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APPROXIMATE ELASTIC MODULI OF LIGHTWEIGHT AGGREGATE

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ABSTRACT

This study presents a method for the estimation of the elastic modulus of lightweight aggregate. In order to investigate the elastic modulus of aggregate and the effect of aggregate content on the elastic modulus of lightweight concrete, cylindrical specimens with different volume ratios (volume ratio of coarse aggregate/total aggregate) and various water/cement ratios were cast and tested. Both single inclusion and double inclusion models were applied to predict the elastic moduli of two-phase and three-phase cement-based composite materials, respectively. The elastic moduli of sand and coarse aggregate were derived from the experimental results using the theoretical models. © *1997 Elsevier Science Ltd*

Introduction

A composite material can be defined as a combination of at least two different materials. Usually the properties of multiphase composite have different properties of the original components. It is appropriate to consider concrete as a cement-based composite which consists of aggregate embedded in a matrix of hydrated cement paste.

The overall mechanical behavior of composite materials has been extensively studied. The elastic moduli of the concrete composite materials are given as a function of properties and volume fraction of the matrix and inclusion. The elastic properties of natural rock aggregate as inclusion can obtained from general information about the rock or directly measured on the bedrock itself, however it is not easy to obtain the elastic properties of artificially-produced small particles such as lightweight aggregates (1). Zhang and Gjϕrv (2) have investigated the characteristics of lightweight aggregate and Nilsen et al. (1) presented a method for estimating the elastic modulus. Chang and Shieh (3) investigated the fracture characteristics of lightweight concrete.

By considering concrete as a two-phase material, Aitcin and Mehta (4), Baalbaki et al. (5) demonstrated that the elastic modulus of concrete is influenced by the elastic properties and volume fraction of aggregates. Hirsch (6) derived an equation to express the elastic modulus of concrete in terms of an empirical constant, and also provided some experimental results for the elastic moduli of concretes with different aggregates.

Voigt's (7) approximation yielded the upper bound and the Reuss's (8) approximation yielded the lower bound of the average elastic moduli. Hashin and Shtrikman (9) proposed the variational principle to find bounds on the average elastic moduli of composite materials which were better than the Voigt and Reuss bounds. Hansen (10) developed mathematical models to predict the elastic moduli of composite materials based on the individual elastic modulus and volume fraction of the components. Mori and Tanaka (11) applied the concept of average field to analyze macroscopic properties of composite materials. The average field in a body contains inclusions with eigenstrain. In addition, the shape effect of dispersoids was introduced in Eshelby's (12) method to assess the properties of composite materials. The recent development of evaluating overall elastic modulus and overall elastic-plastic behavior was reviewed by Mura (13), Nemat-Nasser and Hori (14). Yang and Huang (15) proposed a double inclusion model for approximating elastic modulus of concrete by employing Mori-Tanaka Theory and Eshelby's Method.

In this study, the elastic modulus of cement paste, mortars and lightweight concrete were obtained in the laboratory. Cement paste was considered as matrix. By considering mortar as a two-phase material, single inclusion model (Mori-Tanaka method) (11) was used to evaluate the equivalent elastic modulus of fine aggregate. By considering lightweight concrete as a three-phase composite, double inclusion model (15) was used to evaluate the elastic modulus of lightweight coarse aggregate.

Experimental Program

In this study, mortar was considered as a composite material in which sand particles were embedded in a matrix of hardened cement paste. Concrete was a combination of cement paste, sand, and coarse aggregate. For the test program, three artificial aggregates were selected. The aggregates were made cement and fly ash through a cold-pelletizing process. Water was the wetting agent acting as coagulant so that the wet mixture would be pelletized through the rolling motion in a tilted pan. Type A, type B, and type C aggregates were made of cement and fly ash with fly ash/cement ratio of 0.1, 0.15, and 0.2 (by weight), respectively. The physical properties of the aggregates are shown in Table 1.

