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Extreme vertices design of concrete with combined mineral admixtures

Jian-Tong Ding *, Pei-Yu Yan, Shu-Lin Liu, Jin-Quan Zhu

Department of Civil Engineering, Tsinghua University, Beijing 100084, People's Republic of China

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Abstract

Extreme vertices design method was used to reduce the complexity of the mix proportioning of concrete with combined mineral admixtures. It chooses all vertices, or additionally the centroid, of the whole experimental field, and centroids of the boundary surfaces as experimental points. With only nine experimental points designed by the method, the relationship between concrete strength and binder composition of Portland cement-zeolite-fly ash three-component binder system was established through least square regression with error of the regression estimation less than 6%. It yielded a good prediction on 7-day and 28-day compressive strengths of this concrete.

Keywords: Extreme vertices design; Combined mineral admixtures; Zeolite powder

The combination of two or three kinds of mineral admixtures, such as silica fume and GGBFS, silica fume and fly ash, as well as GGBFS and fly ash, has emerged as a superior choice over single admixture to improve concrete properties [1–2]. Such combinations can produce synergetic effects on concrete performances. For example, silica fume is difficult to handle and expensive, while fly ash is much cheaper and more readily available, but less reactive, especially at early ages. The blending of silica fume and fly ash thus may provide concrete with a somewhat more ideal admixture.

However, the mix proportioning of concrete with combined mineral admixtures is more complicated than normal concrete with only one type of admixture because the binder alone will have at least three components. The currently available mix design methods for concrete generally consider at most only one mineral admixture. It is necessary to find an effective and time-saving proportioning procedure both for trial batches in laboratories and convenient adjustment on site.

Simplex centroid design method [3,4] is semiempirical and gives good prediction of the mortar strength of combined binder system [2,5], but its validity when applied to concrete is yet to be proved. On the other hand, for a system with boundary limits, which is more complex, the method can be derived into a more sophisticated one—the extreme vertices design method (EVD) [3,4], which has been rarely applied in mix design of concrete. In this paper, EVD was adopted to establish the performance equation of concrete with a multicomponent binder system.

1. Extreme vertices design

The composition of most multicomponents systems, such as a binder with combined mineral admixtures, is constricted by upper or lower or both boundary conditions, that is, the variation of the percentage of each component cannot always be 0–100%, as in Eq. (1):

$$0 \leqslant a_i \leqslant x_i \leqslant b_i \leqslant 1 \quad (i = 1, 2, 3\ldots\ n,\ a_i \text{ and } b_i \text{ are constants})$$
$$x_1 + x_2 + x_3 + \ldots\ x_n = 1 \qquad (1)$$

The experimental area here may be irregular polygons of various shapes. For example, the boundary condition for the portland cement-zeolite-fly ash three-component binder system may be: portland cement, 30–100%; natural zeolite powder, 0–15%; fly ash 0–40%. Then the experimental area is a trapezoid as shown in Fig. 1.

The set of points meeting the boundary conditions in Eq. (1) is a $(n-1)$ dimensional convex polyhedron in a $(n-1)$ dimensional normal simplex. The $(n-2)$ dimensional boundary surfaces of the polyhedron are parallel to corresponding coordinate surfaces (for which $x_i = 0$, $i = 1, 2, \ldots, n$) respectively. EVD method chooses all of the vertices, the centroids of the $(n-2)$ dimensional boundary surfaces, or, additionally, the centroid of all the vertices of the polyhedron as experimental points to represent the whole experimental field.

* Corresponding author. Tel.: +86 10 6277 1032; Fax: +86 10 6277 1132; E-mail: dingjt@mail.tsinghua.edu.cn.

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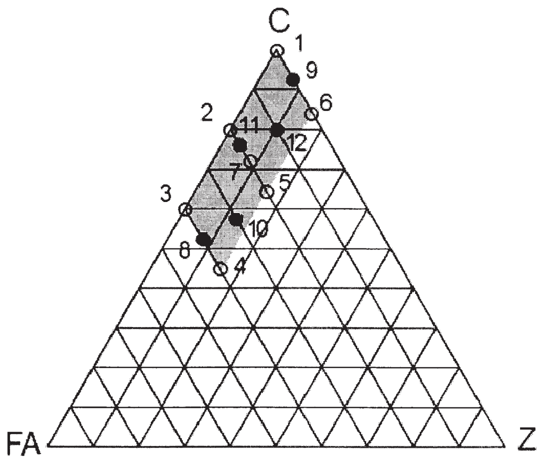

Fig. 1. Mixture design field with boundary limitations (shadowed area) and experimental points of EVD for a three-component system. ○, experimental points (points 1–9); ●, verifying points (points 10–12); C, cement; FA, fly ash; Z, zeolite.

