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## **SIMULATION OF THE EFFECT OF GEOMETRICAL CHANGES OF THE MICROSTRUCTURE ON THE DEFORMATIONAL BEHAVIOUR OF HARDENING CONCRETE**

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### **ABSTRACT**

A constitutive model for hardening concrete is presented, in which the evolution of the microstructure is accounted for explicitly. The concrete is divided into structural components made of anhydrous cement, cement gel, and aggregate with nonchanging elastic and time-dependent stress/strain relations. The division of the microstructure into different structural components and the stiffening of the microstructure by the addition of new components enables the simulation of a deformational behaviour that is highly influenced by the course of the hydration process. In this manner, the creep of concrete on the macro level is described as a joint behaviour of elastic and time-dependent properties and of the effects of redistribution of stress over new and old hydration products on the micro level. The model is able to simulate the elastic modulus and the creep compliance at early ages quite satisfactorily.

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### **Introduction**

In a discussion between Bazant (1) and Rüsch et al. (2) on the reliability and applicability of creep curves, it has been stated that the creep deformations of concrete are determined by over 20 parameters. It is not surprising, therefore, that no single theory proposed so far describes the creep phenomena comprehensively (3). Particular difficulties are encountered when dealing with the time-dependent properties of hardening concrete. The time-dependent behaviour of hardening systems cannot be related straightforwardly to the materials properties at the time of application of the load. Deformations inherent to the hydration process itself have to be superimposed on the true time-dependent deformations. Modelling of such additional phenomena requires a detailed understanding of the microstructural changes of hydrating systems. The need for microstructure-based explanations of time-dependent phenomena has been previously emphasized by Feldman (4). Although many authors support this view, only small steps have been made since that time towards microstructure-based models for the deformational behaviour of hardening systems. "One of the basic problems of these models is," as stated by Jennings (5), "that the properties of concrete depend on phenomena that operate over a wide range of scales. This implies, in the end, the need for the use of computer-based models." The work presented in this paper is to be considered as a step

towards the application of such computer-based models for explaining and quantifying the effect of hydration-caused microstructural changes on the deformational behaviour of cement-based systems at early ages.

## **Creep Theories**

### **Creep and Aging in Hardened Concrete**

Creep mechanisms proposed in the past have in common that they are all related predominantly to the microstructure of the cement paste and to changes therein. Microstructural changes may determine the shape of the creep curves and may also explain why creep is less when the age at loading increases. This age-dependency of creep deformations is called aging. Aging has been attributed to the polymerisation of silicate phases (6), the formation of new bonds with a lower energy state (7), and to highly localized volume changes in the microstructure as considered in the microprestress concept (8).

### **Creep in Hardening Pastes**

With the classical aging concept, the effect of the age at loading on creep can be quantified fairly satisfactorily when dealing with hardened concrete. No good results are obtained when dealing with young hardening concrete. The effect of continuing hydration on the deformational behaviour of young concrete has been studied in detail by Huggins and Timusk (9). They suggested that only gel formed prior to loading would be fully effective as a load-bearing substance. Gel formed after application of a load was assumed to be laid down in a stress-free state and would only gradually begin to participate in carrying the load. This idea has been taken as a reference for further qualitative modelling of the effect of progressive hydration on creep by Timusk and Ghosh (10). A partially hydrated paste was considered to consist of cement grain remnants enveloped by hydration products. Between the anhydrous core and the product layer, a load-bearing water-filled interfacial zone was considered. With the progression of the hydration process the receding cement grains are followed by the gel. In case a paste is loaded with an external load, the latter will be carried by the gel and the anhydrous cement core. When the anhydrous core is gradually dissolved without formation of hydration products in the load-bearing region of the interfacial zone, the deformation of an hydrating system will exceed the deformation in a nonhydrating system.

In 1973, Ghosh (11) discussed four possible mechanisms that might be involved in maturing creep: 1) osmotic pressure effects, 2) drying creep mechanisms, 3) gel following the hydrating cement (Fig. 1, and 4) the deposition of new gel in an unloaded state. Mechanisms 2 and 4 very much depend on the rate of hydration. Quantifying the effect of progressive hydration on the deformational behaviour of hardening pastes presupposes that we are able to model and quantify the hydration process and the associated development of the microstructure on the (cement) particle level. The lack of models for microstructural development at that time is the most plausible reason why this concept was not developed further.

