres-

sion up to 90%  $f_c'$ , the residual fatigue strength could increase by up to 11% in the early stage ( $<30\%$  N, N is the fatigue life) but starts to decrease thereafter. In the final stage ( $>80\%$  N) the strength could decrease by up to 15%. This means that the consolidation and redistribution of stress concentrations only dominate in the early and middle stages but the propagation of microcracks dominates thereafter, especially at high stress levels.

Similar work has been done for pre-tension case [10,13-15]. Under pre-sustained tension, the residual tensile strength is unchanged in the early stage ( $<30\%$  T) but decreases very little thereafter. Pre-sustained bending could reduce the modulus of rupture by up to 10% [16]. Pre-cyclic tension only reduces the residual tensile strength in a very late stage, but such reduction starts in the middle stage under pre-cyclic tension-compression and could be as high as 60%. Young's modulus decreases throughout the whole loading process. Such a decrease is caused by the development of microcracks and other micro-defects within concrete.

Up to now the effect of pre-cyclic bending on the residual bending strength and stiffness has not been studied yet. Also the previous investigations have not quantitatively provided a formula for predicting the residual fatigue strength and stiffness of concrete and these results have their limitations in practical use.

Meanwhile, fatigue of concrete is a continuous damage process. The state of damage can be evaluated by using some stiffness damage parameters such as Young's modulus E, Poisson's ratio  $\nu$  and shear modulus G. These parameters can also be measured using non-destructive testing techniques. Actually non-destructive techniques have been used to investigate the cracking and degeneration within concrete in different loading stages [17-19].

In this paper, based on the fatigue equation proposed by the authors [1,2], a new formula for predicting the residual fatigue strength of ordinary concrete is proposed. This formula will be further verified and modified using flexural fatigue test results on the beam specimens and compared with previous test data. Also residual fatigue stiffness parameters of concrete under bending, E,  $\nu$  and G, will be non-destructively measured using both striking and resonance techniques. Relationships between the residual fatigue strength and these dynamic stiffness parameters will be established and can be used for the future prediction of the residual fatigue states of concrete.

### Fatigue Equation

In the previous studies [1,2], three aggregates (slag, gravel and ceramsite) were used to form heavy, ordinary and lightweight concrete to investigate the effects of some parameters (loading frequency, stress reversal, water-cement ratio, aggregate type, etc.) on the fatigue properties of concrete. Water-cement ratios  $w/c = 0.39, 0.45, 0.53$  and  $0.65$  were examined for the ordinary concrete,  $w/c = 0.45$  for the heavy concrete and  $w/c = 0.53$  for the lightweight concrete. For the ordinary concrete with  $w/c = 0.45$ , thirteen stress levels ( $S_{max}/f_c = 0.5$  to  $0.975$ ) with seven stress ratios ( $R = 0.5, 0.2, 0, -0.2, -0.5, -0.8$  and  $-1$ ) were used. For the rest, six stress levels ( $S_{max}/f_c = 0.95, 0.9, 0.85, 0.8, 0.75$  and  $0.7$  with  $R = 0.2$ ) were examined. Loading frequencies of 1 Hz for low fatigue life ( $N \leq 10^3$ ), 5 Hz for middle fatigue life ( $10^3 < N < 10^5$ ) and 20 Hz for high fatigue life ( $N \geq 10^5$ ) were applied. Over three hundred beams of  $500 \times 100 \times 100$  mm were tested under flexural static and cyclic loading in four-point bending form.

Based on these fatigue test results, a fatigue equation to predict the fatigue life of concrete considering the loading frequency (rate) effect and other factors was proposed as follows:

$$\frac{S_{\max}}{f_c} = C_f [1 - (1 - R')\beta \log N] \quad (1)$$

where  $f_c$  is the static strength;  $\beta$  is a material parameter = 0.0807 for the heavy and ordinary concrete and 0.0700 for the lightweight concrete.  $C_f$  is the loading frequency coefficient,  $C_f = ab^{-\log f} + c$ , where  $f$  is the loading frequency in Hz,  $a$ ,  $b$  and  $c$  are material parameters,  $a = 0.249$ ,  $b = 0.920$  and  $c = 0.796$ .  $R'$  is the specialised stress ratio,  $R' = (S_{\min}/f_{c\min})/(S_{\max}/f_{c\max})$ ,  $S_{\min}$  and  $S_{\max}$  are the minimum and maximum stresses;  $f_{c\min}$  and  $f_{c\max}$  are the static strengths corresponding to the loading states of  $S_{\min}$  and  $S_{\max}$ , respectively.

