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AN EXTRAPOLATION METHOD FOR COMPRESSIVE STRENGTH PREDICTION OF HYDRAULIC CEMENT PRODUCTS

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ABSTRACT

The basis for the AMEBA Method is presented. A strength-time function is used to extrapolate the predicted cementitious material strength for a late (ALTA) age, based on two earlier age strengths—medium (MÉDIA) and low (BAIXA) ages.

The experimental basis for the method is data from the IPT-Brazil laboratory and the field, including a long-term study on concrete, research on limestone, slag, and fly-ash additions, and quality control data from a cement factory, a shotcrete tunnel lining, and a grout for structural repair. The method applicability was also verified for high-performance concrete with silica fume.

The formula for predicting late age (e.g., 28 days) strength, for a given set of involved ages, (e.g., 28, 7, and 2 days) is normally a function only of the two earlier ages' (e.g., 7 and 2 days) strengths. This equation has been shown to be independent on materials variations, including cement brand, and is easy to use also graphically.

Using the AMEBA method, and only needing to know the type of cement used, it has been possible to predict strengths satisfactorily, even without the preliminary tests which are required in other methods.

Introduction

Compressive strength is the most important concrete property because it is the main parameter for quality control. Also, for cements, quality is assessed by testing cylindrical, cubic, or prismatic standard mortar specimens under compressive stress.

Several methods have been proposed for predicting the strength of cement paste, mortar, or concrete at the control age (usually 28 days) from measurements at earlier ages. Such methods give the predicted strength based on a group of cement or concrete characteristics. They can be classified as:

1. Accelerated Curing Methods: The hardening hydration reactions of cement are accelerated by heating the specimens under normal or high pressure or using accelerating admixtures. Previously developed equations correlating specimen strengths with and without the acceleration process are necessary (1–4).
2. Correlations with Early Age Strengths Under Standard Curing Conditions: The test

results obtained on specimens at an early age (e.g., 3 or 7 days) are previously related to the corresponding value at the control age (e.g., 28 days) by regression analysis, and the resulting equation is used for the prediction (1,2).

3. Correlation Methods Using Other Characteristics: This large group of methods includes correlations between the control age strength and chemical composition and fineness of cement, porosity estimated from fresh mixture air content and water-cement ratio, measured heat of hydration, and early-age sonic pulse velocity (1,2); it also includes the prediction with basis as the measured compressive strength with a known maturity at an early age and its projection using a previous strength–maturity equation (5).

All these methods require previous experimental correlations obtained with the same materials and conditions of the particular controlled job.

The present method is based on experiments carried out since 1990 (4–8) and classified approximately into Category 2, with the difference that they do not use a correlation with early age strengths, but extrapolate the control age strength after obtaining two earlier age strengths in the strength-time diagram, with axes adequately transformed to linearize the original curve. This extrapolation has shown independence of a large number of variables which are not negligible in other prediction methods.

Methodology Fundamentals

A linearizable mathematical model for representing concrete compressive strength as a time function was proposed after previous work at IPT (6,7). The AMEBA method (6,8–10) is an application of that model, because the linearization of the strength-time function makes it possible to estimate the strength knowing the coordinates (age, strength) for two points on the curve. This means, basically, extrapolating the compressive strength at a later age from the strengths at medium and early ages. The derived equation is:

$$f_{ca} = f_{cm}^{\mathrm{AMEBA}} / f_{cb}^{(\mathrm{AMEBA}-1)} \tag{1}$$

where: f_{ca} is the concrete compressive strength at a late ("alta") age; f_{cm} is the concrete compressive strength at a medium ("média") age; f_{cb} is the concrete compressive strength at an early ("baixa") age; and AMEBA is a function depending on the late, medium, and early ages, and an adjusting exponent, in Eq. 2:

$$\mathrm{AMEBA} = [(1/t_a^{n}) - (1/t_b^{n})]/[(1/t_m^{n}) - (1/t_b^{n})] \tag{2}$$

where t_a, t_m, and t_b are, respectively, the late, medium and early ages; and n is the adjusting exponent, which depends on material characteristics and test conditions. It has been observed that $n = 0.5$ for Portland cement with or without pozzolanic or limestone additions. When the cement contains an appreciable addition of blast-furnace slag, n appears to be dependent on the fineness and amount.