Cement Paste (Matrix). Cement paste specimens were made of Type I cement and water with different water/cement ratios (w/c = 0.3, 0.4, and 0.5). Cement paste cylinders (ϕ 100 × 200 mm) were cast and cured in water until the time of testing. At the age of 28 days, the elastic moduli of the specimens were measured according to ASTM Test Method for Static Modulus of Elasticity and Poisson's Ratio of Concrete in Compression (C-469-81). All cylinders were ground and polished before testing to achieve smooth end surface. A testing machine with 100 KN load capacity was used.

TABLE 1

Physical Properties of Aggregate

Type of Aggregate	Unit Weight, (SSD) (g/cm^3)	Unit Weight, (OD) (g/cm^3)	Water Absorption, (30 min.) (percent)
Type A	1.62	1.22	33.39
Type B	1.66	1.28	29.97
Type C	1.73	1.43	21.05

TABLE 2

Mix Proportions (kg/m^3)

Materials	w/c=0.3	w/c=0.4	w/c=0.5
Water	178.43	209.77	235.49
Cement	626.06	537.86	470.97
Superplasticizer	9.39	5.38	0
Fine Aggregate			
A, B, C3	1095.8	1095.8	1095.8
A, B, C4	939.2	939.2	939.2
A, B, C5	782.7	782.7	782.7
A, B, C6	626.2	626.2	626.2
Coarse Aggregate			
A3	291.6	291.6	291.6
A4	388.8	388.8	388.8
A5	486.0	486.0	486.0
A6	583.2	583.2	583.2
B3	298.8	298.8	298.8
B4	398.4	398.4	398.4
B5	498.0	498.0	498.0
B6	597.6	597.6	597.6
C3	311.4	311.4	311.4
C4	415.2	415.2	415.2
C5	519.0	519.0	519.0
C6	622.8	622.8	622.8

Mortar and Concrete (Composite). Mortar specimens were made from Type I cement, superplasticizer, water, and natural sand with the same water/cement ratio as cement paste. In this study, mortars with a sand volume ratio of 60% were tested.

Lightweight concrete specimens were cast using Type I cement, superplasticizer, water, natural sand and lightweight coarse aggregate. Three water/cement ratios (w/c = 0.3, 0.4, and 0.5) and four different volume ratios of coarse aggregate (volume ratio of coarse aggregate/total aggregate, $\frac{CA}{TA}$ = 0.3, 0.4, 0.5, and 0.6) were considered in the mix proportions. In order to keep the water/cement ratio as constant as possible, the lightweight coarse aggregate was immersed in water for 30 minutes before mixing and the surface was dried with towels. The lightweight concrete mix design is given in Table 2. Notation for the specimens is that the first letter indicates three different aggregate A, B, and C, and the second number is the volume ratios of coarse aggregates. Mortar and concrete cylinders (ϕ 100 × 200 mm) were cast and cured. At the age of 28 days, the elastic moduli and compressive strength of the specimens were measured according to ASTM C 469-81 and ASTM C 39-81, respectively.

Theoretical Background

In this study, mortar is considered as a two-phase (cement paste and fine aggregate) composite materials and concrete is considered as a three-phase (cement paste, fine aggregate, and coarse aggregate) composite materials. The inclusions are randomly embedded in an infinite matrix. The calculation is divided into two stages. In the first stage, the equivalent elastic modulus of fine aggregate was calculated by single inclusion model for a two-phase composite. The second, double inclusion model for a three-phase composite was used to calculate the elastic modulus of lightweight coarse aggregate.