Actually,  $C_f$  stands for the strength varying with the loading frequency or rate.  $C_f = f_{cf} / f_c$ , where  $f_{cf}$  is the static strength at  $f$ , so  $C_f$  can also be called the relative strength.  $C_f$  decreases with decreasing  $f$ . Specially when  $f$  is near to zero or the loading time becomes infinite,  $C_f$  (near to  $c$ ) becomes the relative long-term strength of concrete.

### Formula for Predicting Residual Fatigue Strength

The residual fatigue strength can be defined as the static strength or the relative static strength (the strength ratio) after known loading cycles. It is a very important parameter in assessing the fatigue damage states of existing concrete structures so that maintenance and repair can be done if necessary. For this purpose, a new formula for predicting the residual fatigue strength is deduced as follows, by means of Eq.(1).

Suppose after  $N_1$  cycles applied onto concrete at  $S_{\max}$ , the residual strength is  $f_{cr}$  and the residual life is  $N - N_1$ , then during  $N - N_1$ , the concrete still obeys the fatigue equation (1), i.e.

$$\frac{S_{\max}}{f_{cr}} = C_f [1 - (1 - R')\beta \log(N - N_1)] \quad (2)$$

Solving simultaneous Eqs.(1) and (2), the relative residual fatigue strength  $\bar{f}_{cr}$  is obtained as:

$$\bar{f}_{cr} = \frac{f_{cr}}{f_c} = \frac{1 - (1 - R')\beta \log N}{1 - (1 - R')\beta \log(N - N_1)} \quad (3)$$

### Experimental Verification and Modification

**Experimental Tests** To verify Eq.(3), flexural fatigue tests were conducted on concrete beam specimens of 500 × 100 × 100 mm to determine the residual strength of concrete after certain loading cycles. The same ordinary concrete with  $w/c = 0.45$  as that in the previous tests was used. The mix design is listed in Table 1. The specimens were cured in the curing room for 28 days, at  $20 \pm 2^\circ\text{C}$  and relative humidity of 90%, before they were moved outdoors. At 90 days, these specimens were used for flexural tests, because by then the strength

TABLE 1  
Mix Design of Concrete

Material	Cement	Water	Fine Aggregate	Coarse Aggregate
Type	#525 OPC	Tap water	Natural river sand	15 mm graded gravel
Ratio	1	0.45	1.18	2.47
Content (kg/m <sup>3</sup> )	456	205	538	1251

and stiffness properties of concrete would be almost unchanged. The material properties of concrete are listed in Table 2, including the compressive strength  $f'_c$ , the modulus of rupture  $f_r$ ,  $E$  and  $\nu$  at both 28 and 90 days.

A total of 40 specimens were subjected to flexural cyclic loading in a four-point bending form in a 250 kN Instron servo testing machine. The effective loading span was 450 mm, and a triangular wave was used for static loading and a sinusoidal wave for cyclic loading. A loading apparatus was specially designed to allow such flexural tests (Fig.1).

In the tests,  $R = 0.2$ ,  $S_{max}/f_r = 0.706$  and  $f = 20$  Hz. Thus the corresponding fatigue life  $N$  is calculated as  $2 \times 10^5$  using Eq.(1). Thirty-two specimens were tested to determine the residual fatigue strength at different fatigue stages and the obtained results are shown in Fig.2.