Inspired by the work of Timusk and Ghosh, van Breugel (12,13) elaborated a model for quantification of the effect of continuing hydration on both creep and the relaxation of hydrating systems. Based on the concept of redistribution of stresses among preloading gel and newly formed gel, van Breugel developed a creep and relaxation formula in which the

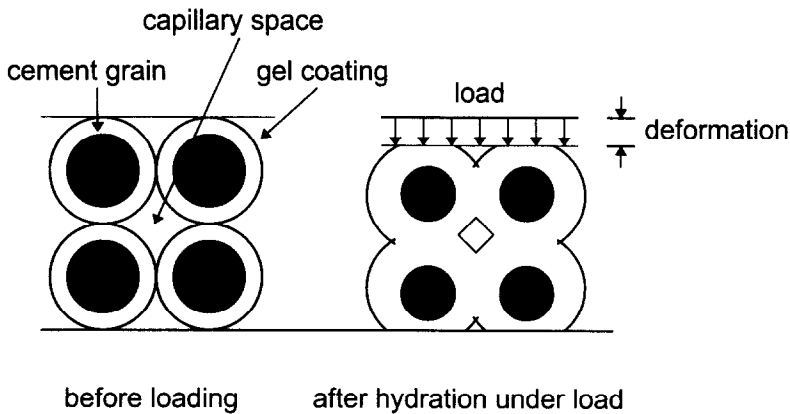


FIG. 1.

Schematic representation of a mechanism for maturing creep: gel following the hydrating cement grain (11).

degree of hydration at the age of loading and the progress of the hydration process afterwards was explicitly accounted for. A creep formula in which the degree of hydration at the age of loading was considered explicitly has also been proposed by Laube (14).

Also worth mention is Bazant's concept of solidification. This theory, proposed for the first time in 1977, was developed further by Bazant and Prasannan (15) and in more detail in a recent paper by Carol and Bazant (16). In the solidification model, the load-bearing structure is formed by parallel layers, of which the number increases with an increase of the hydration process (Fig. 2). Similar to the concept proposed by Timusk (10), a new solidified layer joins the solid microstructure in and at the first stress-free state, while the existing ones will in general carry some stress (16). For each layer, a nonaging visco-elastic behaviour was assumed.

The recent literature on the deformational behaviour of hardening concrete shows convincingly that the microstructural development is considered to play a decisive role. Several authors have proposed creep formulae in which the effect of the progressive hydration and

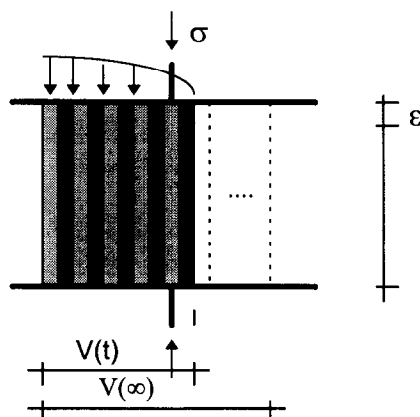


FIG. 2.

Solidification theory (16).

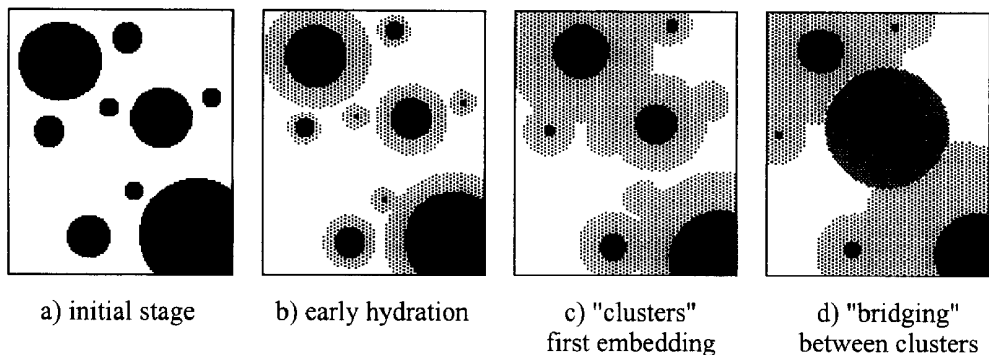


FIG. 3.

Formation of a load-bearing microstructure in hardening cement paste.

the associated microstructural changes has been considered by inserting the degree of hydration as one of the important parameters. In these formulae, the degree of hydration is a "macroscale quantity," indicating the overall progress of the hydration process. It does not describe the microstructure and microstructural changes on the micro level comprehensively. For modelling the consequence of changes in the microstructure on the deformational behaviour, one should start with quantitative modelling of the microstructural development itself. Microstructural models with the potential to describe the stereological/geometrical aspect of microstructural development on the (cement) particle level have been developed by Bache (17), Jennings and Johnson (18), Navi (19), and Garboczi and Bentz (20). In the present research, the HYMOSTRUC-model, developed at the Delft University of Technology, will be used. The main features of the model have been discussed previously (21).

