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ON PREDICTING COMPRESSIVE STRENGTHS OF MORTARS WITH TERNARY BLENDS OF CEMENT, GGBFS AND FLY ASH

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ABSTRACT

The simplex-centroid design with upper and lower bounds of component proportions was adopted to study the compressive strengths of mortars made with ternary blends of cement, ground granulated blast-furnace slag, and fly ash in this paper. Based on the results of a minimum of seven design points, three special cubic polynomial models were used to establish the strength-predicting equations at different ages for the mortars. Five experimental checkpoints were also designed to verify the precision of the equations. With few exceptions, the errors of the predicted values were within 10%, and most were within 5%. At last, the ternary diagrams of compressive strengths at different ages were also drawn, and the compressive strengths could also be predicted from the iso-strength contour lines on them. The paper provides a rational way for determining the optimal proportions of two mineral admixtures when they are used in concrete. © *1997 Elsevier Science Ltd*

Introduction

The incorporation of ggbfs or fly ash in concrete can lead to many technical advantages[1]. When the two mineral admixtures are used together, better results can always be achieved[2]. However, the ternary blends of cement, ggbfs and fly ash used to be developed for massive concrete with the purpose of avoiding the development of thermal cracking due to the heat of cement hydration[3]. In fact, high performance concrete with the principal characteristics of strength, dimensional stability, impermeability, and high workability can also be produced using the ternary blends[4]. When the ternary blends are used for producing HPC, determining the proportions of cement, ggbfs and fly ash is essential. A rational way with better accuracy and a minimum of test mixes should be found to evaluate the strength effect of the ternary blends.

There are a few ways to study the strength effect of the ternary blends. The entire simplex-centroid design used by E. Douglas *et al.* is encouraging[5]. The complex-centroid design is also used in other fields, for example, N. Standish *et al.* once used it on porosity calculations of ternary mixtures of particles[6].

However, the proportions of cement, ggbfs and fly ash in the ternary blends should have upper and lower bounds in order to meet the requirement for early strength of concrete.

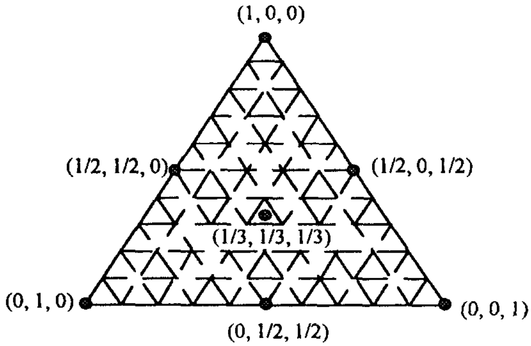

FIG. 1.
Test points of simplex-centroid design for ternary mixtures.

Either fly ash or ggbfs, especially the former, may hydrate fully only when $Ca(OH)_2$ which is released by cement hydration exists. The paper is intended to establish the strength-predicting equations of mortars with the ternary blends by the simplex-centroid design with upper and lower bounds, which is without doubt more accurate than those by entire simplex-centroid design.

The author combined the recommendations by ACI and JSCE, and determined the following bounds: cement: 30% ~ 100%; ggbfs: 0% ~ 70; fly ash: 0% ~ 40%[7][8].

Theory

The simplex-lattice design was introduced by Scheffe in 1958. Based on the simplex-lattice design, some modified designs have since been developed, including the simplex-centroid design[9].

Let n be the number of components and x_i the proportion of the ith component in a mixture, so that there are constraints:

$$x_i \geq 0 \ (i = 1, 2, \ldots\ldots n); \quad x_1 + x_2 + \ldots\ldots + x_n = 1 \tag{1}$$

In the simplex-centroid design, $2^n - 1$ measurements are taken, one on each of the following: the n pure components, the C_n^2 binary mixtures with equal proportions, the C_n^3 ternary mixtures with equal proportions, and the n-nary mixture with equal proportions. A polynomial which has as many coefficients as the number of measured points in this design is adopted to regress the relationship between any component proportions and the corresponding response:

$$Y = \sum_{1 \leq i \leq n} \beta_i x_i + \sum_{1 \leq i < j \leq n} \beta_{ij} x_i x_j + \sum_{1 \leq i < j < k \leq n} \beta_{ijk} x_i x_j x_k + \ldots\ldots + \beta_{12\ldots\ldots n} x_1 x_2 \ldots\ldots x_n \tag{2}$$

If the mixture is composed by three components, then

$$Y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \beta_{123} x_1 x_2 x_3 \tag{3}$$

TABLE 1

Seven Test Points for Mortars of Ternary Blends

	1	2	3	4	5	6	7
x_1	1.00	0.30	0.45	0.65	0.65	0.30	0.54
x_2	0.00	0.70	0.15	0.35	0.00	0.35	0.23
x_3	0.00	0.00	0.40	0.00	0.35	0.35	0.23

Fig. 1 shows the corresponding points on the ternary diagram.

