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# ON THE VALIDITY OF THE KATZ-THOMPSON EQUATION FOR PERMEABILITIES IN CONCRETE

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**ABSTRACT**

Hydraulic permeabilities for concrete and for concrete incorporating slag or fly ash have been determined at w/c = 0.65 and 0.55 at 7, 14, 21, and 28 days. Hydraulic permeabilities were also calculated using the Katz-Thompson equation. In all cases, the calculated hydraulic permeabilities were two orders of magnitude lower than experimentally measured hydraulic permeabilities. *© 1998 Elsevier Science Ltd*

## Introduction

Katz and Thompson used percolation concepts to derive an equation relating the permeability of saturated random porous media to microstructural descriptors and conductivities (1):

$$K' = \frac{1}{226}\left(\frac{\sigma}{\sigma_o}\right)(l_c)^2 \quad (\mathrm{m}^2) \tag{1}$$

where $K'$ is the hydraulic permeability (m$^2$), $\sigma$ and $\sigma_o$ are bulk and porewater conductivities respectively ($\Omega^{-1}\mathrm{m}^{-1}$), and $l_c$ is the critical pore diameter (m). Equation 1 was successfully demonstrated for sandstone and carbonate sedimentary rocks (1). Garboczi reasoned that Eq. 1 applies to cementitious materials (2). El-Dieb and Hooton reported that Eq. 1 did not hold for cement paste or concrete (3). The conclusion was recently challenged by Christensen, Mason, and Jennings (4). However, there are limitations for both experimental reports. El-Dieb and Hooton did not directly measure $\sigma/\sigma_o$ (3). Instead, $\sigma/\sigma_o$ was calculated from mercury intrusion curves using the equation (5),

$$\left(\frac{\sigma}{\sigma_o}\right) = \frac{l_{max}{}^{e}}{l_c}\cdot\phi\cdot S(l_{max}{}^{e}) \tag{2}$$

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TABLE 1
Concrete mix designs (kg $m^{-3}$).

| | S1 | | S2 | | S3 | |
|---|---|---|---|---|---|---|
| | M3 | M2 | M3 | M2 | M3 | M2 |
| Cement[1] | 363 | 352 | 126 | 122 | 127 | 123 |
| Slag | 0 | 0 | 234 | 226 | 0 | 0 |
| Flyash | 0 | 0 | 0 | 0 | 236 | 228 |
| Aggregate[2] | 1101 | 1066 | 1092 | 1055 | 1099 | 1064 |
| Sand | 727 | 704 | 721 | 696 | 725 | 702 |
| Water | 200 | 229 | 198 | 226 | 199 | 228 |
| w/c[3] | 0.55 | 0.65 | 0.55 | 0.65 | 0.55 | 0.65 |

[1]Type 50 cement.
[2]Aggregate was crushed limestone which passed 10 mm and was restained on #16 screen.
[3]w/c = water/(cement + slag + flyash).

TABLE 2
Results for mix 2 (w/c = 0.65).