### Microstructural Development

#### Basic Features of the Numerical Simulation Model HYMOSTRUC

Several authors have simulated the microstructural development in cement-based systems as a process of outward growth of the hydrating cement grains (17,18,22). In the simulation program HYMOSTRUC, the formation of interparticle contacts is simulated numerically, taking account of the water/cement ratio and the particle size distribution of the cement (23). Different stages in the formation of interparticle contacts are presented schematically in Figure 3. With the progression of the hydration process, small particles become embedded in the outer shell of bigger growing particles (Fig. 3c). As long as a particle is not embedded in the outer shell of a larger particle, it is called a "free" particle. A free particle with smaller particles embedded in its outer shell is called a "cluster." In this model, the clusters are regarded as the basic "structural elements" of the microstructure.

The contacts between clusters are formed by cement particles that are not embedded completely in a cluster. These particles, called "bridging particles," exceed the shell thickness around the free particle of a cluster and can also be partly embedded in another neighbouring cluster (the dark coloured particle in Fig. 3d). The minimum size of a particle that is able to

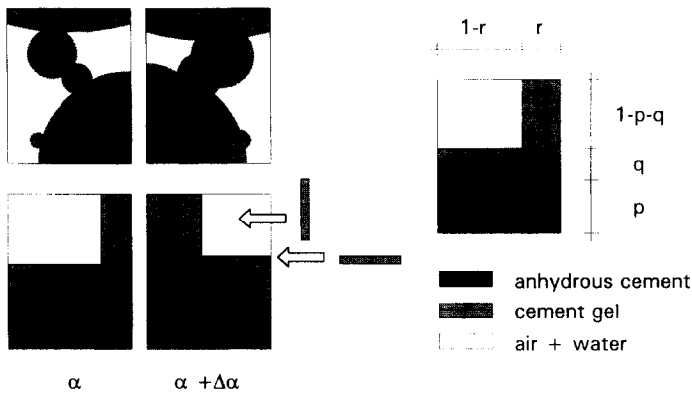


FIG. 4.

Composite model for hardening cement paste.

bridge the distance between two (or more) neighbouring clusters depends on the thickness of the outer shell of the free particles of the clusters and the distance between the clusters.

### Towards Mathematical Composite Models of Cement Paste and Concrete

The numerically generated information about the development of clusters and bridging particles constitutes the basis for “composite material models” for cement paste and concrete. For this purpose, the classical distinction between anhydrous cement, gel, and capillary pores was adopted. The step to a composite model for hardening cement paste is shown schematically in Figure 4. In the top left panel of Figure 4, the microstructure is shown for a degree of hydration  $\alpha$ , with the corresponding composite model at the bottom left panel. In this composite model, the horizontal layers represent the radial expansion of the larger cement particles. Vertical bars represent the bridging particles. Because the bridging particles become more embedded in the outer shell of larger particles, their geometrical change causes an increase of the contact area with adjacent particles and a shortening of the distance to be bridged.

With an increase in the degree of hydration by  $\Delta\alpha$ , the remaining volume of the anhydrous cement, the capillary pore volume, and the number of layers and bars representing the gel are adjusted. These volumes and numbers are calculated directly from the output of the numerical simulations with HYMOSTRUC. The resulting composite model is identified with the parameters  $p$ ,  $q$ , and  $r$ :

$$p = \frac{(1 - \alpha)V_{0,\text{cem}}}{V_{0,\text{paste}}} \quad (1)$$

$$q = \frac{V_{\text{clus}}(\alpha) - V_{\text{br}}(\alpha) - (1 - \alpha)V_{0,\text{cem}}}{V_{0,\text{paste}}} \quad (2)$$

$$r = \frac{1}{(1 - p - q)} \cdot \frac{V_{\text{br}}(\alpha)}{V_{0,\text{paste}}} \quad (3)$$

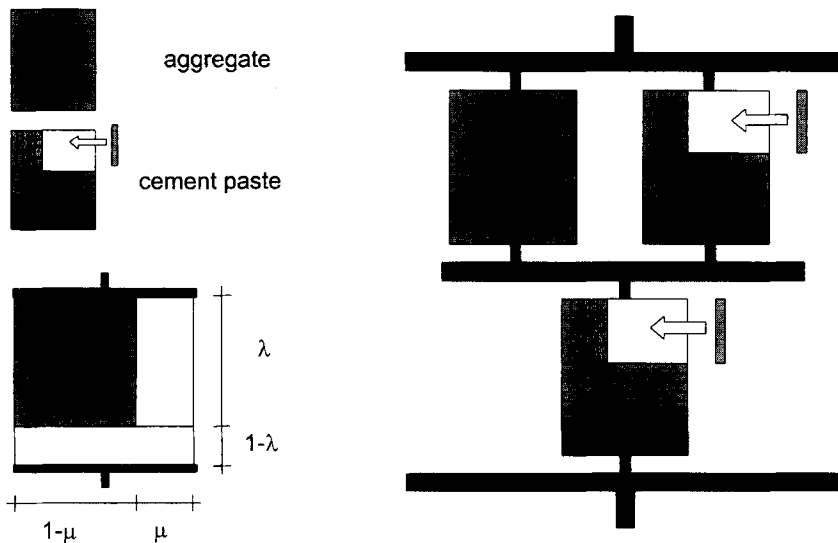


FIG. 5.