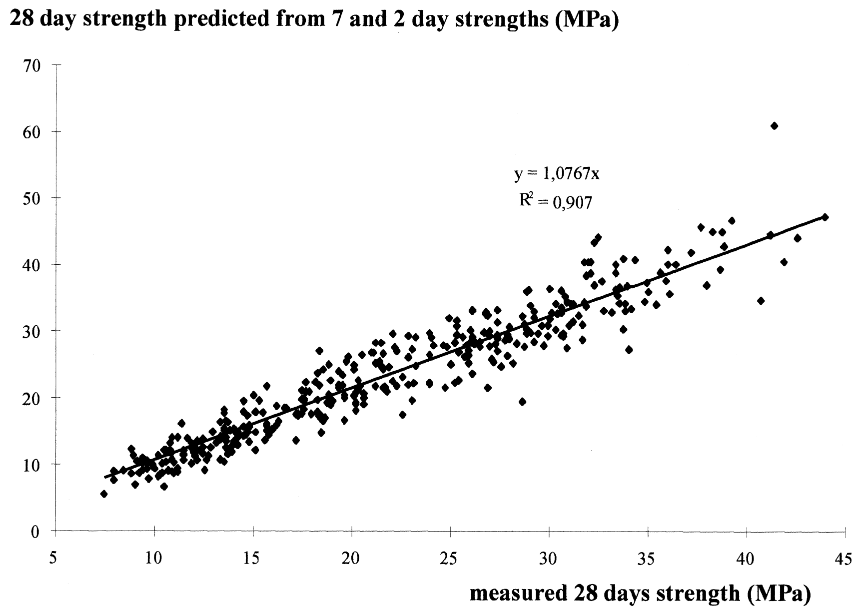

FIG. 1.

Predicted and measured compressive strengths of 75 samples from 5 different Brazilian cement brands, 5 different cement contents and levels of water-cement ratio. Late age was 28, medium 7, and early 2 days.

Experimental Verification and Examples of Applications

Portland Cement Concretes without Additions

References (8–10) show results from a comprehensive long-term study conducted since 1933, in which specimens were molded with variable mix proportions and standard aggregates, using 5 different brands of Brazilian cements in 75 samples collected between 1933 and 1965; the compressive strength tests were carried out at 13 ages varying from 2 days up to 50 years. The study is not yet complete, since the last test is scheduled for 2015, but the available results are sufficient to verify the applicability of the AMEBA method for several combinations of late, medium and early ages, and to show independence from concrete mix proportions, including water-cement ratio, and the cement's chemical composition, physical properties, manufacturer, and batch. Some comparisons between predicted and measured strengths are shown in Figures 1–3.

Portland Cement with Limestone Filler—A Study on Mortars

Campiteli (11) conducted a study to verify the performance of Portland cement with different amounts and finenesses of a limestone filler. Cylindrical mortar specimens 5 cm diameter and 10 cm height were cast, using a Brazilian clinker ground with limestone. Results of 7, 28, 91, and 182 days' compressive strengths permitted comparison of predicted and measured values

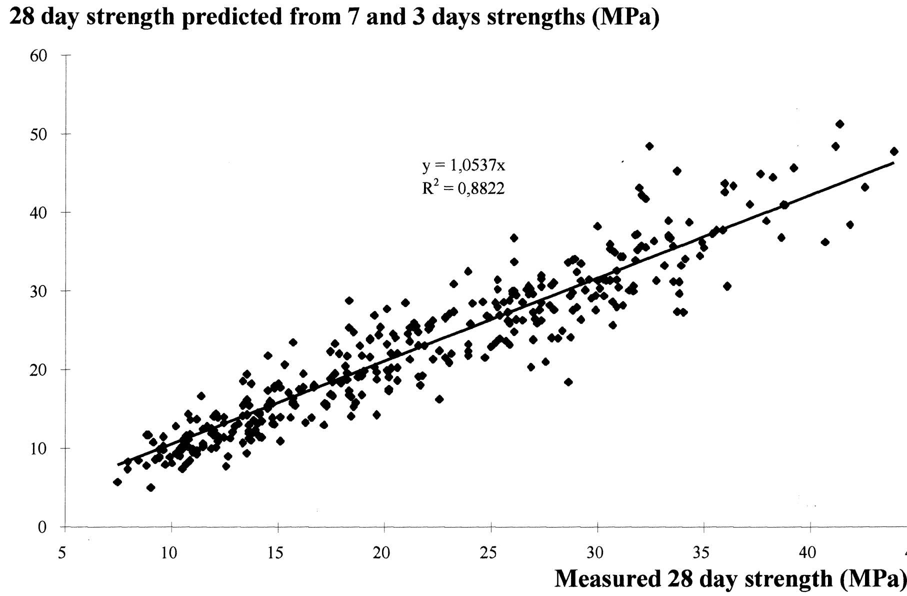

FIG. 2.
As in Figure 1, late age was 28, medium 7, and early 3 days.

as shown in Figure 4. From Figure 4 it can be seen that, using a *n* value = 0.5 in Eq. 2, the correlation is independent of the filler amount and cement fineness.

Portland Cement with Slag—Research on *n* Value in Equation 2

Marques (12), with the objective of obtaining economical concrete with slag additions at the job site, performed mortar and concrete tests on two Brazilian cements; one was blended cement with limestone filler, and the other was high early strength; the amount and fineness of the slag were varied. It was possible to use Marques' compressive strength test results on cylindrical mortar specimens to observe the linearity of the transformed-axis strength-time diagram with log f_c as y axis and t^{-n} as x axis. To find the optimal *n* value that resulted in straight-line graphical representation for each condition, regression plots as illustrated in Figures 5–7 were made for obtaining the maximum correlation coefficients corresponding to various *n* values in Table 1.

Table 1 and Figures 5 to 7 show how the amount and fineness of the slag influence the optimal value of *n* to use in Eq. 2.

In Figures 5 and 6, it is observed that the curvature increases with an increase in the amount of slag. The curves for the higher slag amounts could be straightened by using a higher value of *n*.