| Time (d) | $l_c$ (m) $\times 10^{-7}$ | $l_{max}^e$ (m) $\times 10^{-7}$ | $S(l_{max}^e)$ | $\phi$ | $(\sigma/\sigma_o)_{calc}$* | $(\sigma/\sigma_o)_{meas}$ | $K_{calc}$ (m $s^{-1}$) $\times 10^{11}$** | $K_{meas}$ (m $s^{-1}$) $\times 10^{9}$ |
|---|---|---|---|---|---|---|---|---|
| | | | | S1 | | | | |
| 7 | 2.48 | 0.7 | 0.090 | 0.170 | 0.0044 | 0.0096 | 2.57 | 6.01 |
| 14 | 1.83 | 0.44 | 0.083 | 0.157 | 0.0032 | 0.0105 | 1.53 | 3.76 |
| 21 | 1.53 | 0.35 | 0.088 | 0.145 | 0.0029 | 0.0054 | - | - |
| 28 | 1.34 | 0.35 | 0.073 | 0.145 | 0.0028 | 0.0073 | 0.57 | 2.08 |
| | | | | S2 | | | | |
| 7 | 0.83 | 0.18 | 0.091 | 0.144 | 0.0029 | 0.0125 | 0.38 | 0.75 |
| 14 | 0.58 | 0.18 | 0.088 | 0.133 | 0.0037 | 0.0048 | 0.07 | 0.82 |
| 21 | 0.43 | 0.10 | 0.107 | 0.134 | 0.0035 | 0.0051 | 0.04 | 0.55 |
| 28 | 0.38 | 0.11 | 0.088 | 0.127 | 0.0032 | 0.0041 | 0.03 | 0.08 |
| | | | | S3 | | | | |
| 7 | 3.16 | 1.04 | 0.090 | 0.172 | 0.0051 | 0.0297 | 12.9 | 18.6 |
| 14 | 0.83 | 0.29 | 0.097 | 0.147 | 0.0051 | 0.0152 | 0.45 | 11.2 |
| 21 | 1.45 | 0.43 | 0.089 | 0.162 | 0.0044 | 0.0090 | 0.83 | 7.28 |
| 28 | 0.94 | 0.42 | 0.039 | 0.130 | 0.0023 | 0.0071 | 0.27 | 6.51 |

*Calculated with Eq. 2.
**Calculated with Eq. 1 and the relation,

$$K = K' \cdot \left( \frac{\rho \cdot g}{\eta} \right) \ (\mathrm{ms}^{-1})$$

where $K'$ is the hydraulic permeability given in Eq. 1, $\eta$ is the viscosity of the permeating fluid (Pa s), $\rho$ is the density of water (kg $m^{-3}$), and $g$ is gravitational acceleration (m $s^{-2}$),

TABLE 3
Results for mix 3 (w/c = 0.55).

| Time (d) | $l_c$ (m) $\times 10^{-7}$ | $l_{max}^e$ (m) $\times 10^{-7}$ | $S(l_{max}^e)$ | $\phi$ | $(\sigma/\sigma_o)_{calc}$* | $(\sigma/\sigma_o)_{meas}$ | $K_{calc}$ ($m\ s^{-1}$) $\times 10^{11}$** | $K_{meas}$ ($m\ s^{-1}$) $\times 10^{10}$ |
|---|---|---|---|---|---|---|---|---|
| | | | | S1 | | | | |
| 7 | 1.27 | 0.35 | 0.085 | 0.159 | 0.0037 | 0.0060 | 4.16 | 13.1 |
| 14 | 1.27 | 0.44 | 0.070 | 0.149 | 0.0035 | 0.0047 | - | - |
| 21 | 1.00 | 0.35 | 0.075 | 0.147 | 0.0039 | 0.0047 | 2.01 | 6.08 |
| 28 | 0.66 | 0.23 | 0.081 | 0.127 | 0.0037 | 0.0056 | 1.06 | 5.62 |
| | | | | S2 | | | | |
| 7 | 0.73 | 0.24 | 0.072 | 0.136 | 0.0032 | 0.0068 | 1.58 | 49.7 |
| 14 | 0.65 | 0.24 | 0.070 | 0.156 | 0.0040 | 0.00417 | 0.76 | 4.62 |
| 21 | 0.43 | 0.15 | 0.082 | 0.128 | 0.0036 | 0.0038 | - | - |
| 28 | 0.51 | 0.18 | 0.082 | 0.139 | 0.0041 | 0.0031 | 0.35 | 5.96 |
| | | | | S3 | | | | |
| 7 | 5.04 | 1.59 | 0.051 | 0.172 | 0.0028 | 0.0094 | 109 | 7.28 |
| 14 | 1.75 | 0.50 | 0.066 | 0.155 | 0.0029 | 0.0087 | 11.6 | 0.36 |
| 21 | 1.44 | 0.44 | 0.082 | 0.158 | 0.0039 | 0.0064 | - | - |
| 28 | 1.67 | 0.50 | 0.064 | 0.133 | 0.0025 | 0.0071 | 8.59 | 0.53 |