**Modification of Residual Fatigue Strength Formula.** The comparison of Eq.(3) with the experimental results shows a big difference between the residual strength predicted by the formula and the experimentally observed values. The theoretical prediction does not tend to be conservative. Hence Eq.(3) is further modified as follows:

$$\bar{f}'_{cr} = \bar{f}_{cr} \cdot f(N_1/N) = \frac{f_{cr}}{f_c} \cdot f(N_1/N) \quad (4)$$

where  $\bar{f}'_{cr}$  is the relative modified residual strength, and  $f(N_1/N)$  is a modifying function which is chosen as

$$f(N_1/N) = \exp[-A(N_1/N)^B(1 - N_1/N)^C] \quad (5)$$

where  $A$ ,  $B$  and  $C$  are material parameters. Eq.(5) satisfies  $f(N_1/N) \equiv 0$  for  $N_1/N = 0$  and  $N_1/N = 1$ . By differentiating Eq.(5), an extreme value of  $f(N_1/N)$  can be obtained at

TABLE 2  
Material Properties of Concrete

28 days		90 days						
$f'_c$ (MPa)	$f_r$ (MPa)	$f'_c$ (MPa)	$f_r$ (MPa)	$E$ (GPa)	$E_{dB}$ (GPa)	$\nu$	$\nu_d$	$G_d$ (GPa)
50.7	7.19	57.4	7.88	41.1	46.2	0.17	0.23	18.9

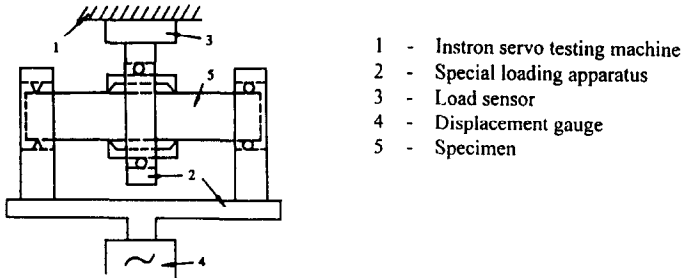


FIG. 1.  
General sketch of static and cyclic testing apparatus.

$$\frac{N_1}{N} = \frac{B}{B+C} \quad \text{or} \quad \frac{B}{C} = \left( \frac{N_1}{N} \right) / \left( 1 - \frac{N_1}{N} \right) = \frac{N_1}{N - N_1} \quad (6)$$

This means that A, B and C have very clear physical meanings: A stands for the amplitude of  $f(N_1/N)$ ; B and C express the eccentricity and flatness and B/C is exactly equal to the ratio of the used fatigue life to the residual fatigue life as  $f(N_1/N)$  has an extreme value.

Using the least-square method, these parameters can be nonlinearly estimated as:  $A = 0.144$ ,  $B = 1.890$  and  $C = 0.224$  with a regression coefficient  $r = 0.724$ . In this study,  $B/(B+C) \approx 0.9$ , i.e. the maximum deviation between the theoretical prediction and experimental results occurs at 90% of fatigue life. Thus Eq.(4) can be finally rewritten as

$$\bar{f}_{cr}' = \frac{1 - (1 - R')\beta \log N}{1 - (1 - R')\beta \log(N - N_1)} \cdot \exp[-A(N_1/N)^B (1 - N_1/N)^C] \quad (7)$$

Using Eq.(7), the residual fatigue strength of concrete can be predicted.

**Discussion of Predicting Formula.** The test results show that the residual fatigue strength gradually drops down with increasing cyclic number until about 90% of the fatigue life is

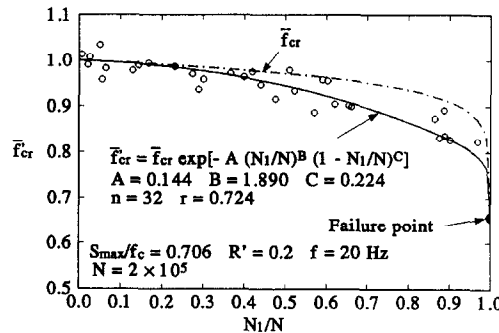


FIG. 2.  
Relationship between residual fatigue strength  $\bar{f}_{cr}$  and relative cyclic number  $N_1/N$ .

