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SIZE EFFECT ON FRACTURE ENERGY OF CONCRETE DETERMINED BY THREE-POINT BENDING

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ABSTRACT

The three-point bending test of a notched specimen is a common method of determining the fracture energy of concrete. Because there exist size effects on the test results, its validity is in doubt. Theoretical analysis shows that the size effect is mainly caused by inaccuracy in the formula used to calculate the fracture energy, but not due to shortcoming of the test method. The theoretical analysis has been verified with some experimental results on mortar. © *1997 Elsevier Science Ltd*

Introduction

The fracture energy G_F used in fracture mechanics of concrete and rock comes from the concept of specific fracture energy. It is defined as the energy to create a unit area of fracture surface. There are many experimental results on the fracture energy of different materials, and most of these were obtained using the three-point bending test (TB). This method is widely used, because it is easily performed and needs less equipment. Many researchers have studied TB theoretically and have developed the method in practice. When Petersson first studied the fracture energy of concrete, he utilized TB(1). On the basis of many results by different researchers, RILEM TC-50FMC has recommended a series of rules on determination of the fracture energy of concrete by TB. TB has a good base in determining the fracture energy of concrete.

However most researchers have found that there exist size effects of varying extent when TB is used to determine the fracture energy of concrete, rock and other materials. As a material parameter, fracture energy should be a constant and independent of specimen size. Therefore, some researchers have doubted about the validity of TB.

Theoretically, this paper discusses further the size effect on fracture energy determined by TB. This paper also gives experimental results of the fracture energy of mortar determined by us with TB. We find that these results decrease with increasing specimen size, which is contrary to the usual size effect on fracture energy of concrete. Based on our theoretical analysis and experimental results, the size effect on fracture energy in TB can be better interpreted.

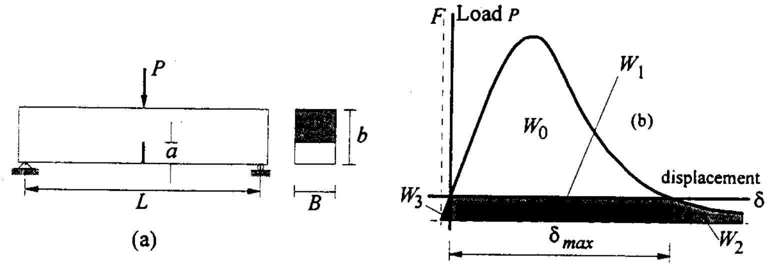

FIG. 1.
(a) Shape of specimen and type of loading; (b) Typical load-displacement curve.

Three-Point Bending Test

In Fig. 1, the shape of the specimen, the type of loading and experimental results of typical load vs. loading point displacement (P-δ) curve are shown. The work dissipated in completely fracturing a beam with a notch in the middle is:

$$w = w_0 + w_1 + w_2 + w_3 \tag{1}$$

where w_0 is the work done by load P; w_1, w_2 , and w_3 is the work done by self-weight of beam. Generally, w_3 is too small to consider. After proposing many hypotheses, Petersson deduced that $w_2 = w_1$, since $w_2 = 1/2mg\ \delta_{max}$, so the fracture energy can be calculated as Eq. 1.

$$G_F = \frac{W}{A_{lig}} = \frac{\left(\int_0^{\delta_{\max}} P d\delta + mg\delta_{\max}\right)}{A_{lig}} \tag{2}$$

where A_{lig} is the fracture area $[B(b-a)]$; m is the specimen mass, mg is the self-weight of specimen; b, B are the height and width of the beam, respectively; a is the depth of the notch.

Size Effect on Fracture Energy Determined by TB

As a material parameter, it was first thought that fracture energy was independent of loading type, specimen size, etc. While TB is a common method for measuring fracture energy, people naturally think of inspecting the validity of this method. Many experimental results showed that there was a size effect in determining fracture energy with TB. Since for concrete materials, the specimen size is usually larger, the size effect is more significant. There is no agreement upon whether the size effect is caused by inaccuracy of the calculation method or is a characteristic of the fracture energy. Considering experimental results now available, there are always size effects in the experimental results of the fracture energy of concrete determined by TB, and these results generally show that fracture energy increases as specimen size increases. Hillerborg (2) came to this conclusion based on some experimental results of different size beams tested by TB in 14 laboratories from 9 countries. However, it was be seen from our experimental results of G_F that there is a contrary size effect in mortar, and fracture energy decreases as the specimen size increases. These results are given in Table 1. Mechanical properties of the mortar were: tensile strength = 3.10 MPa,

TABLE 1

Test Results of Mortar

Specimen size (mm) (height × width × length)	Number of specimen	G_F (N/m)
40 × 40 × 160	18	94.2
70 × 70 × 280	12	89.8
100 × 100 × 400	6	76.6

compressive strength = 41.8 MPa, elastic modulus = 19.6 GPa, Poisson's ratio = 0.176. The sand was Chongqing superfine sand, the cement was 525# Portland cement and the notch-height ratio was 0.5.