In Figure 7, it is observed that the curvature diminishes with an increase in the fineness of the slag. The curves for the lower finenesses could be straightened by using a higher value of *n*.

The observed trend is an increase in the optimal *n* value with an increase of the amount of slag or a decrease in slag fineness. However, satisfactory linearity also occurs in the

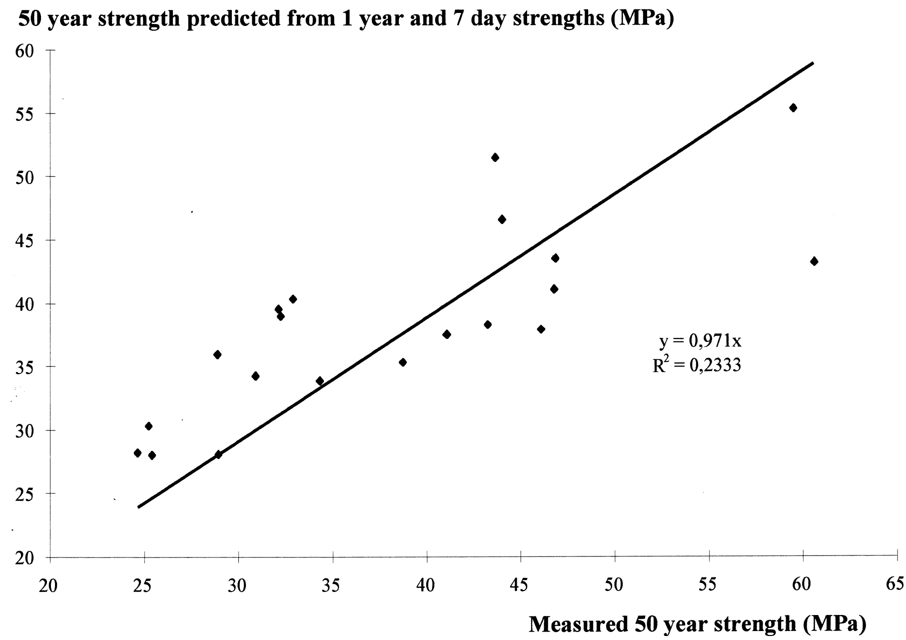

FIG. 3.
Predicted by Eq. 1 vs. measured 50-year concrete strengths for 3 different Brazilian cement brands, samples obtained between 1933 and 1939. Medium age used was 1 year and early age 7 days in Eq. 2.

neighborhood of the optimal *n* value, as shown by the squared correlation coefficients in Table 1.

Marques' data show also that the finer the slag, the closer the optimal *n* is to the corresponding *n* for cement without addition (see Fig. 7).

Portland Blast-Furnace Slag Cement Quality Control

In this case (9), quality control data from a high early strength cement manufacturer were used. The cement was obtained from clinker and slag intensively ground in special mills. Figure 8 shows a plot comparing predicted 28-day strengths with the measured values. The predicted strengths were calculated from Eq. 1, using an *n* value of 1/1.5 in Eq. 2, and 1 and 7 days being the ages for the early and medium age strengths. The prediction error was less than 3% of the mean strength.

Portland Cement Shotcrete with Limstone Filler, High-Range Water-Reducing Admixture (HRWRA), and Accelerating Admixtures

Use of AMEBA to predict 28-day strengths was possible in a tunnel lined with wet-applied shotcrete, where the quality control technicians used diamond-drilled cylinders from in situ shotcreted plates. Previous experiments for correlation and adjustment were not required; a

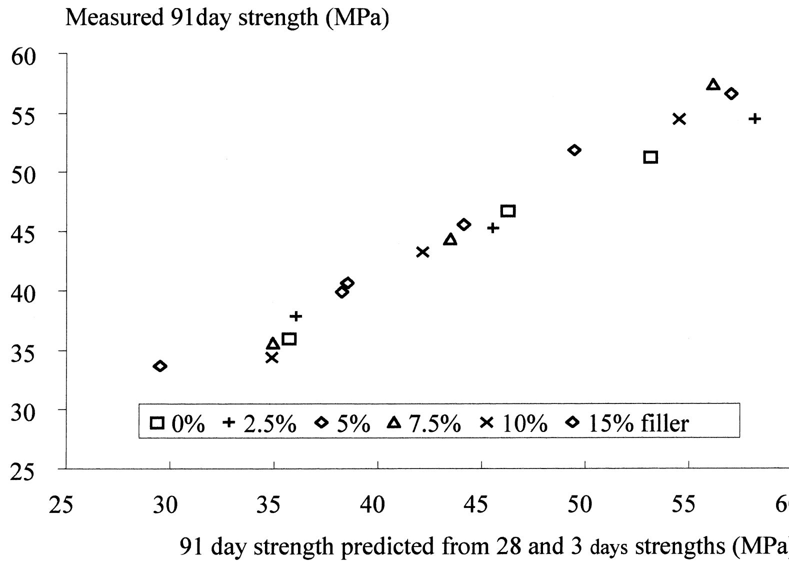

FIG. 4.
90-day compressive strength predicted from Eq. 1 and measured mortar strengths using laboratory cements with several amounts and finenesses (300, 400, and 500 m^2/kg, Blaine) of a limestone filler. Medium age 28 days and early age 7 days used in Eq. 2, with $n = 0.5$.

n value of 0.5 was used in Eq. 2, with spreadsheet software being used for the calculations (10). Figure 9 shows a plot comparing predicted and measured strengths for a time series of quality control samples.