Proposed composite model for concrete, i.e., the bar model.

in which  $p(\alpha)$  is the volume of the anhydrous cement,  $q(\alpha)$  the cluster volume minus the anhydrous cement volume and the bridge volume,  $r(\alpha)$  the width of the bridge volume,  $V_{br}(\alpha)$  the total bridge volume,  $V_{clus}(\alpha)$  the volume of the clusters including the bridge volume,  $V_{0,paste}$  the initial volume of the cement paste, and  $V_{0,cem}$  the initial volume of the anhydrous cement.

The composite model for cement paste has been extended to a composite model for concrete by combining an aggregate element with two cement paste elements. The aggregate is modelled as an elastic solid element. One cement paste model is placed in series and one in parallel with the aggregate element. The resulting "bar model" is shown in Figure 5. The dimensions of the aggregate and the paste elements are defined by the volumetric parameters  $\lambda$  and  $\mu$ . The product of the height  $\lambda$  and the width  $1-\mu$  of the aggregate element must equal to aggregate volume  $V_a$ . Apart from the continuous adjustment of the volumes of the different paste components, the model is similar to the model proposed by Counto (24) in case  $\lambda = \sqrt{V_a}$ . A composite model similar to the one shown in Figure 5 was also applied by Granger and Bazant (25). They, however, used a different cement paste element to study the effect of composition and aging in concrete.

By assigning elastic and time-dependent properties to the different elements of the bar model, the elastic and time-dependent behaviour of the model as a whole can be obtained, provided that the model parameters  $p$ ,  $q$ ,  $r$ ,  $\lambda$  and  $\mu$  are known.

### Elastic Modulus

In the cement paste element three volumes are placed in series. The elastic modulus of the cement paste  $E_{cp}$  is calculated with:

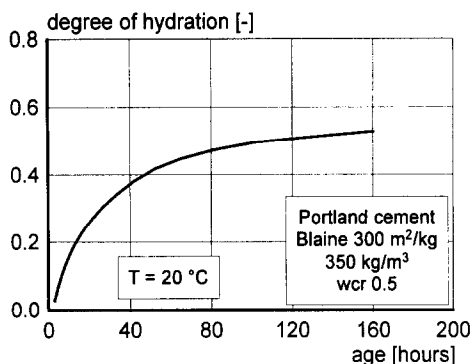


FIG. 6.

Calculated development of degree of hydration of a Portland cement-based concrete mixture.

$$\frac{1}{E_{cp}} = \frac{p}{E_{cem}} + \frac{q}{E_{gel}} + \frac{1-p-q}{rE_{gel}} \quad (4)$$

in which  $E_{cem}$  and  $E_{gel}$  are the elastic moduli of the anhydrous cement and cement gel, respectively. For the elastic modulus of the concrete model  $E_c$ , it holds:

$$\frac{1}{E_c} = \frac{\lambda}{\mu E_{cp} + (1-\mu)E_{agg}} + \frac{1-\lambda}{E_{cp}} \quad (5)$$

with  $E_{agg}$  the elastic modulus of the aggregate.

For four different concrete mixtures, the evolution of the elastic modulus was measured and calculated with Eq. 4 and 5. All mixes contained 350 kg/m<sup>3</sup> cement. One concrete was made with Portland cement and had a water/cement ratio of 0.5. The other three mixtures were made with blast furnace slag cement and had water/cement ratios of 0.4, 0.5 and 0.6. River sand and river gravel were used as aggregate. The mixtures were all cured under sealed conditions at 20°C. The elastic modulus of the anhydrous cement, the cement gel, and the aggregate (60, 30, and 60 kN/mm<sup>2</sup>, respectively) were chosen, which agree well with literature (26,27). The parameters  $\lambda$  and  $\mu$  were determined by assuming that the aggregate element is a square, i.e.,  $\lambda = \sqrt{V_a}$ . The values of  $p(\alpha)$ ,  $q(\alpha)$  and  $r(\alpha)$  are calculated according to the Eq. 1, 2, and 3, the degree of hydration  $\alpha$  being determined with HYMOSTRUC. For the Portland cement concrete, the calculated degree of hydration and the corresponding cluster and bridge volumes (per m<sup>3</sup> concrete) are shown in the Figures 6 and 7. In Figure 8, the calculated elastic modulus is shown together with the measurements. For the three blast furnace slag cement mixtures, the elastic moduli are presented in Figure 9. It appears that the evolution of the elastic moduli can be simulated quite well with the bar model. Only in the early stage of hydration, up to a degree of hydration  $\alpha \approx 0.2$ , the elastic modulus is overestimated. The good agreement between calculated and measured elastic deformations are considered a promising starting point for investigations into the potential of the bar model for simulating time-dependent behaviour of hardening systems.