Hilsdorf and his coworkers (3) also have studied the size effect on G_F of mortar with TB. Their results are identical to ours. Their test results are shown in Table 2. The size effect on fracture energy of mortar may increase the difficulty of interpreting the size effect on G_F.

Analysis of Size Effect on G_F Determined by TB

The author has previously discussed the size effect on G_F of concrete (4). The present paper discusses the size effect on G_F mainly in term of the calculating formula of G_F in TB, Eq.2.

The formula for G_F used in TB includes an (mg δ_{max}) term, in which m is self-weight of specimen, δ_{max} is the displacement when completely broken, i.e., load is zero at that point, as shown in Fig. 1(b). This term relates to the self-weight of the specimen. To better analyze the size effect on G_F, we first consider the proportion of this term to the whole energy. From the test results of Mindess (5) shown in Table 3, we can see that when the specimen size is 400 × 400 × 3360 mm, the self-weight of the specimen is 7.5 times the maximum load, so (mg δ_{max}) in Eq. 2 constitutes above 90 percent of the entire energy.

Let us consider the physical meaning of (mg δ_{max}) in Eq. 2. Petersson assumed that $w_2 = w_1$ when he proposed Eq. 2. w_1 is the work done by self-weight of specimen from zero deformation to δ_{max}, its value is $1/2mg\ \delta_{max}$, so the second term in Eq. 2 is $mg\ \delta_{max}$. But if $w_2 \neq w_1$, then the second term in Eq. 2 must be multiplied by a coefficient, k. It is obvious that when $w_2 > w_1$, $k > 1$; when $w_2 < w_1$, $k < 1$. The formula for G_F is then modified as follows:

$$G_F = \frac{\int_0^{\delta_{max}} P d\delta + kmg\delta_{max}}{A_{lig}} \quad (3)$$

TABLE 2

Test Results of Mortar by Hilsdorf (3)

Specimen size (mm) (height × width × length)	Number of specimen	G_F (N/m)
100 × 100 × 400	8	53
400 × 200 × 2000	7	49
800 × 400 × 4000	4	44

TABLE 3

Test Results on GF of Concrete (5)

Specimen size (cm) (height × width × length)	Specimen weight (kg)	Maximum load(N)	δ_{max} (mm)	G_F (N/m)	w_0 (Nm)	$mg\,\delta_{max}$ (Nm)	$\frac{mg\,\delta_{max}}{mg\,\delta_{max}+w_0}$
	18.43	623	0.76	78.0	0.251	0.137	0.356
10 × 10 × 84	18.12	623	1.00	104.3	0.325	0.181	0.358
(average)	17.48	356	0.89	58.8	0.128	0.153	0.546
		534	0.883	80.3	0.235	0.157	0.401
	147.7	712	0.64	59.1	0.238	0.927	0.824
20 × 20 × 168	147.0	1560	1.00	116.8	0.836	1.442	0.693
(average)	147.9	712	0.76	71.8	0.298	1.103	0.787
		995	0.800	82.6	0.457	1.157	0.717
	1198	1710	0.41	64.9	0.582	4.819	0.892
40 × 40 × 336	1194	1000	0.64	98.9	0.241	7.496	0.969
(average)	1182	2110	1.10	177.1	1.168	12.750	0.912
		1607	0.717	114.0	0.664	8.355	0.926

Note: compressive strength = 48.5 MPa (38 day), maximum aggregate size = 32mm

When the specimen size is small, whether or not $k = 1$ does not notably influence the calculated value of G_F. However when the specimen size is large, the value of k will notably influence the calculated value of G_F.

When Petersson proposed $w_2 = w_1$ he was actually assuming that after the mid-point displacement exceeded δ_{max}, $F(\delta)$ took the form, Eq.4.

$$F(\delta) = \frac{c}{\delta^2} \tag{4}$$

$$\text{since: } F(\delta_{max}) = \tfrac{1}{2} mg \tag{5}$$

$$\text{so: } c = \tfrac{1}{2} mg\delta_{max}^2 \tag{6}$$

$$W_2 = \int_{\delta_{max}}^{\infty} F(\delta) d\delta = \int_{\delta_{max}}^{\infty} \frac{mg\delta_{max}^2}{2\delta^2} d\delta = \frac{mg\delta_{max}}{2} \tag{7}$$

therefore $w_2 = w_1$. If the F(δ) vs. δ relationship is not accordance to Eq. 4, the function is still assumed to be a power function, and can be written as,

$$F(\delta) = \frac{c}{\delta^n} \tag{8}$$

$$\text{since: } F(\delta_{max}) = \tfrac{1}{2} mg, \text{so}: c = \tfrac{1}{2} mg\delta_{max}^n \tag{9}$$

$$\text{so:}\quad W_2 = \int_{\delta_{max}}^{\infty} F(\delta)d\delta = \int_{\delta_{max}}^{\infty} \frac{mg\delta_{max}^{n}}{2\delta^{n}}d\delta = \frac{mg\delta_{max}}{2(n-1)} \tag{10}$$

From Equation 10, when $n > 2$, $w_2 < w_1$, and the fracture energy calculated by Eq. 2 will show an increasing size effect with increasing specimen size; when $n < 2$, then $w_2 > w_1$, and G_F calculated by Eq. 2 will show a decreasing size effect with increasing specimen size.