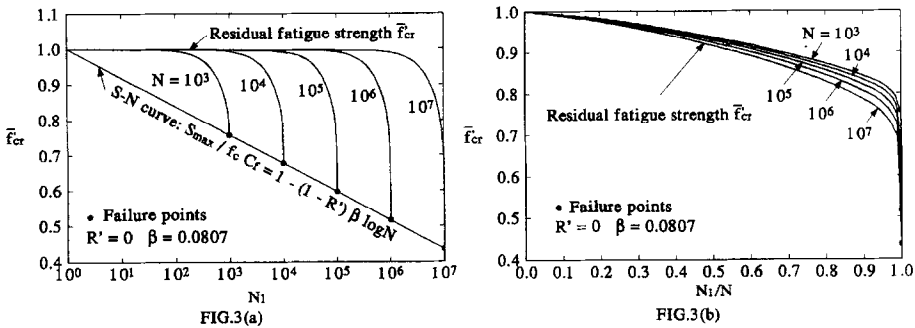


FIG. 3

Family of residual fatigue strength–cyclic number curves for different fatigue lives at  $R' = 0$ .

reached. Thereafter the residual strength drops much more quickly until the final failure occurs.

Fig.3a shows a family of  $\bar{f}_{cr}$  versus  $N_1$  curves (Eq.(7)), for different fatigue lives,  $N$ , at  $R' = 0$  with the corresponding S-N curve. The same curves with the abscissa of  $N_1/N$  shown in Fig.3b indicate that for the same  $R'$ , the residual strength at any  $N_1/N$  decreases more quickly for higher fatigue life.

Fig.4a shows a family of  $\bar{f}_{cr}$  versus  $N_1$  curves (Eq.(7)), at different  $R'$  for  $N = 10^6$ . The same curves with the abscissa of  $N_1/N$  shown in Fig.4b demonstrate that for the same  $N$ , the residual strength decreases more quickly for lower stress ratio  $R'$ . This also confirms that pre-cyclic tension-compression will cause more reduction in the residual strength than pre-cyclic tension.

**Comparison With Previous Tests Under Pre-Cyclic Loading.** Fig.5 compares Eq.(7) with the test data from the only two studies on pre-cyclic tension or tension-compression.

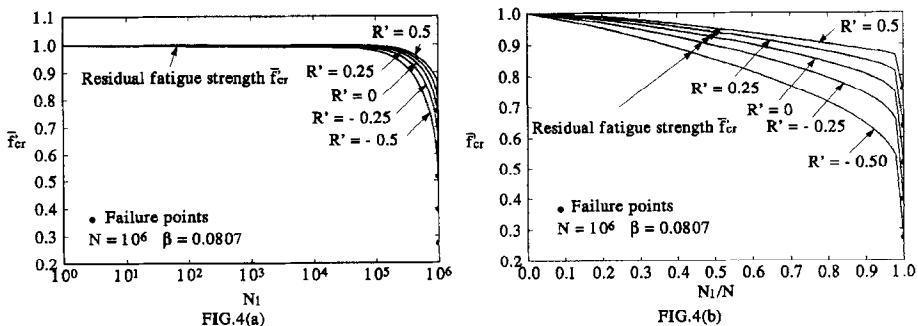


FIG. 4.

Family of residual fatigue strength–cyclic number curves for  $N = 10^6$  at different stress ratios.

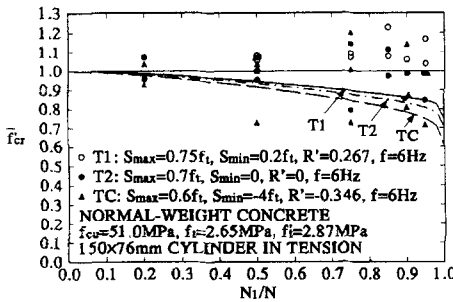


FIG.5(a) Cornelissen and Reinhardt [15]

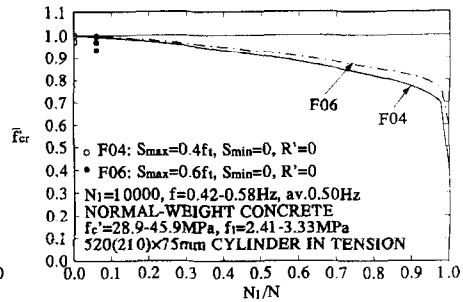


FIG.5(b) Cook and Chindaprasirt [13]

FIG. 5.

Comparison of Eq. (7) with residual tensile fatigue strength tests.