Single Inclusion Model. The theoretical model based on Mori-Tanaka theory and Eshelby's method which the stress disturbance in the applied compressive stress, due to inhomogeneities can be simulated by the eigenstress caused by the fictitious misfit strain. The fictitious misfit strain (eigenstrain), was introduced to simulate the inhomogeneity effect. This model can provide an evaluation of average elastic relationships of cement-based materials with spherical inhomogeneities. The overall average elastic moduli of cement-based composite $\bar{\underset{\sim}{C}}$ was given by

$$\bar{\underset{\sim}{C}}=\left\{\underset{\sim}{C}^{-1}+f\left[\left\{(1-f)\left(\underset{\sim}{C}^{*}-\underset{\sim}{C}\right)\underset{\sim}{S}-f\left(\underset{\sim}{C}-\underset{\sim}{C}^{*}\right)+\underset{\sim}{C}\right\}^{-1}\right]^{-1}\left(\underset{\sim}{C}-\underset{\sim}{C}^{*}\right)\underset{\sim}{C}^{-1}\right\}, \quad (1)$$

where $\underset{\sim}{C}$ and $\underset{\sim}{C}^{*}$ are the elastic moduli tensor of matrix and aggregate, respectively. f is the volume ratio of inclusion, and $\underset{\sim}{S}$ is the Eshelby's tensor. The Eshelby tensor is a function of the geometry of the inclusion and Poisson's ratio of the matrix (see Appendix A).

Double-Inclusion Method. The double-inclusion method is applied to calculate the equivalent elastic modulus of coarse lightweight aggregate. The overall elastic moduli of concrete composite materials are investigated in this study by employing the theory of micromechanics. The inclusions are divided into two groups: fine aggregate and coarse aggregate. The overall elastic moduli of the concrete composite materials are given as a function of properties and volume ratio of the following three components: fine aggregate, coarse aggregate, and cement paste. In the pervious work (15), a composite material is simulated by a homogeneous material with uniform stiffness $\underset{\sim}{C}$ and distributing eigenstrains $\underset{\sim}{\varepsilon_1}^{*}$ in the domain of fine aggregate and $\underset{\sim}{\varepsilon_2}^{*}$ in the domain of coarse aggregate, respectively. The distributing eigenstrains $\underset{\sim}{\varepsilon_1}^{*}$ and $\underset{\sim}{\varepsilon_2}^{*}$ are calculated as

$$\left\langle\underset{\sim}{\varepsilon_1}^{*}\right\rangle=\alpha\underset{\sim}{\sigma}^{o}, \quad (2)$$

$$\left\langle\underset{\sim}{\varepsilon_2}^{*}\right\rangle=\beta\underset{\sim}{\sigma}^{o}. \quad (3)$$

α and β are shown in the Appendix B. $\underset{\sim}{\sigma}^{o}$ is an applied uniform stress. The average elastic moduli tensor of concrete composite materials, $\bar{\underset{\sim}{C}}$, for three-phase is given by

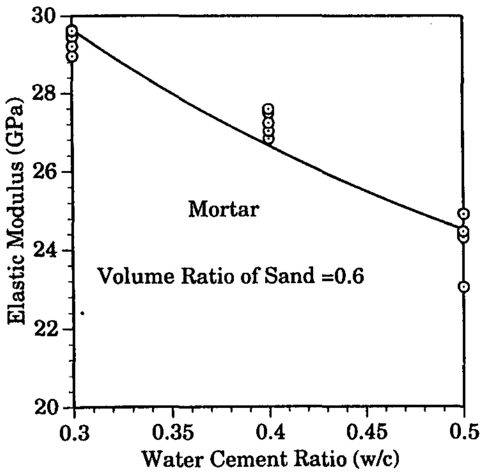

FIG. 1.
Elastic modulus vs. w/c ratio curve.

$$\bar{\underset{\sim}{C}} = \left(\underset{\sim}{C}^{-1} + f_1\alpha + f_2\beta\right)^{-1}, \tag{4}$$

where f_1 and f_2 are the volume fraction of fine aggregate and coarse aggregate, respectively.

Results and Discussions

Considering mortar as a two-phase composite materials, i.e., cement paste is considered as matrix and fine aggregate is considered as inclusion. Figure 1 is the elastic moduli vs. water/cement ratio curve for the mortar with a fine aggregate volume ratio of 0.6. Test results show that the elastic modulus of mortar decreases as water-cement ratio increases.