If we know the functional relationship of the descending part of load-displacement [P(δ)–δ] curve, it is easy to verify that the above-mentioned analysis is reasonable.

Since one can not get experimentally the part of curve when $\delta \geq \delta_{max}$, then, the part of the curve nearest to δ max could be taken to approximately express the functional relationship of $\delta \geq \delta_{max}$. At the same time it should be noticed that when $\delta \leq \delta_{max}$, there is an approximate relationship between $P(\delta)$ and $F(\delta)$, Eq. 11,

$$P(\delta) + \frac{1}{2}mg = F(\delta) \tag{11}$$

To make the functional relationship of specimens of different sizes more comparable, for every size we select the area as,

$$\delta_1 \,|_{P(\delta_1)=0.1(P_{max}+0.5mg)} \leq \delta \leq \delta_{max} \tag{12}$$

We select usually about 10 points on each curve, group the selected points for the same specimen size, assume that $F(\delta)$ fits to Eq. 8, and then determine the value of n by means of regression.

The results for mortar are shown in Table 4, in which the coefficient of correlation is obtained when the relationship of $F(\delta)$ in Eq. 8 is changed into a linear relationship.

For mortar, it is obvious that when the F(δ) is assumed as the functional relationship of Eq. 8, n decreases as specimen size increases and is less than 2; then k in Eq. 3 is greater than 1. So, the size effect will inevitably decrease with increasing specimen size when G_F calculated by Eq. 2 (see Table 1). Because $w_2 = 1/(n\text{-}1)$, so, $k = n/2(n\text{-}1)$. When Eq. 3 is used to calculate G_F, the corrected values of G_F are shown in Table 5.

Conclusion and Discussion

If fracture energy is purely a material property, determined by a standard or agreed upon test method, then the values of G_F are comparable. For cement concrete, the fracture models used in analyzing most nonlinear problems need an exact value of G_F., and there is a concern

TABLE 4

The Approximate Functional Relation F(δ) of $\delta \geq \delta_{max}$ of Mortar

Specimen size (mm) (height × width × length)	Functional relation of $F(\delta)$	Value of n	Coefficient of correlation
40 × 40 × 160	$0.51\,\delta^{-1.740}$	1.740	0.91
70 × 70 × 280	$2.36\,\delta^{-1.479}$	1.479	0.73
100 × 100 × 400	$3.73\,\delta^{-1.411}$	1.411	0.88

Note: the unit of $F(\delta)$ is kg, the unit of δ is mm.

TABLE 5

The Corrected Value of G_F in Table 1 Calculated by Equation 3

Specimen size (mm) (height × width × length)	G_F calculated by equation (2)(N/m)	G_F calculated by equation (3)(N/m)	Value of k
40 × 40 × 160	94.16	95.03	1.18
70 × 70 × 280	89.82	95.15	1.54
100 × 100 × 400	76.61	87.25	1.72

whether there is size effect on G_F or not. At present it can not be concluded whether the size effect on G_F is a experimental phenomenon, is an attribute of this property itself. From the above-mentioned analysis, it appears that the inaccuracy of the formula used in the calculations is one of the principal reasons that result in size effect on G_F determined by TB. From the results in Table 5, the size effect on G_F of mortar is remarkably reduced after correcting the formula for fracture energy. Though the values of n in Table 4 are approximate and thus the corrected values in Table 5 are approximate too, this gives a better explanation of size effects theoretically. Further studies must be performed in analyzing G_F of concrete. If n in Eq. 8 appears to increase with an increase of specimen size, then it is easier to interpret the two contrary phenomena of size effect on G_F of concrete and mortar. Since it is only an approximation to get the part of $F(\delta)$-δ curve beyond $\delta > \delta_{max}$, it is very important to have enough experimental results to verify of the theoretical analysis.

The authors think that TB is a feasible method in determination of fracture energy. The above-mentioned analysis shows that the principal reason for the size effect on G_F determined by TB is the inaccuracy of the formula, which results from not considering exactly the influence of self-weight. Whatever method is used, the influence of the self-weight of the specimen is not eliminated; however, the influence of self-weight in TB is easier to calculate. TB may be used as a standard test method of determining fracture energy if one fully considers its shortcomings.

If size effect is a real phenomenon, then the fracture energy of small specimens will be less influenced by size. Relatively, smaller specimen will induce less errors and cause less experimental difficulty. Therefore, a smaller specimen should be use so long as it can meet homogeneity requirements.

Acknowledgments

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