When the component proportions have upper and lower bounds, the simplex-centroid design should be modified. For the case of lower bounds, we can transform it into entire simplex-centroid design by pseudo-components as follows:

Let $a_i \geq 0$ be the lower bound for component i, and let $L = \sum_{i=1}^{n} a_i$. Note that a_i could be zero for some components and L must be less than 1, since otherwise there would be no region satisfying the constraints. The pseudo-components $x_i^* = \dfrac{x_i - a_i}{1 - L}$ are linear transformations of the original components such that there are no constraints on the pseudo-components. All the techniques for modelling over the entire simplex can be used with the pseudo-components. Once a design has been chosen in the pseudo-components, the settings of the original components can be obtained using the equation

$$x_i = a_i + (1 - L)x_i^* \tag{4}$$

For the case with both upper and lower bounds on the component proportions, one approach starting with a design using pseudo-components that takes into account the lower bounds can be used. Any design points having component proportions that exceed their upper bounds are modified by replacing those component proportions with their upper bounds and increasing the remaining component proportions by equal amounts. With several upper bounds the procedure may require some iteration since the component proportions that are raised could exceed their upper bounds.

In the ternary blends of mortars, the proportion bounds of cement, ggbfs and fly ash are: $0.3 \leq x_1 \leq 1.0$, $0.0 \leq x_2 \leq 0.7$, and $0.0 \leq x_3 \leq 0.4$, respectively. According the design discussed above, the seven design points were obtained (given in Table 1). Based on the results of the seven design points, the compressive strength equations of mortars made with portland cement, ggbfs and fly ash blends could be regressed by special cubic polynomial models.

Materials and Test

The 525R portland cement produced by Jidong Cement Plant in Hebei Province was used in all the mortar mixtures. The ground granulated blast-furnace slag was obtained from steelworks in Beijing. The fly ash produced in Yuanbaoshan Power Plant in Inner Mongolia was used. The chemical and physical characteristics of them are described in Table 2.

TABLE 2

Chemical and Physical Properties of the Cementitious Materials

	Cement	GGBFS	Fly ash
Chemical analysis (%)			
SiO_2	24.36	33.56	58.64
Al_2O_3	5.40	11.40	19.78
Fe_2O_3	3.72	0.33	9.56
TiO_2	—	1.34	0.91
CaO	62.48	40.39	4.42
MgO	2.44	11.20	2.08
SO_3	0.14	—	0.41
K_2O	0.92	0.57	2.64
Na_2O	0.33	0.57	0.87
MnO	—	0.09	—
P_2O_5	—	0.05	—
f-CaO	0.60	—	—
Loss on ignition	—	—	0.25
Density (Kg/m^3)	3110	2910	2200
Fineness: 80μm retained (%)	5.0	3.2	2.0

The mortar specimens were prepared according to "The Standard for Cement Testing in China", which specifies a water to cementitious material ratio of 0.44 and a sand to cementitious material ratio of 2.5 by weight.

The specimens were stored in water with temperature of 20 ± 2°C for curing one day after being cast. Compressive strengths of the mortar specimens were determined at 7, 28 and 56 days. The strength-predicting equations could be established by solving three 7-variable linear equations. The compressive strengths at different ages of any mortar of the ternary blends can be obtained by putting the component proportions into the corresponding cubic polynomial models.

TABLE 3

The Component Proportions and Compressive Strengths of Mortars with Ternary Blends

No.	Component proportions			Compressive strengths (MPa)		
	Cement	GGBFS	Fly ash	7 days	28 days	56 days
1	1.0	0.0	0.0	37.3	51.9	57.0
2	0.3	0.7	0.0	12.4	33.2	42.0
3	0.45	0.15	0.4	12.0	24.2	40.1
4	0.65	0.35	0.0	22.2	40.9	55.6
5	0.65	0.0	0.35	21.5	33.1	43.8
6	0.3	0.35	0.35	10.5	21.4	39.0
7	0.54	0.23	0.23	17.8	34.4	48.4
8	0.8	0.0	0.2	29.3	40.3	49.0
9	0.5	0.3	0.2	15.2	34.1	48.5
10	0.5	0.5	0.0	16.9	38.5	49.9
11	0.7	0.15	0.15	25.4	41.6	52.1
12	0.5	0.2	0.3	17.5	31.2	45