*Calculated with Eq. 2.
**Calculated with Eq. 1 and the relation,

$$K = K' \cdot \left(\frac{\rho \cdot g}{\eta}\right) \ (ms^{-1})$$

where $K'$ is the hydraulic permeability given in Eq. 1, $\eta$ is the viscosity of the permeating fluid (Pa s), $\rho$ is the density of water (kg $m^{-3}$), and $g$ is gravitational acceleration (m $s^{-2}$),

where $\phi$ is the porosity, $l_{max}^e$ is the pore width that produces the maximum electrical conductivity, and $Sl_{max}^e$ is the fractional volume of connected pore space involving pore widths $l_{max}^e$ or larger. The validity of Eq. 2 has not been proved for cementitious systems. Christensen, Mason, and Jennings (4) did not measure permeabilities; previously reported permeabilities for similar pastes were used (6).

Therefore, there is a twofold purpose to this investigation:

- to confirm the validity of Eq. 1 and Eq. 2 for concrete systems, including those incorporating pozzolans, when permeabilities, electrical conductivities and porosities are measured on specimens from the same mix, and
- to confirm the validity of Eq. 2 for concrete systems.

## Experimental

The concrete mix designs are given in Table 1. Samples were cured at 100% RH until required. The bulk concrete and extruded porewater conductivities were measured using the procedure and apparatus described previously (7,8). Porosities were determined using the