- (a) Cornelissen and Reinhardt [15]: measured the residual tensile strength at  $N_1/N$  varying from 0.2 to 0.95 on 3 groups of ordinary concrete cylinders after pre-cyclic tension and tension-compression. Eq.(7) well predicts the decreasing trend of the residual tensile strength with decreasing stress ratio  $R'$ . Especially Eq.(7) reasonably fits the results under the pre-cyclic tension-compression. Eq.(7), however, gives a conservative prediction of the residual strength under direct tension.
- (b) Cook and Chindaprasirt [13]: measured the residual tensile strength at  $N_1 = 10000$  cycles on two groups of varied sectional concrete cylinders after pre-cyclic tension at  $S_{max} = 0.4 f_t$  and  $0.6 f_t$ . The actual relative cyclic numbers are very small and the tests data points are very few, but the decreasing trend with increasing cyclic number is captured by Eq.(7).

The above comparisons indicate that Eq.(7) can be used to quantitatively predict the residual bending or tensile strength of concrete whose failure is dominated by the development of microcrack. Such prediction is of essential importance for some concrete structures such as concrete pavements, airport runways, etc. Theoretically Eq.(7) can also predict the residual compressive fatigue strength specially in later fatigue stage because it is deduced from a general fatigue equation, but more tests should be done to further confirm this.

### Non-Destructive Testing

**Striking Method.** When a specimen is excited by a striking force, its response comprises of a free damped vibrating state. Consequently its fundamental natural frequencies, corresponding amplitudes and damping characteristics can be measured and transformed into the corresponding properties of the material, e.g. the dynamic Young's modulus  $E_{dB}$  in bending and the dynamic shear modulus  $G_d$ , etc.

The striking tests were conducted using a DEQ-1 Dynamic Striking Instrument, schematically shown in Fig.6. Both bending and torsional modes were examined and from these  $E_{dB}$  and  $G_d$  were obtained based on the following formulae [20]:

$$E_{dB} = 0.94644(L^3 / bh^3)Wf_B^2T_i \quad (\text{Pa}) \quad (8)$$

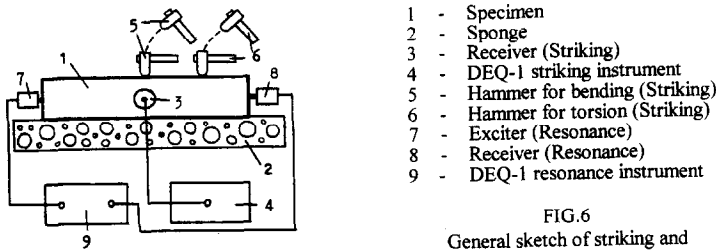


FIG.6  
General sketch of striking and  
resonance testing arrangements

FIG. 6.

General sketch of striking and resonance testing arrangements.

$$G_d = 3.92(L / bh)Wf_T^2 T_2 \quad (\text{Pa}) \quad (9)$$

where  $L$ ,  $b$  and  $h$  in m are the length, breath and height of the specimen;  $W$  is the mass in kg of the specimen,  $f_B$  and  $f_T$  in Hz are the bending and torsional fundamental frequencies,  $T_1 = 1.2600$  and  $T_2 = 1.1834$  are modification factors taking into account of the specimen geometry.

**Resonance Method.** Similar to the striking method, the resonance method also detects the inherent natural frequencies of the specimen. An audio-frequency oscillator can be used to excite different resonant states, say longitudinal, bending and torsional vibrations. The obtained natural frequencies are then transformed into the dynamic elastic modulii. A DEG-1 Dynamic Elastic Parameter Instrument (Fig.6) was used and the appropriate dynamic parameters were obtained using the following formulae [20]:

$$E_{dL} = 3.92(L / bh)Wf_L^2 \quad (\text{Pa}) \quad (10)$$

$$\nu_d = (E_{dL} / 2G_d) - 1 \quad (11)$$

where  $E_{dL}$  is the longitudinal dynamic Young's modulus,  $\nu_d$  is the dynamic Poisson's ratio, and  $f_L$  is the longitudinal frequency in Hz.  $G_d$  is the same as in Eq.(9).