Coarse Grout for Structural Concrete Repair or Reinforcement

In the manufacture of a semi-prepared grout, maximum aggregate size 9.5 mm, it was possible to use the factory's quality control data at 1 and 7 days to predict the 28-day strengths, using Eq. 1 and an *n* value = 0.5 in Eq. 2. The cement used was a blended cement with less than 34% slag, HRWR and expansive admixtures, and pozzolan. Figure 10 is a plot for an *n* value of 0.5, and Figure 11 is the control chart for predicted and measured 28-day strengths. Because the squared correlation coefficients in Figure 10 are close to 1, showing good linearity, the predictions should be accurate with that *n* value; however, this is demonstrated only after a certain time in Figure 11, probably, in part, because of the control laboratory variability.

Blended Pozzolance Cement Paste, Mortar, and Concrete

Krizan et al. (14) performed a study on blended fly ash cement whose results demonstrated the applicability of the AMEBA method using the exponent $n = 0.5$ for paste, mortar, and concrete with 0, 35, and 55% pozzolan-binder proportions. Cement used was a Brazilian high early strength type with maximum 5% limestone filler, and fly ash was from Tubarão Coal Power Station, Brazil. Cylindrical paste specimens, 20 mm diameter and 40 mm height, were

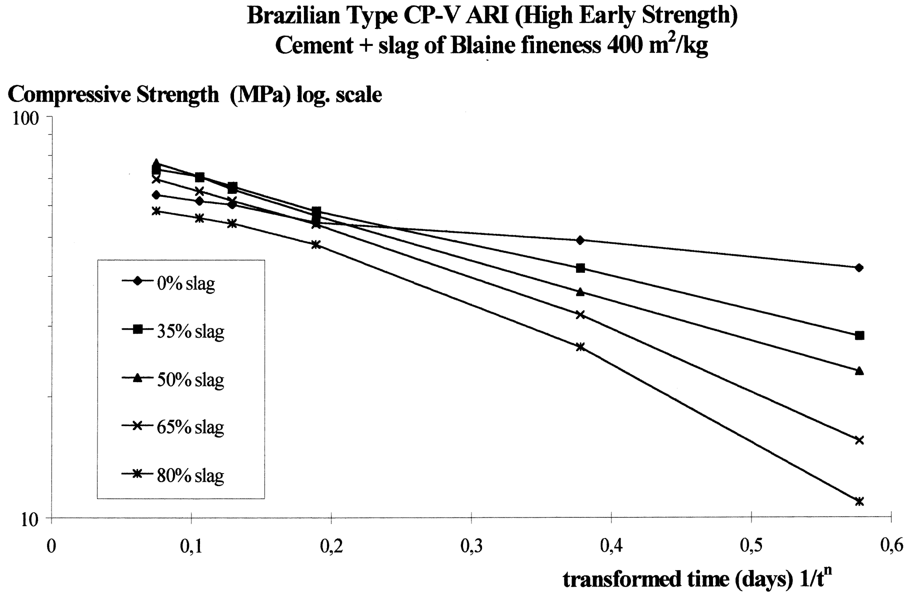

FIG. 5.
Plots of transformed strength vs. transformed time, $n = 0.5$, for high early strength cement consisting of clinker and limestone filler, with different addition levels of 400 m^2/kg ground slag.

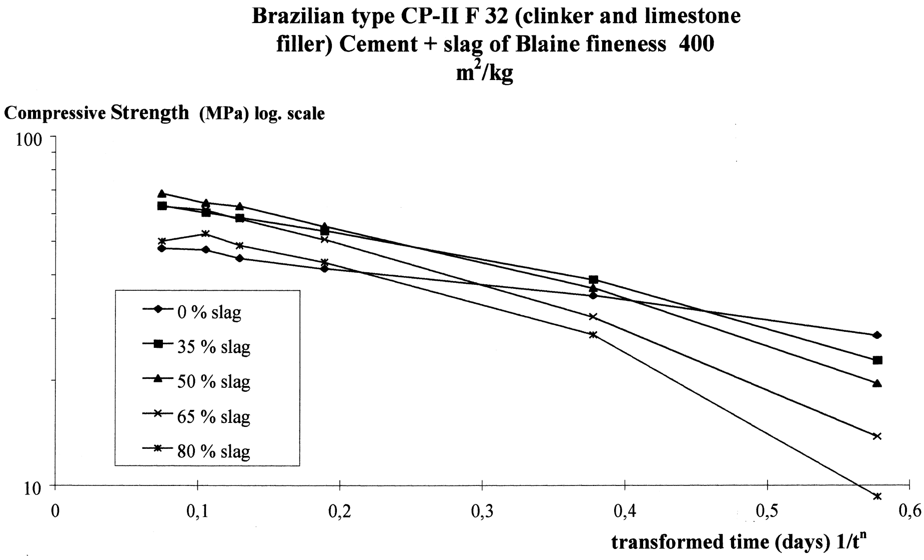

FIG. 6.
Plots of transformed strength vs. transformed time, $n = 0.5$, for a blended cement consisting of clinker and limestone filler, with several different levels of 400 m^2/kg ground slag.