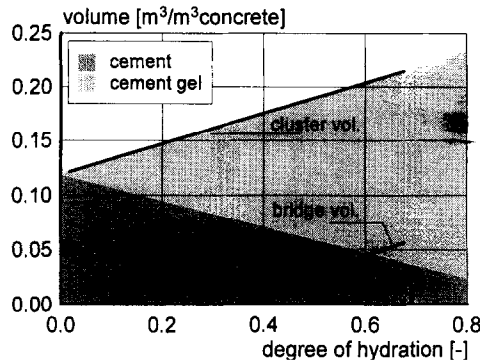


FIG. 7.

Cluster and bridge volume calculated with HYMOSTRUC. The concrete contains 30% cement paste by volume.

### Deformational Behaviour of Hardening Systems

#### Time-dependent Stress-Strain Relation

In the previous paragraph, elastic stress-strain relations were assigned to the different components of the bar model. To model the time-dependent deformations, a time-dependent stress-strain relation is assigned to the components of the model to represent the cement gel, i.e., the layers and bars.

For the time-dependent strain  $\epsilon_c(t)$  of the cement gel under a constant stress  $\sigma(\tau)$  applied at age  $\tau$ , and a constant temperature  $T$ , it was proposed (28):

$$\epsilon_c(t) = a \cdot V(T) \cdot \frac{(t - \tau)^n}{t_0} \cdot \sigma(\tau) \quad (6)$$

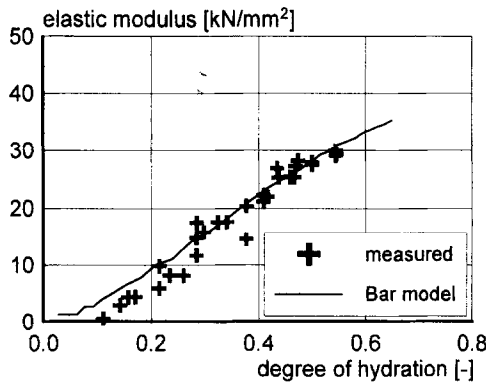


FIG. 8.

Simulated and measured evolution of the elastic modulus of the Portland cement-based concrete ( $wcr = 0.5$ ).



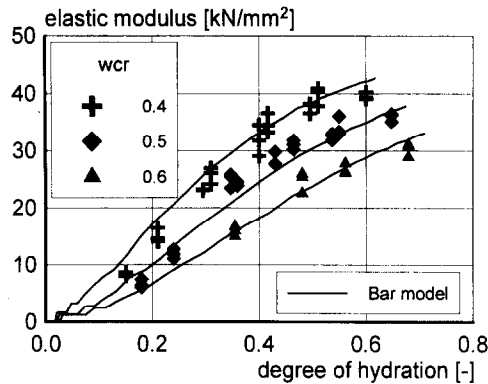


FIG. 9.

Simulated and measured evolution of the elastic modulus of three blast furnace slag cement-based concretes with different water/cement ratios (wcr).

where  $t - \tau$  is the time under load  $h$ ,  $t_0$  is the reference time (1 hour),  $a$  is a creep constant ( $\text{mm}^2/\text{N}$ ),  $n$  is a constant (approximately 0.3).  $V(T)$  is a factor describing the effect of temperature on the rate of deformation according to:

$$V(T) = \exp\left(\frac{Q}{R} \cdot \frac{T - T_0}{T \cdot T_0}\right) \quad (7)$$

with  $Q$  the activation energy for creep,  $R$  the universal gas constant (8.31 J/mol K), and  $T_0$  the reference temperature (293 K). Note that  $V(T) = 1$  for  $T = T_0$ .

The creep constant  $a$  is in fact the aging factor, which is not a real constant but a function of the age at loading. In the present project, emphasis is on the effect of continuing hydration on the deformational behaviour of hydrating systems. In the very early stage of hardening, the effect of hydration on time-dependent deformations is assumed to be dominant. In this stage, the age-dependency caused by the previously mentioned mechanisms of polymerisation or stabilisation is considered to be of less importance. For this reason,  $a$  is assumed to be a constant. The values of the parameters  $Q$  and  $n$  are assumed to be constant for all the bars and layers representing the cement gel. For the activation energy a value of 20,000 J/mol will be applied.