**Results and Discussion.** Non-destructive tests were performed on the same specimens as those on which residual fatigue strength tests were conducted. Before cyclic loading, all initial values of,  $E_{dB}$ ,  $\nu_d$  and  $G_d$  were measured as the references. The average values at 90 days are listed in Table 2. These values were slightly greater than the static ones because they were measured in the dynamic states. The residual values of these stiffness parameters for each specimen were measured again just before the residual fatigue strength was determined. Here the relative dynamic stiffness parameters  $\bar{E}_{dB}$ ,  $\bar{\nu}_d$  and  $\bar{G}_d$  are used which are the ratios of the measured values at different  $N_1/N$  to their own individual initial values, so the results should be more accurate and representative.

Figs.7(a) to (c) show graphs of this data. Even though there is some scatter, the overall trends suggest a linear decrease of all three quantities with  $N_1/N$  until the concrete fails. Regression relationships were obtained as follows:



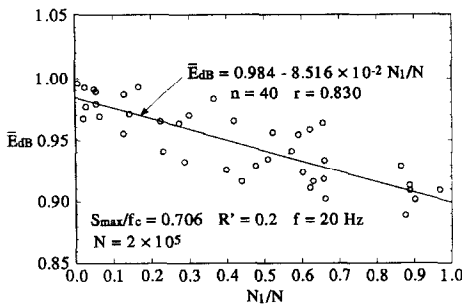
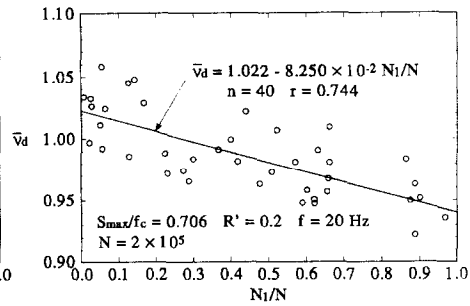
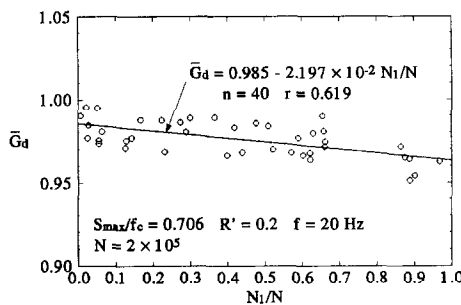
FIG. 7(a)  $\bar{E}_{dB}$  versus  $N_1/N$ FIG. 7(b)  $\bar{v}_d$  versus  $N_1/N$ FIG. 7(c)  $\bar{G}_d$  versus  $N_1/N$ 

FIG. 7.

Relationship between relative dynamic stiffness parameters and relative cyclic number.

$$\bar{E}_{dB} = 0.982 - 8.760 \times 10^{-2} (N_1 / N) \quad (r = 0.890) \quad (12)$$

$$\bar{v}_d = 1.022 - 8.338 \times 10^{-2} (N_1 / N) \quad (r = 0.700) \quad (13)$$

$$\bar{G}_d = 0.985 - 2.342 \times 10^{-2} (N_1 / N) \quad (r = 0.680) \quad (14)$$

The cut-off point for  $N_1/N = 0$  in Eq.(12) is less than one indicating that the concrete suffers some initial instantaneous damage. Eq.(13) suggests that Poisson's ratio has a small instantaneous increase. This may be due to a slightly larger release of internal stress along the transverse direction than along the longitudinal direction during the early fatigue stage. In compressive fatigue, Poisson's ratio  $v$  increases with increasing cycles, but in bending or tension  $v$  continuously decreases because the longitudinal tensile damage is dominant and the lateral compressive damage is secondary. The decrease in  $\bar{G}_d$  is less than  $\bar{E}_{dB}$  and  $\bar{v}_d$  but Eq.(14) does show that damage along the main direction in bending will cause shear resistance to reduce slightly. The fact that the initial values of  $\bar{E}_{dB}$  and  $\bar{G}_d$  for  $N_1/N = 0$  are almost identical suggests that the concrete suffers isotropic instantaneous fatigue damage.

The dynamic stiffness parameters of concrete measured by non-destructive testing are not so big as those measured by the mechanical method [21]. The trends, however, are similar. This is because the latter measures only the fatigue damage within the pure bending region, i.e. local damage zone, whereas the former also includes the effects of sub-main damage and non-damage regions. Nevertheless, the results show that it is possible to predict

the residual fatigue stiffness properties at any fatigue stage using non-destructive testing techniques.