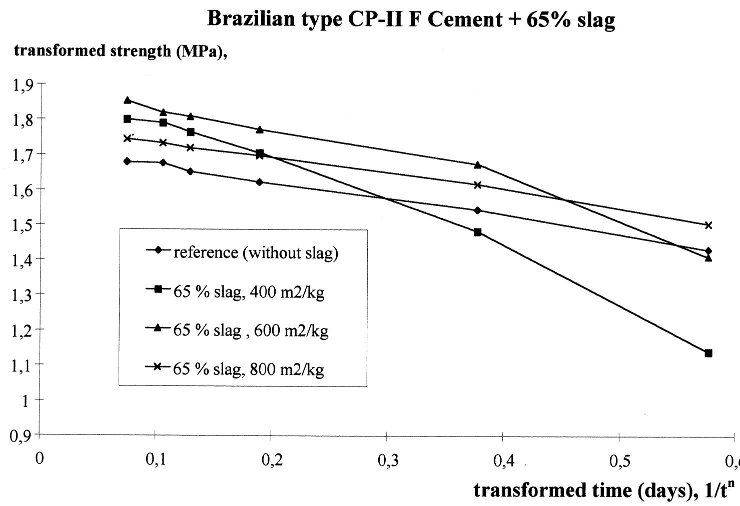

FIG. 7.
Plot of the transformed strength vs. transformed time, $n = 0.5$, for a blended cement consisting of clinker and limestone filler, with 65% by mass of several finenesses of ground slag.

cast with water-binder ratio 0.6. Mortar specimens were cylinders, 50 mm diameter, 100 mm height, and the binder-river sand weight proportions were 1:2, 1:3, and 1:4; the fixed-range Brazilian flow table index was used with the necessary variable water-binder ratios. Concrete specimens were 100 mm diameter and 200 mm height cylinders, cast with river sand and

TABLE 1
Correlation coefficients (r^2) between log f_c and $1/t^a$, obtained with Marques (10) data for slag-containing cements by varying the n value, (maximum absolute values are under lined)

type of cement	slag amount (%)*	$n = 0.5$	$n = 0.6$	$n = 0.7$	$n = 0.8$	$n = 0.9$	$n = 1.0$	$n = 1.1$	$n = 1.2$
high early strength	0	−0.9943	<u>0.9949</u>	−0.9925	-	-	-	-	-
	35	-	−0.9923	−0.9971	<u>−0.9988</u>	−0.9979	-	-	-
	50	-	-	−0.9988	<u>−0.9989</u>	−0.9965	-	-	-
	65	-	-	−0.9982	<u>−0.9995</u>	−0.9983	-	-	-
	80	-	-	-	-	-	−0.9969	<u>−0.9980</u>	−0.9975
blended, with limestone filler addition	0	<u>−0.9785</u>	−0.9697	-	-	-	-	-	-
	35	<u>−0.9988</u>	−0.9965	-	-	-	-	-	-
	50	<u>−0.9994</u>	−0.9957	-	-	-	-	-	-
	65	-	−0.9979	<u>−0.9998</u>	−0.9986	-	-	-	-
	80	-	-	-	−0.9995	<u>−0.9998</u>	−0.9977	-	-

Amounts of slag ground to 400 m^2/kg Blaine fineness as a percentage of the total binder mass.

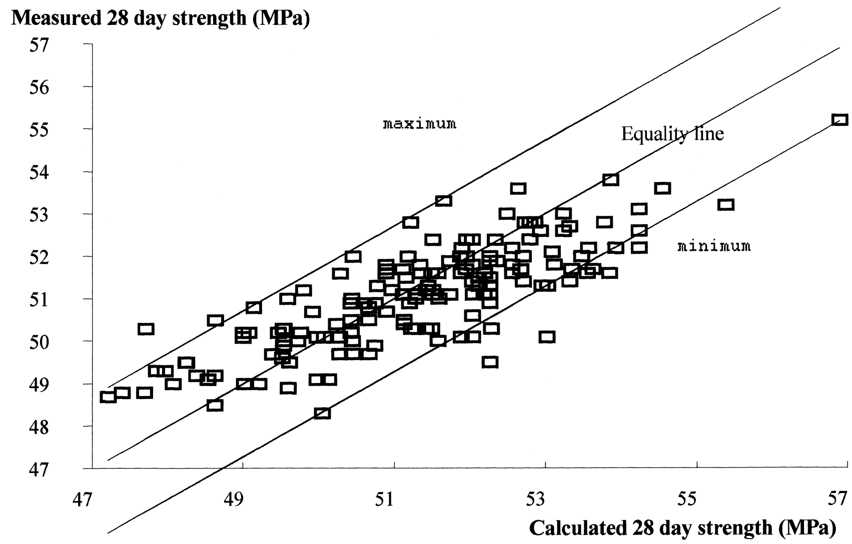

FIG. 8.
Calculated (AMEBA method) and measured strengths from a Brazilian cement manufacturer's quality control data. Cement with slag addition. Used *n* value was adjusted to 0.667, late age 28 days, medium age 7 days, and early age 1 day.