### Time-dependent Deformational Behaviour of the Bar Model

The key feature of the proposed model is that the time-dependent response of the model not only depends on the number of layers and bars in the cement paste elements at the time of the application of load, but also on the changes of these numbers while under load. These changes take place in the two cement paste elements and are governed by the addition of bars and layers. After insertion, a layer will be stressed immediately and starts to deform in correspondence with the actual stress in the cement paste element. Equilibrium between internal stresses and the external loading requires that the stress in all layers of a paste element is the same.

A new bar, however, is inserted stress free. The total stress in the bars, therefore, will

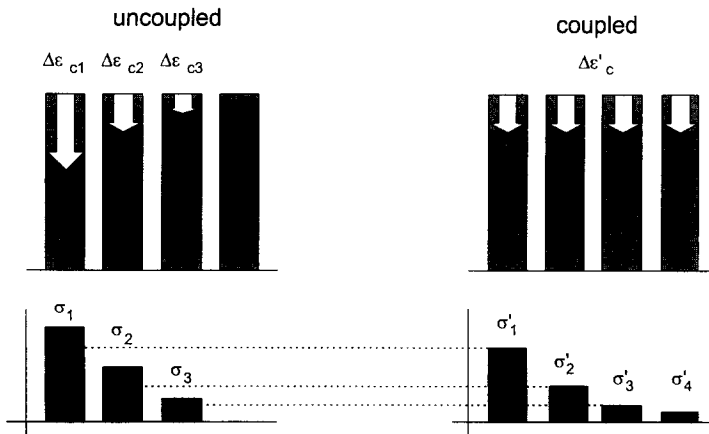


FIG. 10.

The coupling of four vertical bars with different ages, different stresses, and corresponding time-dependent strain increments results in a redistribution of stress over the individual bars.

generally not be the same because of their different age and, consequently, different stress- and strain-history. While under load, the bars will tend to deform with a rate that depends on the stress in the bar and the time under load. Compatibility in the cement paste models requires that the deformation of all bars is the same, and therefore a redistribution of stress will take place (see Fig. 10). In this way, new bars that were initially inserted stress free (see bar 4 in Fig. 10) gradually become load-bearing. This redistribution of stresses causes a restraining effect on the rate of the time-dependent deformation on the macro level. This is illustrated in Figure 11. The dashed line represents the creep deformations in time in case no bars and layers were added after the application of load (i.e., no hydration while under load). The shape of this curve is identical to the shape of the power function of Eq. 6. The effect of the addition of bars and layers on the creep deformations is shown by the solid line. It is clear that the restraining effect of the addition of bars plays a dominant role on the deformations.

For the dimensions of the layers and bars, arbitrary values were chosen. Each horizontal

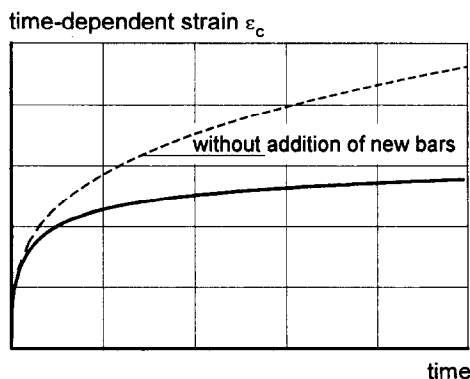


FIG. 11.

Restraining effect on the deformation of the bar model caused by the addition of bars.

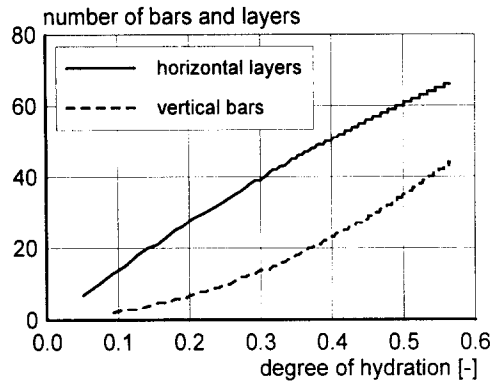


FIG. 12.

Number of bars and layers in the cement paste elements for the Portland cement-based concrete as a function of the degree of hydration.

layer has a constant volume of 0.5% of the paste volume. Because the length of the bars changes with the degree of hydration, their volume is not constant. Therefore, instead of their volume, their width is defined as 0.5% of the width of the cement paste model. The number of layers and bars is calculated by dividing both  $q(\alpha)$  and  $r(\alpha)$  by 0.005. For the Portland cement-based concrete, the increase of the number of layers and bars is shown in Figure 12. The stress and strain history of each of the layers and bars in the two cement paste elements is accounted for in the calculation of the deformations of the bar model.