### Relationship Between Residual Strength and Dynamic Stiffness Parameters

$\bar{f}_{cr}$ - $\bar{E}_{dB}$  Relationship. An exponential expression is assumed for  $\bar{f}_{cr}$ - $\bar{E}_{dB}$  relationship:

$$\bar{f}_{cr}' = A_{d1} \bar{E}_{db}^{B_{d1}} \quad (15)$$

where  $A_{d1}$  and  $B_{d1}$  are material constants. Using logarithmic linear regression on the 32 sets of experimental results,  $A_{d1}$  and  $B_{d1}$  were determined:  $A_{d1} = 1.016$  and  $B_{d1} = 1.490$  with  $r = 0.789$ . A linear expression can also be used here:

$$\bar{f}_{cr}' = A_{d2} + B_{d2} \cdot \bar{E}_{db} \quad (16)$$

where  $A_{d2}$  and  $B_{d2}$  can be determined using linear regression:  $A_{d2} = -0.435$  and  $B_{d2} = 1.450$  with  $r = 0.791$ . Fig.8(a) shows that both relationships (15) and (16) fit the experimental results quite well. Actually Eqs.(15) and (16) can be directly deduced from some well-known relationships among  $f_c'$ ,  $f_t$ ,  $f_t$ ,  $E$  and  $E_d$  in the existing concrete design codes [22-24].

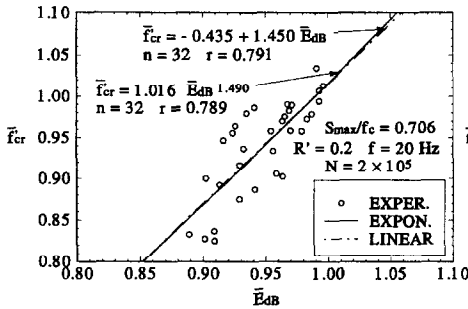


FIG.8(a)  $\bar{f}_{cr}$  versus  $\bar{E}_{db}$

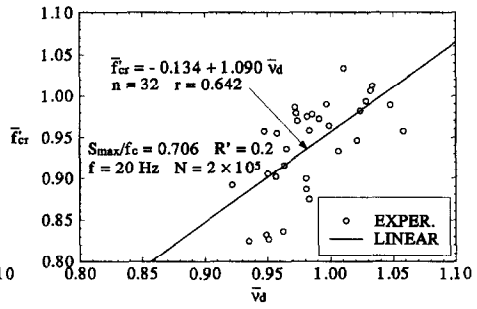


FIG.8(b)  $\bar{f}_{cr}$  versus  $\bar{v}_d$

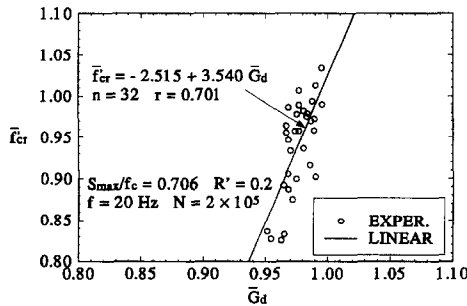


FIG.8(c)  $\bar{f}_{cr}$  versus  $\bar{G}_d$

FIG. 8.

Relationship between residual fatigue strength and relative dynamic stiffness parameters.

$\bar{f}_{cr}' - \bar{v}_d$  Relationship. The total 32 sets of test results show a good linear relationship between  $\bar{f}_{cr}'$  and  $\bar{v}_d$  as follows (Fig.8(b)):

$$\bar{f}_{cr}' = A_{d3} + B_{d3} \cdot \bar{v}_d \quad (17)$$

where  $A_{d3}$  and  $B_{d3}$  were determined using linear regression:  $A_{d3} = -0.134$  and  $B_{d3} = 1.090$  with  $r = 0.642$ .

$\bar{f}_{cr}' - \bar{G}_d$  Relationship. Similarly, based on the same sets of test results, a linear relationship between  $\bar{f}_{cr}'$  and  $\bar{v}_d$  is also obtained as follows (Fig.8(c)):

$$\bar{f}_{cr}' = A_{d4} + B_{d4} \cdot \bar{G}_d \quad (18)$$

where  $A_{d4}$  and  $B_{d4}$  can be determined using linear regression:  $A_{d4} = -2.515$  and  $B_{d4} = 3.540$  with  $r = 0.701$ .