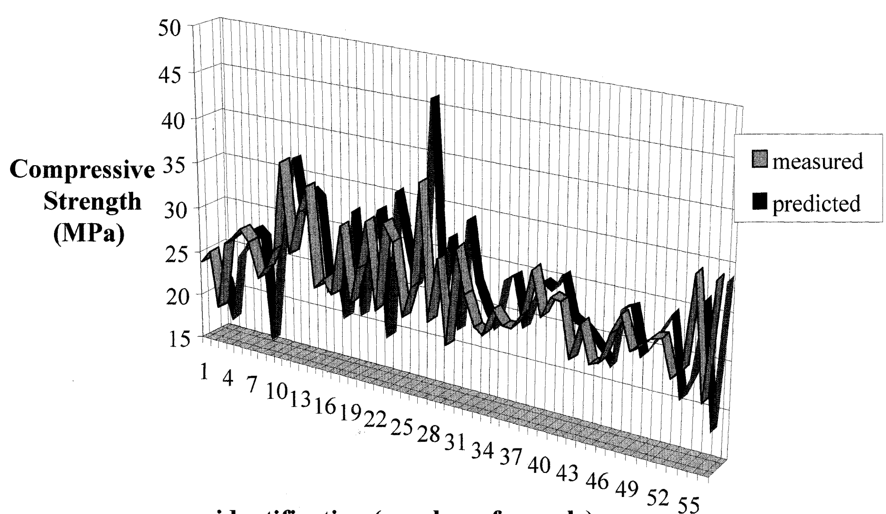

FIG. 9.
Measured and predicted 28-day strengths on a quality control chart, for wet-applied shotcrete used in a tunnel lining, with several amounts of HRWR and accelerating admixtures. Brazilian ordinary portland cement. Medium and early ages were 7 and 1 days respectively.

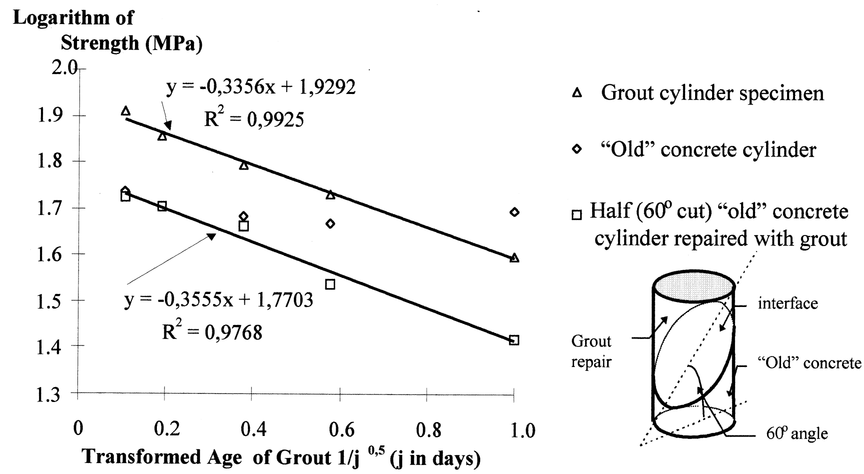

FIG. 10.

Plots of strength vs. time curves, for grout and concrete repaired with grout specimens. Concrete strength remained approximately constant during the study. *n* value used was 0.5.

maximum size 9.5 mm crushed granite with weight binder-aggregate proportions 1:3.5, 1:5.0, and 1:6.5, having fixed-range slump index and necessarily variable water-cement ratios. In Figure 12 the linearity of the transformed mortar strength vs. transformed time curve (for ages 7, 28, 63, 91, and 182 days) is visible for several mix proportions and pozzolan amounts.

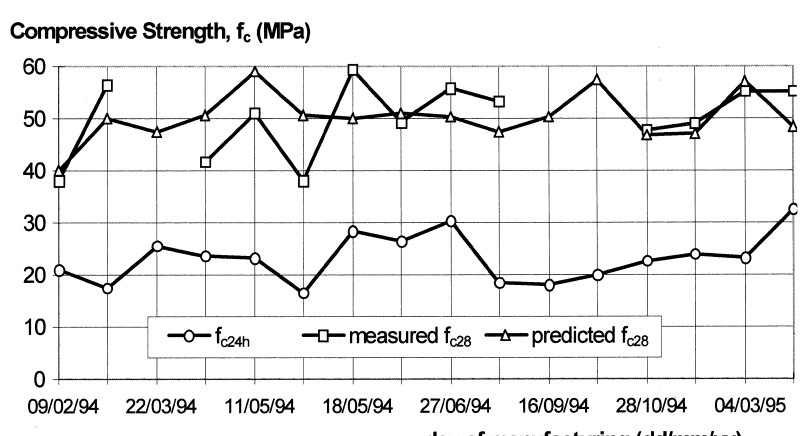

FIG. 11.

Predicted and measured 28-day and 24-h compressive strengths for non-shrinking and high-strength grout with maximum aggregate size, 6.4 mm. Quality control data from the factory, using 7- and 1-day ages for the predictions.