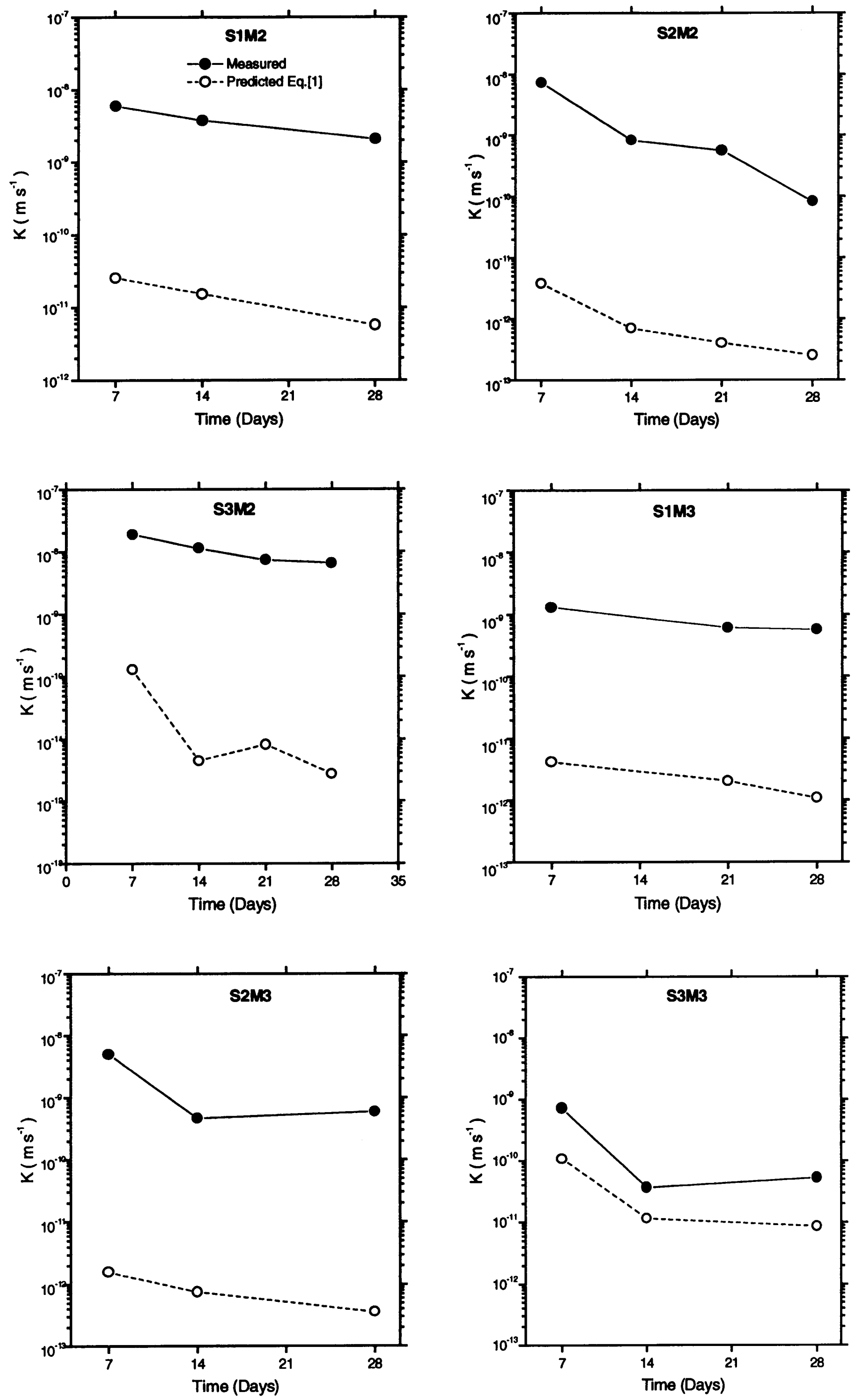


FIG. 1.
Measured and calculated hydraulic permeabilities as a function of time for all systems.

mercury intrusion technique (9). Hydraulic permeabilities were measured with an apparatus similar to that described by Daw (10).

## Results and Discussion

Values for $l_c$, $l_{max}^e$, $S(l_{max}^e)$, and $\phi$ were determined from mercury intrusion curves as described by El-Dieb and Hooton (3). The values are presented in Tables 2 and 3 for concrete with w/c = 0.65 and w/c = 0.55, respectively. Subsequently, $(\sigma/\sigma_o)_{calc}$ was calculated with Eq. 2. Values for $(\sigma/\sigma_o)_{meas}$ are also presented in Tables 2 and 3. In all cases, $(\sigma/\sigma_o)_{meas}$ are larger than $(\sigma/\sigma_o)_{calc}$, but $(\sigma/\sigma_o)_{calc}$ are within one order of magnitude. Considering the uncertainty in values from mercury intrusion curves, the agreement between $(\sigma/\sigma_o)_{calc}$ and $(\sigma/\sigma_o)_{meas}$ is good. Therefore, Eq. 2 appears to be valid for concrete.

Values for $K_{meas}$ are summarized in Tables 2 and 3 for concrete with w/c = 0.65 and w/c = 0.55, respectively. The data are also been plotted in Figure 1. As expected, the $K_{meas}$ data show that permeabilities decrease as hydration time increases. Further, with the exception of the flyash concrete at w/c = 0.65, permeabilities for the pozzolan and cement concrete are less than concrete made without pozzolans. $K_{meas}$ for flyash concrete at w/c = 0.65 depends markedly on the hydration time. It is expected that eventually, $K_{meas}$ will decrease below the control concrete value.

Values for $K_{calc,}$ calculated with Eq. 1, are presented in Tables 2 and 3 for concrete with w/c = 0.65 and w/c = 0.55, respectively. The calculated results are also plotted in Figure 1. It can be seen that $K_{meas}$ is about two orders of magnitude greater than $K_{calc}$. There were no cracked specimens and there was no leakage around the confining gasket which may have compromised the experimental results. Therefore, Eq. 1 does not appear to hold for concrete specimens including those which incorporate supplementary cementitious materials.

## Conclusions

Within experimental error, values of $(\sigma/\sigma_o)$, or formation factor, can be calculated with Eq. 2 and the mercury intrusion curves for concrete systems. However, for the conditions of this experiment, K, or hydraulic permeabilities, calculated with the Katz-Thompson equation, Eq. 1, do not correspond with experimental values for concrete systems.

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