TABLE 4

Strength-Predicting Equations at Different Ages

Ages (days)	Strength-predicting equations
7	$Y = 37.3x_1 + 8.2x_2 - 38.6x_3 - 21.6x_1x_2 + 47.3x_1x_3 + 98.6x_2x_3 - 131.7x_1x_2x_3$
28	$Y = 51.9x_1 + 29.2x_2 - 46.3x_3 - 13.5x_1x_2 + 68.4x_1x_3 + 28.7x_2x_3 + 68.4x_1x_2x_3$
56	$Y = 57.0x_1 + 20.6x_2 - 10.5x_3 + 49.8x_1x_2 + 45.9x_1x_3 + 72.1x_2x_3 - 14.0x_1x_2x_3$

Test Results and Discussion

The component proportions and compressive strengths at different ages of the twelve test mortars are listed in Table 3. The strength-predicting equations based on the 1-7 test mixes are listed in Table 4, where x_1, x_2, and x_3 are weight proportions of portland cement, ggbfs, and fly ash, respectively. On the basis of the strength-predicting equations, we can calculate the compressive strengths of mortars of ternary blends with component proportions satisfying specific bounds. The comparison of the predicted values and experimental ones for checkpoints with number of 8-12 is shown in Table 5. It can be seen that the accuracy of most predicted values is 95% or better.

The ternary diagrams for compressive strengths based on the strength-predicting equations are shown by Fig. 2-4. The ternary diagrams can be used to predict the compressive strengths of mortars visually.

From the test results the following observations can be made:

1. The compressive strengths at 7 days of mortars with water to cementitious material ratio of 0.44 are almost proportional to the proportions of portland cement.
2. The contribution of ggbfs on the strength gain from 7 to 28 days is the largest, and the one of fly ash is the smallest.
3. The contribution of ggbfs on the strength gain from 28 to 56 days is the largest, and the one of cement is the smallest.

It is conceivable that the contribution on the later strength gain (after 56 days) of fly ash will be the largest one.

TABLE 5

The Predicted and Experimental Compressive Strength Values of Checkpoints

No.	Experimental values (MPa)			Predicted values (MPa)		
	7 days	28 days	56 days	7 days	28 days	56 days
8	29.3	40.3	49.0	29.7	43.2	50.8
9	15.2	34.1	48.5	16.9	34.0	48.6
10	16.9	38.5	49.9	17.4	37.2	51.3
11	25.4	41.6	52.1	24.4	41.3	52.9
12	17.5	31.2	45.0	15.6	30.6	45.2

A----Cement
B----GGBFS
C----Fly ash

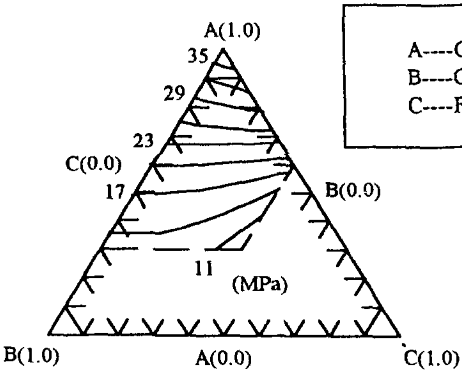

FIG. 2.
Ternary diagram with the iso-strength contour lines at 7 days.

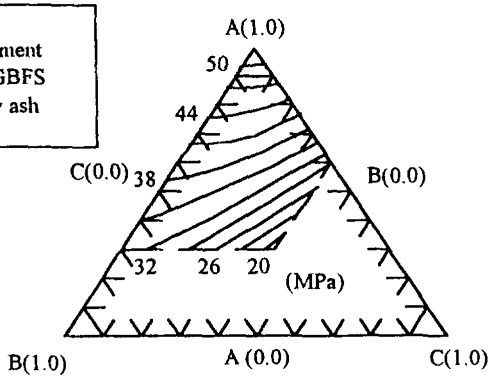

FIG. 3.
Ternary diagram with the iso-strength contour lines at 28 days.

Conclusions

1. The simplex-centroid design with upper and lower bounds of component proportions can be used to study the compressive strengths of mortars made with ternary blends of portland cement, ground granulate blast-furnace slag, and fly ash. Based on the results of a minimum of seven design points, the strength-predicting equations can be established.
2. The compressive strength values at different ages of mortars of ternary blends can be calculated with an excellent accuracy by the strength-predicting equations, which are no doubt more accurate than those from entire simplex-centroid design.

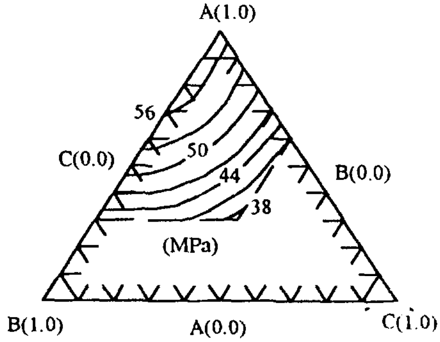

FIG. 4.
Ternary diagram with the iso-strength contour lines at 56 days.

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