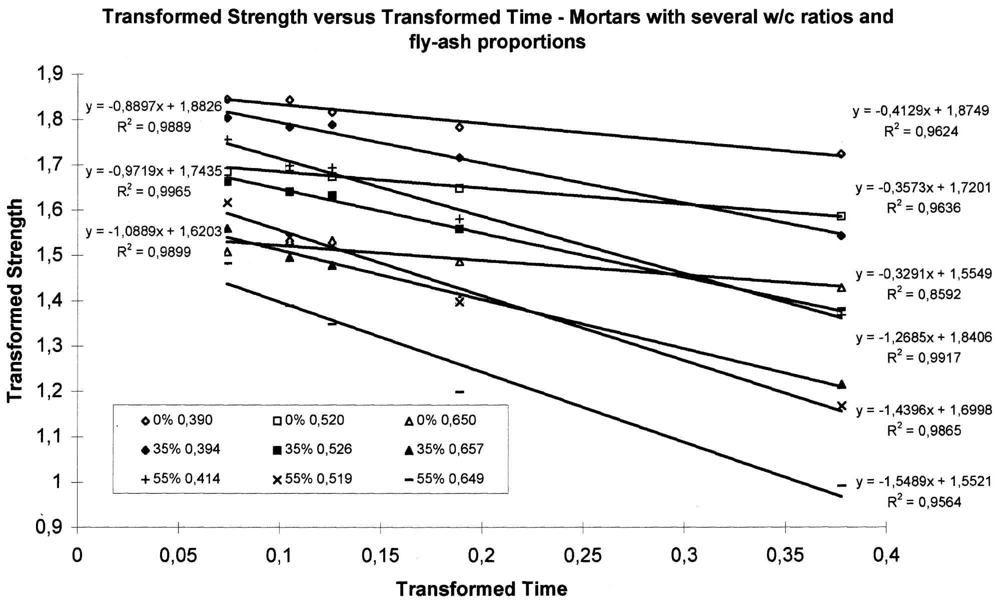

FIG. 12.

Transformed strength log f_c vs. transformed time $1/j^{0.5}$ for mortars with variable aggregate-binder proportions and fixed consistency range, three levels of water-cement ratio and three levels of pozzolan-binder proportion. Plotted transformed ages are 7, 28, 63, 91, and 163 days.

High-Performance Concrete with Silica Fume

The linearity of transformed strength log f_c vs. transformed age $1/j^{0.5}$ curve was verified for high-performance concretes using silica fume with or without HRWR admixtures, with data from Almeida (15), who studied concrete cubes with silica fume from Norway and Portugal, as shown in Figure 13. Results from Larrard and Malier (16), cited by Isaía (17) as typical strength evolution data for high-strength concrete, also showed linearity in the same conditions; but these required attention for verifying what was a minimum age for linearity in the curve, perhaps due to stabilization of the reaction rate between silica fume, or other additions, and Portlandite, after initial Portland cement normal hydration reactions, as shown in Figure 14.

AMEBA Method Uncertainties

It is obvious that the accuracy of the predictions depends on the selection of the appropriate n value for the cement used. The value 0.5 for n has been satisfactory in practically all the cases; the only observed exception is when certain slag-blended cements were used. If the correct n value is used, the precision of predicted values around the effectively measured value is a function of the precision of the measured medium and early age strengths and also of the time-intervals between the three ages. An analogy with the expected dispersion in target shooting can be made with the distance from the target (transformed time between late

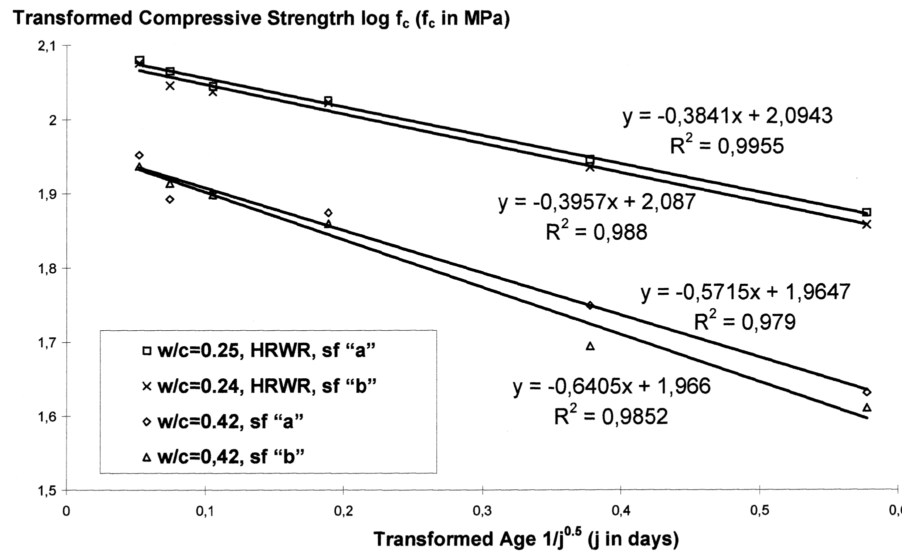

FIG. 13.
Linearization of compressive strength-time plots for high-performance concretes with and without HRWR admixture, with silica fume. Cubes 15 × 15 × 15 cm^3, silica fume "a" from Norway, silica fume "b" from Portugal,.