The experimental results in this study show good correlations between the residual fatigue strength and the dynamic stiffness parameters in bending. These relationships can be used to determine the residual fatigue strength of existing concrete structures. As soon as these dynamic stiffness parameters are measured using non-destructive testing techniques, they can be directly transferred to the corresponding residual fatigue strength.

### Conclusions

- Based on the fatigue equation proposed by the authors, a new formula Eq.(7) to predict the residual fatigue strength of concrete has been proposed and further modified by the flexural fatigue tests on concrete beam specimens. Eq.(7) predicts the previous test data reasonably well. Eq.(7) can also be used for tension and even compression cases if more research work is done because it is deduced from a general fatigue equation.
- At any fatigue stage, the stiffness parameters such as Young's modulus, Poisson's ratio and shear modulus can be dynamically measured using non-destructive testing techniques. The test results show that under bending all these parameters decreased fairly linearly with increasing cycles.
- The experimental results also show good relationships between the residual fatigue strength and other stiffness parameters measured using non-destructive testing techniques. These relationships can be used to predict the residual fatigue strength in the existing concrete structures.

### References

1. B. Zhang, D.V. Phillips and K. Wu (in press) *Mag. Concr. Res.*, 1996.
2. B. Zhang, D.V. Phillips and K. Wu, accepted by *Mag. Concr. Res.*, 1996.
3. Comite Euro-Internationale du Beton, *CEB Bulletin D'Information* 188 (1988).
4. G.P. Mallett, *Fatigue of Reinforced Concrete*, HMSO, London, 1991.
5. E.L. Nelson, R.L. Carrasquillo and D.W. Fowler, *ACI Materials Jour.* **85** (4), 248 (1988).
6. M.-T. Do, O. Chaallal and P.-C. Aitcin, *Jour. Mate. in Civil Eng., ASCE* **5** (1), 96 (1993).
7. W. Yin and T.C. Hsu, *ACI Materials Jour.* **92** (1), 71 (1995).
8. A.M. Fredenthal and F. Roll, *Proc. ACI Jour.* **54** (66), 1111 (1958).

9. S.P. Shah and S. Chandra, *Proc. ACI Jour.* **67** (10), 816 (1970).
10. S.A.A. Coutinho, *Mate. and Struct.: Res. and Testing* **10** (55), 3 (1977).
11. M.E. Awad and H.K. Hilsdorf, Fatigue of Concrete, ACI SP-41, Detroit, 1 (1974).
12. D.J. Cook and P. Chindaprasirt, *Mag. Concr. Res.* **32** (111), 89 (1980).
13. D.J. Cook and P. Chindaprasirt, *Mag. Concr. Res.* **33** (116), 154 (1981).
14. C. Tinic and E. Bruhwiler, *Inter. J. Cem. Comp. & Lightweight Concr. Res.* **7**(2), 103 (1985).
15. H.A.W. Cornelissen and H.W. Reinhardt, Fracture Control of Eng. Struct., *Proc. Euro. Conf. on Fracture - 6*, **3**, Amsterdam, 2087 (1987).
16. R.Sri Ravindrarajah, *Inter. J. Cem. Comp. & Lightweight Concr. Res.* **4** (4), 251 (1982).
17. Y. Sakata and M. Ohtsu, *ACI Materials Jour.*, **92** (6), 686 (1995).
18. S.K. Mantrala and C. Vipulanaadan, *ACI Materials Jour.* **92** (6), 660 (1995).
19. N. Krstulovic-Opara, R.D. Woods and N. Al-Shayea, *ACI Materials Jour.* **93** (11), 75 (1996).
20. W. Li, Nondestructive Testing and Electrical Measurement, Tongji Univ. Press, China, 1987.
21. B. Zhang and K. Wu, *Jour. Tongji Univ.* **17** (1), China, 59 (1989).
22. ACI 318-89, Building code requirements for reinforced concrete, Detroit, 345 (1994).
23. CP110 Part 1 : 1972. Code of Practice for the structural use of concrete, 142 (1972).
24. A.M. Neville, Properties of Concrete, Longman Group Ltd., London, 309 & 596 (1995).