Elastic Modulus of Fine Aggregate. The Poison's ratio of cement paste and mortar was assumed to be 0.2 for the computation of the elastic modulus tensor of the matrix and the mortar. The volume ratio of sand is 0.6. The elastic moduli of cement paste and mortar were determined experimentally and presented in Table 3. Equation 1 was used to calculate the

TABLE 3

Elastic Moduli of Cement Paste, Mortar, and Fine Aggregate

W/C	Elastic modulus, (GPa)		
	*Cement Paste (Matrix), (Experimental)	*Mortar (Two-Phase Composite), (Experimental)	Fine Aggregate (Inclusion), (Theoretical)
0.3	21.43	29.38	36.56
0.4	18.41	27.45	36.43
0.5	13.85	24.23	36.90
			Average= 36.63

* Average of five specimens

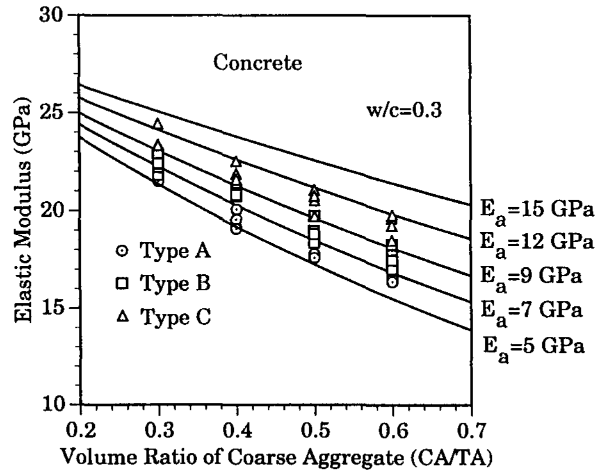

FIG. 2.
Elastic modulus vs. volume ratio curves (w/c = 0.3).

elastic modulus of the fine aggregate based on single inclusion model. Table 3 shows the elastic modulus of the fine aggregate obtained from the single inclusion model. The average elastic modulus of fine aggregate is 36.63 GPa.

Elastic Modulus of Coarse Aggregate. A three-phase composite with spherical inclusions was considered in the theoretical approach. The elastic moduli of cement paste and fine aggregate are shown in Table 3. The Poisson ratio of cement paste, fine aggregate, and lightweight coarse aggregate are assumed to be 0.2. The volume ratios of fine aggregate and lightweight coarse aggregate are calculated from Table 2. The elastic modulus of lightweight coarse aggregate is computed from Eq. 4 based on the elastic moduli, Poisson ratios, and volume ratios of cement paste, fine aggregate, and lightweight concrete. Figures 2, 3 and 4

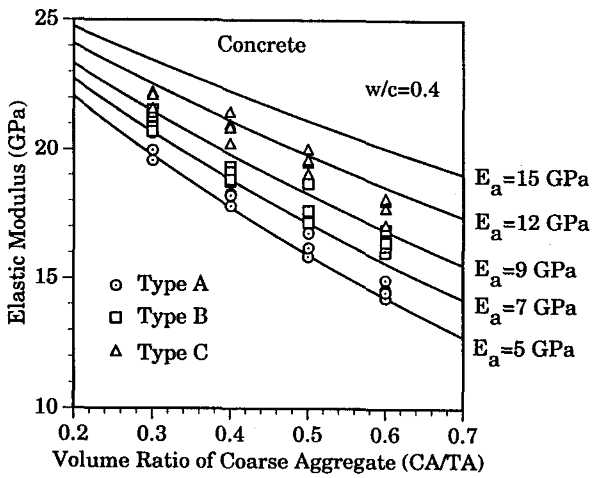

FIG. 3.
Elastic modulus vs. volume ratio curves (w/c = 0.4).