### Underlying Calculation Procedures

The redistribution of stresses over a set of  $k$  vertical bars follows by solving  $k - 1$  equations referring to compatibility, stating that the sum of the changes of the elastic strain  $\epsilon_c$  and time-dependent strain  $\epsilon_c$  must be equal for all bars:

$$\frac{d\epsilon_{c,1}}{dt} + \frac{d\epsilon_{c,1}}{dt} = \frac{d\epsilon_{e,b}}{dt} + \frac{d\epsilon_{c,b}}{dt} \quad b \in [2, k] \quad (8)$$

and one equation referring to equilibrium stating that the sum of the stress increments  $\Delta\sigma_{r,b}$ , due to redistribution, must remain zero:

$$\frac{d\sigma_{r,1}}{dt} + \frac{d\sigma_{r,2}}{dt} + \frac{d\sigma_{r,3}}{dt} + \dots + \frac{d\sigma_{r,k}}{dt} = 0 \quad (9)$$

From the above described mechanism, it follows that even if the external stress on the cement paste or concrete remains constant throughout the loading, the stress in a bar will vary in time. Therefore the accountancy of the stress history should be given due attention. Only in case the number of bars and layers in the model does not change, the stress in a bar remains constant.

The effect of a variable stress history of a bar or a layer on the time-dependent deformation is taken into account with the superposition principle. In correspondence with the process of addition of discrete layers and bars, an incremental calculation procedure is used:

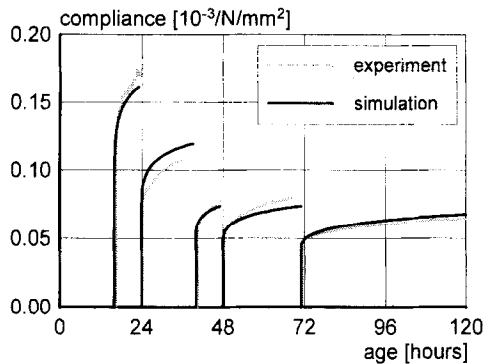


FIG. 13.

Measured and simulated creep compliance up to loading age of 72 hours.

$$\varepsilon_{\text{tot}}(t_{i+1}) = \varepsilon_{\text{tot}}(t_i) + \Delta\varepsilon_e(t_i) + \sum_{j=1}^i \Delta\varepsilon_c(t_i, \tau_j, T_i) \quad (10)$$

or:

$$\varepsilon_{\text{tot}}(t_{i+1}) = \varepsilon_{\text{tot}}(t_i) + \frac{\Delta\sigma(t_i)}{E} + \sum_{j=1}^i a \cdot V(T_i) \cdot \Delta\sigma(\tau_j) [(t_{i+1} - \tau_j)^n - (t_i - \tau_j)^n] \quad (11)$$

Thus the actual total strain  $\varepsilon_{\text{tot}}(t_{i+1})$  at time  $t_{i+1}$ , is obtained by the summation of three components:

1. the total strain at (the previous) time  $t_i$ ;
2. the new elastic strain increment  $\Delta\varepsilon_e(t_i)$  resulting from a new stress increment  $\Delta\sigma(\tau_i)$ ; and
3. the sum of the time-dependent strain increments  $\Delta\varepsilon_c(t_i, \tau_j, T_i)$  occurring in the time interval  $[t_i, t_{i+1}]$  resulting from all stress increments  $\Delta\sigma(\tau_j)$ .

### Comparison of Simulations with Experiments

The time-dependent deformational behaviour of the Portland cement-based concrete mentioned earlier was determined experimentally with two series of sustained loading tests. The loading ages varied between 16 and 168 hours. The duration of the tests was relatively short (compared to traditional creep tests) and varied between 7 and 70 hours. The loading level at the age of loading was 30% of the actual compressive prism strength.

For the tests, specimens with sizes of  $400 \times 100 \times 100 \text{ mm}^3$  were used. After casting, the specimens were stored in a climate room ( $20^\circ\text{C}$ ) and remained in their mould until about 30 min before the start of the test. After demoulding, the specimens were sealed in a plastic foil to prevent drying. Each specimen was accompanied by a sealed dummy specimen to monitor the deformations due to temperature changes and autogenous shrinkage. The specimens were tested in the laboratory, where the temperature varied between 20 and  $25^\circ\text{C}$ .