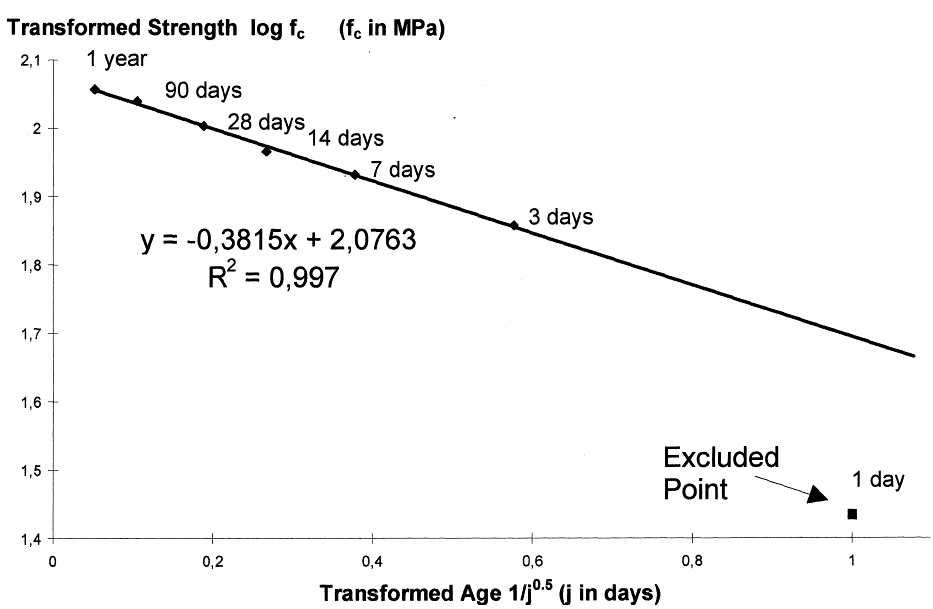

FIG. 14.
Linearization of typical compressive strength-time plots for high strength concrete in which it is shown the needing of waiting for a minimum early age for linearity.

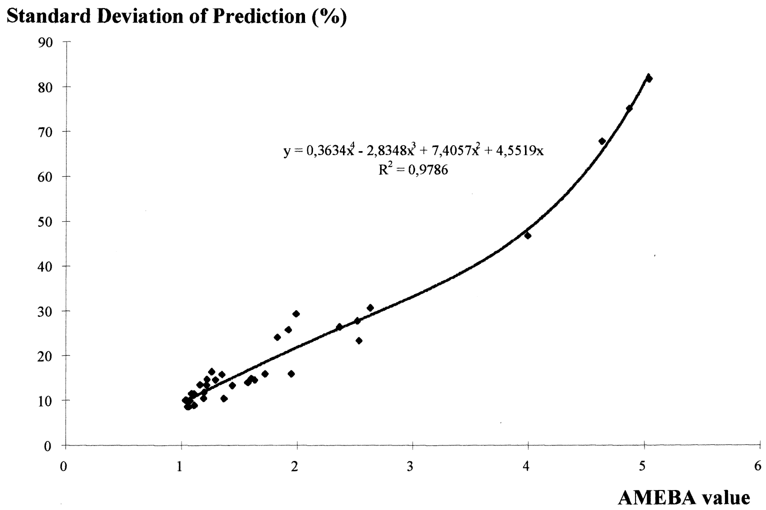

FIG. 15.
Precision of the predicted compressive strength as a function of the AMEBA value from Eq. 2, for IPT conditions and $n = 0.5$ (data from reference 6).

and medium ages) and the length of the rifle (transformed time between medium and early ages). The AMEBA value, as defined in Eq. 2, is the ratio between two transformed time intervals, and a good index of expected precision. The greater the calculated AMEBA value, the less is the expected precision, as shown in Figure 15 for cylindrical specimens 30 cm (height) × 15 cm (diameter) tested under IPT laboratory conditions.

Conclusions

The AMEBA method has limitations, but has been shown to be useful in quality control of cement, mortar, concrete (normal or high performance), and grout strength. As in all extrapolations, the variability of the predictions increases with the length of the extrapolation to the control age, and decreases as the time between the two earlier ages used as basis increases.

The main advantage is a relatively small dependence on materials variations which occur during the quality control. There are no adjustable constants, except the value of n, which is the same for all cements of the same type, and varies from the normal value of 0.5 only for cements with a certain amount of slag.

For ordinary Portland and pozzolanic cements and other hydraulic cements which contain only clinker, gypsum, pozzolan, or limestone filler, and for high performance concretes with silica fume for early ages after 3 days, an n value of 0.5 showed good results without need of preliminary verifications. When blended blast-furnace cements are used, it is preferable to establish experimentally the optimal n value. With spreadsheet software, these verifications can be made using the normal quality control data.

The main difficulty with the AMEBA method, the involved calculations, is not a problem if a micro-computer with a spreadsheet software is available. Spreadsheet software specially prepared for the use of AMEBA concrete quality control is always available with control diagram programs (8) and will be in software in preparation by the author for adjusting *n* using actual field data.

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