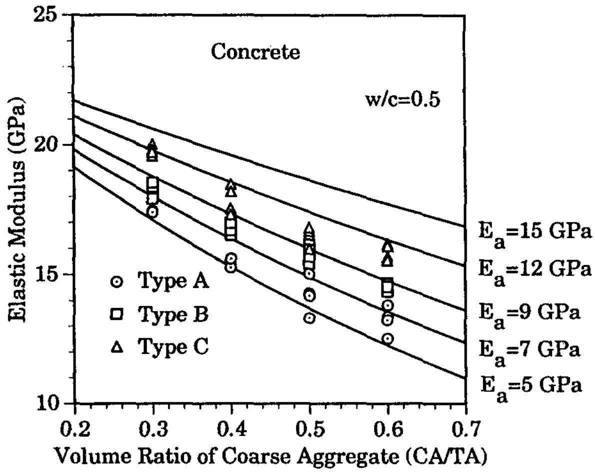

FIG. 4.
Elastic modulus vs. volume ratio curves (w/c = 0.5).

show the relationships between the elastic modulus of lightweight concrete and the coarse aggregate volume ratio ($\frac{CA}{TA}$) for water/cement ratios of 0.3, 0.4, and 0.5, respectively. The corresponding theoretical results are also illustrated in the figures. The graphs correlate the volume ratio of the aggregate with the elastic modulus of the concrete for various elastic modulus of coarse aggregate (E_a). For the elastic modulus of Type A aggregate, the experimental results lie between the curves at which E_a are assumed to be 5 GPa and 7 GPa. Comparing the experimental data with the theoretical results, the elastic modulus of Type B and C aggregate are 7 GPa to 9 GPa and 9 GPa to 12 GPa, respectively. It also appears that the influence of the lightweight concrete elastic modulus decreases as the volume ratio of aggregate and w/c increased.

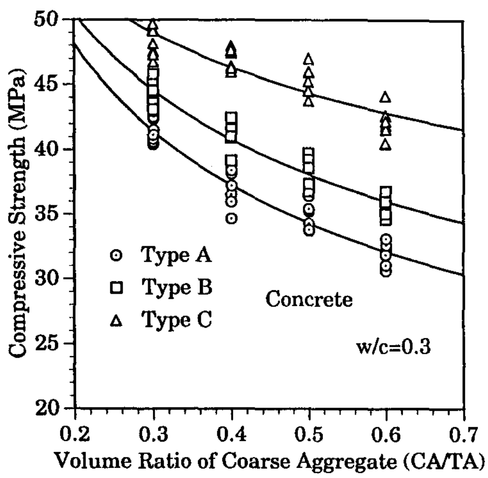

FIG. 5.
Compressive strength vs. volume ratio curves (w/c = 0.3).

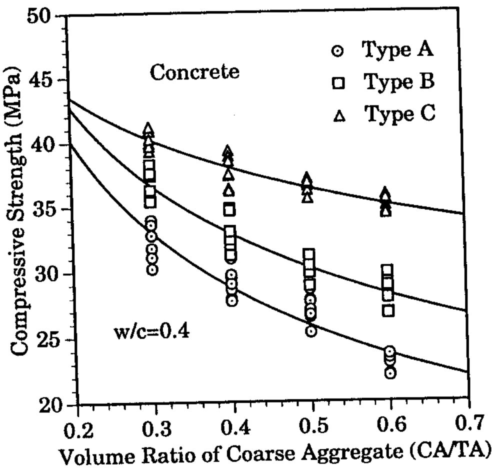

FIG. 6.
Compressive strength vs. volume ratio curves (w/c = 0.4).

Figures 5, 6 and 7 show the compressive strength of lightweight concretes vs. $\frac{CA}{TA}$ ratio for water/cement ratios of 0.3, 0.4, and 0.5, respectively. The compressive strength of lightweight concrete decreases as $\frac{CA}{TA}$ ratio increases and the compressive strength of lightweight concrete decreases as water/cement ratio increases.