The results of the sustained loading tests are presented as creep compliance, which is the ratio of the total deformations and the applied stress. The results of the first series of tests are shown as gray lines in Figure 13. All specimens were taken from one batch of concrete. The

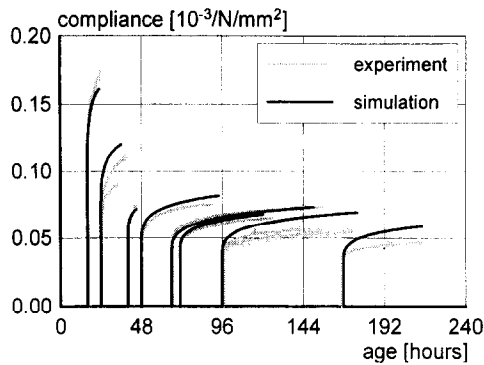


FIG. 14.  
Measured and simulated creep compliance.

loading age varied between 16 and 72 hours after mixing. The black lines refer to the compliance as calculated with the bar model. It shows that the model is capable of simulating the age-dependency of the elastic and time-dependent deformations in the early stage of hardening (up to a degree of hydration of 0.45) quite well. For the creep constant  $a$ , a value of  $1.5 \cdot 10^{-5}$  was used.

In the second series of tests, the range of the loading age was between 24 and 168 hours. The specimens originated from several batches of concrete. The experimental results of the first and second series of tests are plotted in Figure 14 (gray lines). The results of the simulations are plotted as black lines. The results were obtained with the same model parameters used for simulations of the first series. The figure shows that for loading ages of 96 and 168 hours the bar model overestimates the compliance.

Because the evolution of the elastic modulus of the concrete can be simulated quite well, the differences between measured and simulated compliance originate from the time-dependent deformations predicted by the model. The main reason for this might be that "traditional" aging was not considered in the model. The results of the simulations are obtained solely by adjusting the number of layers and bars of the model with progress of the hydration process. In the early stage of hydration, the changes of these numbers are very large and have a distinct influence on the deformations. When the hydration process is very slow, the number of bars and layers varies only little and the model hardly predicts age-dependency anymore.

### Discussion and Conclusions

The many parameters that determine the time-dependent behaviour of concrete make it impossible to describe this behaviour with simple formulae. This holds particularly for young hardening concrete. In order to allow for the effect of hydration while under load, Emborg (29), and more recently Westman (30), have extended the triple power law proposed by Bazant & Chern (31) with additional factors. The work of those authors has provided convincing evidence that for a satisfactory description of the time-dependent behaviour of hydrating cement-based materials the effect of the hydration process on this behaviour must be taken into account. In this paper, an attempt is made to model the effect of hydration and of the associated microstructural development on the deformational behaviour of hardening concrete in a more explicit way. The approach has been inspired on the work of Ghosh (11),

who postulated that “maturing creep” could be explained by assuming that newly formed hydration products are laid down stress free and gradually become load-bearing. This idea has been worked out in more detail with the help of the simulation model HYMOSTRUC. With this model, the increase of contact areas between hydrating cement particles can be quantified. This information about the developing microstructure has been the basis for the proposed composite model for hardening concrete, called the “bar model.” With this model, the microstructural changes in the cement paste are modelled as an increase of layers and bars, representing the increasing amount of cement gel. The subdivision between bars and layers is determined by the microstructural information generated by HYMOSTRUC. Simple material laws were assigned to the layers and bars. No “traditional” aging was considered.

With the bar model, the evolution of the elastic modulus and the deformational behaviour of hardening concrete, i.e., the creep compliance, in the early stage of the hydration process can be simulated quite satisfactorily. The discrepancy between simulated and measured compliance becomes larger with increasing age at loading and increasing time under load. This finding was to be expected, because at later ages and longer periods under load the classical aging phenomenon becomes decisive. This aging was not considered in the time-dependent law that was assigned to the components of the bar model. A similar observation has been made by Bazant (8), who found that his solidification theory gave good results for very young concrete, but that some modification was required in order to cover the time-dependent deformations at later ages.

The deformational behaviour of hardening concrete has been attributed to the solid material only. It was shown that the addition of bars and the associated mechanism of the redistribution of stress has a restraining influence on the rate of time-dependent deformation. However, it is likely that this mechanism is only one of the mechanisms involved in early-age creep. As suggested by Oshita & Tanabe (32), the water in the capillary pores may strongly influence the short term deformational behaviour of hardening concrete, whereas the long term deformations may be attributed to visco-elastic behaviour of the solid material.

The fact that both the elastic modulus and the creep compliance at early ages can be simulated quite satisfactorily with one and the same model constitutes a promising starting point for further research. Such research should focus on the specific features of hydration and microstructural development that have been ignored in the present configuration of the model. The role of water in the continuously changing pore system has to be considered in more detail. Also the formation of hollow shells, an often observed phenomenon in the hydration of cement grains, requires more attention. This phenomenon would violate the assumption that anhydrous cement and the surrounding cement gel act together in the way assumed in the model. This, however, would also be the case for many other composite models in which gel and anhydrous cement are considered to act together in carrying an external load. Nevertheless, the present model shows the potential of numerical models to simulate the effect of microstructural changes on the deformational behaviour of hardening cement-based systems.

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