The set of experimental points is sought in the following procedure:

1. List all possible combinations of the upper and lower boundary values of $(n - 1)$ components. This will give $C_n^{n-1} \cdot 2^{n-1} = n2^{n-1}$ combinations.
2. For any one of the $n2^{n-1}$ combinations, if the addition of the boundary values of the combination and the amount of the rest component equals 100%, then the point with the combination as its coordinates is a vertex.
3. For vertices to be those located at a same $(n - 2)$ dimensional boundary surface, it is necessary that they have $(n - 3)$ common upper or lower boundary coordinates.
4. The coordinates of the boundary centroid will be the average values of the coordinates of all vertices on the boundary surface.

For a three-component system ($n = 3$), the design points can be found quite directly, as shown in points 1 to 9 in Fig. 1.

The relation between the performance index Y and compositions x_i of an n-component system can be regressed by least-square method in the form shown in Eq. (2):

$$Y = \sum_{1\leq i\leq n} \beta_i x_i + \sum_{1\leq i\leq j\leq n} \beta_{ij} x_i x_j + \sum_{1\leq i\leq j\leq k\leq n} \beta_{ijk} x_i x_j x_k + \beta_{12\ldots n} x_1 x_2 \ldots x_n \quad (2)$$

2. Experimental

2.1. Materials

The cement was Grade 525# PII Portland cement, which had a clinker content of no less than 95%. Fly ash was Grade I according to Chinese National Standard GB 1596-91. The main mineral of natural zeolite powder was clinoptilolite, which generally had a chemical formula of $(Na, K)_6(Al_6Si_{30}O_{72}) \cdot 24H_2O$. The chemical and physical analyses of cement, fly ash, and zeolite powder are given in Table 1. A superplasticizer of sulfonated naphthalene formaldehyde base was used in powder form. Its content of Na_2SO_4 was less than 3%. The fine aggregate was siliceous sand with a fineness modulus of 3.2 and specific gravity 2650 kg/m^3. The coarse aggregate was crushed limestone with a nominal maximum size of 20 mm and specific gravity of 2750 kg/m^3.

2.2. Design

The selected binder system in this investigation was portland cement-zeolite-fly ash with boundary conditions mentioned above. The experimental section and the nine experimental points chosen by EVD are shown in Fig. 1 and Table 2. The nine experimental points were, respectively, the four vertices of the trapezoid section (points 1, 3, 4, and 6), the midpoints of the boundary lines (points 2, 5, 8, and 9), and the centroid of the whole area (point 7). As a matter of fact, points 3, 8, and 4 as well as points 1, 9, and 6 are already very close to each other respectively. The so-called point-clustering problem is a significant flaw of EVD. Another type of modified EVD—symmetric simplex design method [5]—can avoid the problem, but is less simple and direct than EVD.

In addition, three points (points 10–12 as shown in Fig. 1) were also chosen to verify the effectiveness of EVD. The binder compositions of these points are given in Table 2.

3. Results and discussion

7-day and 28-day compressive strengths of the concretes were tested and are given in Table 3. The relationship between compressive strength and binder composition was regressed by least square fit and is given by Eq. (3) and Eq. (4):

$$f_{cu,7d} = 66.1x_1 - 58.1x_2 - 640x_3 + 141x_1x_2 + 1316x_2x_3 + 792x_1x_3 - 1{,}083x_1x_2x_3 \quad (3)$$

$$f_{cu,28d} = 72.7x_1 + 24.4x_2 - 938x_3 + 59.4x_1x_2 + 764x_2x_3 + 1{,}235x_1x_3 + 596x_1x_2x_3 \quad (4)$$

Table 1
Chemical and physical analyses of cement, fly ash, and zeolite

	Chemical (percent by mass)		
Property	525# PII cement	Fly ash	Zeolite
CaO	62.48	3.87	3.67
SiO_2	24.36	57.57	65.01
Al_2O_3	5.40	21.91	11.85
Fe_2O_3	3.72	7.72	0.63
MgO	2.44	1.68	0.48
Na_2O	0.33	1.54	–
K_2O	0.92	2.51	–
SO_3	0.14	0.41	–
Bulk density (kg/m^3)	3110	2300	2600
Specific surface area (m^2/kg)	360	580	500

Table 2
Binder composition of the experimental points by EVD and verifying points

	Experimental points									Verifying points		
Percentage of components	1	2	3	4	5	6	7	8	9	10	11	12
x_1	1	0.8	0.6	0.45	0.65	0.85	0.725	0.525	0.925	0.588	0.75	0.8
x_2	0	0.2	0.4	0.4	0.2	0	0.2	0.4	0	0.3	0.2	0.1
x_3	0	0	0	0.15	0.15	0.15	0.075	0.075	0.075	0.112	0.05	0.1

Note: Other mixture proportioning parameters were, respectively: water-binder ratio, 0.3; total binder content, 550 kg/m^3; and sand to aggregate ratio, 40%. The dosage of superplasticizer was adjusted in the range of 1–1.5% to give slumps of concrete mixtures around 20 ± 2 cm.