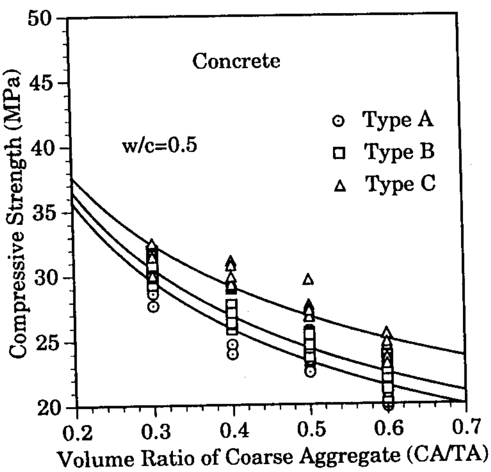

FIG. 7.
Compressive strength vs. volume ratio curves (w/c = 0.5).

Conclusions

The elastic modulus of concrete is influenced by the elastic properties and the volume fraction of the matrix (cement paste), fine aggregate, and coarse aggregate. Based on the single inclusion model, the average elastic modulus of fine aggregate can be derived from the testing results. In this study, the average elastic modulus of fine aggregate in Taiwan is about 36.63 GPa. In addition, based on double inclusion model, the estimated elastic moduli of the three different lightweight aggregates, Type A, B, and C, is about 5 GPa to 7 GPa, 7 GPa to 9 GPa, and 9 GPa to 12 GPa, respectively.

Acknowledgment

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Appendix A

The Eshelby's tensor $\underset{\sim}{S}$ for sphere inclusion is listed below (16).

$$S_{1111} = S_{2222} = S_{3333} = \frac{7-5\nu}{15(1-\nu)}.$$

$$S_{1122} = S_{2233} = S_{3311} = S_{1133} = S_{2211} = S_{3322} = \frac{5\nu-1}{15(1-\nu)}.$$

$$S_{1212} = S_{2323} = S_{3131} = \frac{4-5\nu}{15(1-\nu)}.$$

Appendix B

The calculation of parameters α and β

$$A = \left[(1-f_1)\underset{\sim}{C} + f_1 \underset{\sim}{C_1^*}\right]\left(\underset{\sim}{S} - \underset{\sim}{I}\right) - \underset{\sim}{C_1^*}\underset{\sim}{S}$$

$$B = \left[(1-f_2)\underset{\sim}{C} + f_2 \underset{\sim}{C_2^*}\right]\left(\underset{\sim}{S} - \underset{\sim}{I}\right) - \underset{\sim}{C_2^*}\underset{\sim}{S}$$

$$M = \left(\underset{\sim}{C_1^*} - \underset{\sim}{C}\right)\left(\underset{\sim}{S} - \underset{\sim}{I}\right)$$

$$N = \left(\underset{\sim}{C_2^*} - \underset{\sim}{C}\right)\left(\underset{\sim}{S} - \underset{\sim}{I}\right)$$

$$\begin{aligned}\left\langle \underset{\sim}{\varepsilon_1}^* \right\rangle &= -\left(\underset{\sim}{I} - f_1 f_2 A^{-1} M B^{-1} N\right)^{-1} A^{-1}\left[f_2 M B^{-1}\left(\underset{\sim}{C_2^*}\underset{\sim}{C} - \underset{\sim}{I}\right) + \left(\underset{\sim}{C_1^*}\underset{\sim}{C} - \underset{\sim}{I}\right)\right]\underset{\sim}{\sigma}^o \\ &= \alpha \underset{\sim}{\sigma}^o\end{aligned}$$

$$\begin{aligned}\left\langle \underset{\sim}{\varepsilon_2}^* \right\rangle &= B^{-1}\left\{f_1 N\left(\underset{\sim}{I} - f_1 f_2 A^{-1} M B^{-1} N\right)^{-1} A^{-1}\left[f_2 M B^{-1}\left(\underset{\sim}{C_2^*}\underset{\sim}{C} - \underset{\sim}{I}\right) - \left(\underset{\sim}{C_1^*}\underset{\sim}{C} - \underset{\sim}{I}\right)\right] + \left(\underset{\sim}{C_2^*}\underset{\sim}{C} - \underset{\sim}{I}\right)\right\}\underset{\sim}{\sigma}^o \\ &= \beta \underset{\sim}{\sigma}^o\end{aligned}$$