Table 3
Comparison of calculated and tested compressive strength at 7 and 28 day of portland cement-zeolite-fly ash concrete

	7 days			28 days		
	Calculated	Tested	Error (%)	Calculated	Tested	Error (%)
Experimental points						
1	66.1	66.2	−0.1	72.7	71.1	2.3
2	63.8	62.3	2.4	72.6	72.5	0.1
3	50.2	51.7	−2.8	67.6	69.3	−2.4
4	39.0	39.9	−2.4	57.8	58.2	−0.7
5	49.2	48.7	0.9	74.1	76.5	−3.1
6	61.1	60.6	0.8	78.5	75.7	3.7
7	59.7	61.7	−3.2	80.9	78.6	3.0
8	46.6	44.2	5.5	71.0	68.9	3.0
9	68.1	68.5	−0.6	82.6	87.0	−5.1
Verifying points						
10	49.5	51.4	−3.6	74.3	77.9	−4.7
11	61.8	62.4	−0.9	79.8	78.1	2.2
12	62.1	65.6	−5.3	82.8	82.6	4.1

The comparison between the tested results and the calculated results of the three verifying points are given in Table 3, in which the comparison between those results of experimental points are also shown to analyze the accuracy of the regression equations.

From Table 3 it can be seen that for 7-day and 28-day compressive strengths, the erratum for all experimental points and that for verifying points were less than 6%. The prediction error of verifying points are within the range of regression estimation error. The values calculated by Eq. (3) and (4) are quite congruous with those tested.

According to strength vs. binder composition equation, the strength contour graph can be drawn, the binder composition at the highest strength calculated, and the influence of the binder composition on strength discussed. Fig. 2 shows the effects of fly ash content and zeolite content on 28-day compressive strength of concrete. It demonstrates that concrete strength would remain almost the same with 10–20% of Class I fly ash, but would decrease with further increment of fly ash content; with 10% zeolite, the concrete achieved the highest strength. These analyses are in accordance with test results in the literature. This provides further evidence in support of the EVD method.

Subsequently, the mixture design of a four- or six-component system can be applied to concrete with combined mineral admixtures, when water and fine aggregate or coarse aggregate are incorporated into the consideration. Other con-

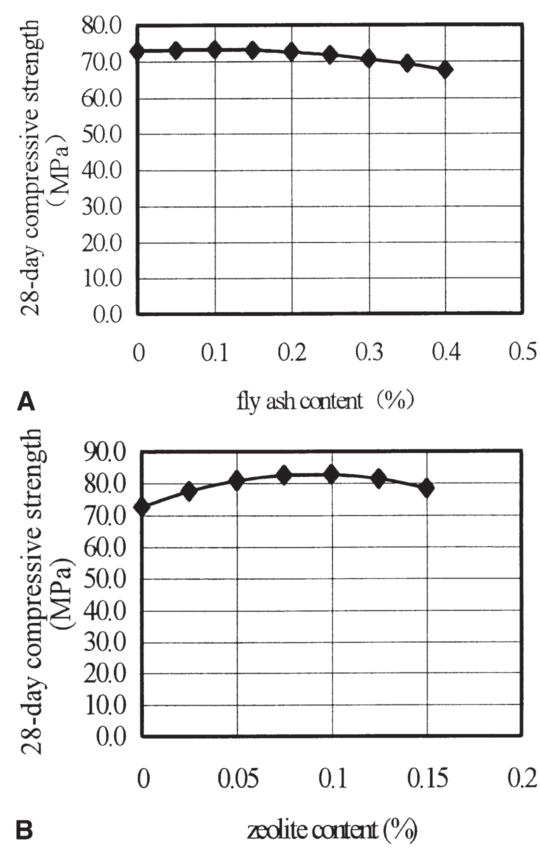

Fig. 2. (*A*,*B*) Calculated influence of fly ash or zeolite content on compressive strength of concrete with combined mineral admixtures.

crete performances, such as durability and deformation, can also be taken as indexes.

4. Conclusions

The EVD method can be applied on concrete incorporating combined mineral admixtures with boundary restriction on the contents of binder components. The compressive strength of the kind of concrete can be calculated accurately by the performance equation regressed from limited experimental points, in this case, only nine points. This was supported by the comparison of the calculated and tested compressive strengths of Portland cement-zeolite-fly ash concrete.

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