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Determination of quasibrittle fracture law for cohesive crack models

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Abstract

A basic and necessary ingredient of cohesive crack models, so widely used for numerical quasibrittle fracture simulations, is the softening function which characterizes the property of concrete-like materials. In this paper, a simple procedure to determine the softening parameters for such functions is proposed. In addition to knowledge of the tensile strength—typically obtainable within about 20% from a cylinder splitting strength (Brazilian) test—the only other requirement is knowledge of the load-displacement curve, as would be obtained from a standard three-point bending test. The method is particularly suitable for materials, such as fiber reinforced concrete, which exhibit a long tail in their load-displacement response. Although suitable for identifying both nonlinear and piece-wise linear relations, we illustrate applications of the scheme using a bilinear law.

Keywords: Cohesive crack; Quasibrittle fracture; Material identification; Softening law

1. Introduction

The cohesive crack, as originally conceived by Hillerborg, is currently widely accepted as the constitutive model that appears suitable for describing the mechanical behavior of quasibrittle materials like cementitious materials, ceramic, rock and other concrete-like materials. In particular, it is able to capture—in a simple and elegant—way the essential features of a progressively fracturing surface and its evolution until failure. This two-law model consists of an ascending stress-strain relationship and a descending softening function (or cohesive law), expressing a stress-crack opening relationship within a fracture process zone or craze. Unfortunately, as emphasized in numerous papers (for instance, the extensive work and related references reported in the FRAMCOS2 [1] proceedings), the determination of the material parameters characterizing the cohesive crack model is not straightforward. Moreover, there are neither standard nor recommended methods capable of achieving this difficult task.

The tensile strength should be measured in a direct tensile test, but it is usually practiced to estimate it via a cylinder splitting strength or Brazilian test, which gives results within 20% of actual values. In this paper, we assume that the tensile strength is known.

A uniaxial test is not suitable to obtain information about the actual shape of the softening function. Most methods to date [2–9] rely on indirect procedures based on some parameterization of the softening curve and its best fitting to measured load-deflection curves of stable tests on notched specimens in a three-point bending test. All such methods have their own advantages and disadvantages, including the fact that the size dependence of some parameters—particularly the specific fracture energy or work needed to bring completely apart the two faces of a unit surface of crack (represented by the area under the softening curve)—for some types of concrete is not yet fully understood.

This paper presents a procedure for inferring the parameters of the softening function from load-deflection curves obtained from three-point bending tests. This simple and novel proposal is particularly suitable for those materials, such as fiber reinforced concrete, which exhibit a long load-displacement tail. Any softening function, including piece-wise linear ones, can be obtained. In essence, we introduce the concept of a partial or "quasi-fracture" energy whose determination enables us, in conjunction with the tail of a three-point bending load-displacement curve, to obtain the softening function.

The organization of this paper is as follows. In the next section, we define the quasi-fracture energy and detail its calculation. We then briefly describe how, starting from a determination of its tail, the entire softening function can be obtained. As an illustrative example, we apply the procedure to a set of experimental data performed on short fiber reinforced concrete beams to determine a suitable bilinear softening law. The results are then verified by using this

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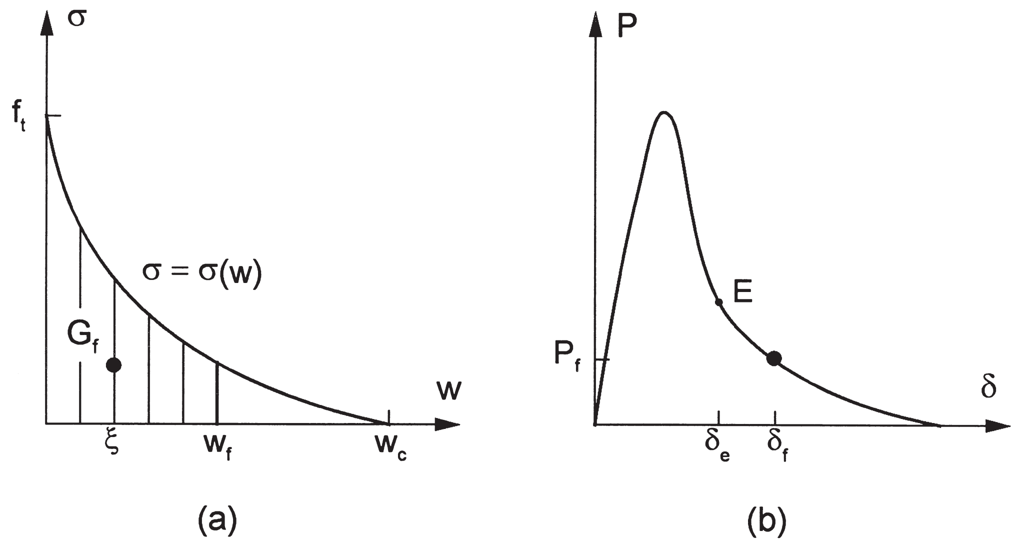

Fig. 1. (a) Softening function. (b) Load-deflection curve.

identified law to analyze, through a discrete crack boundary element scheme, the test beams for their load-displacement response, which can then be compared with the experimentally obtained counterpart.

A brief word concerning notation is appropriate. Parentheses are used to denote both functional dependence and product; their meanings should, however, be clear from the context.

2. Concept and calculation of quasi-fracture energy

The problem of determining an appropriate constitutive law for use with the cohesive crack model can be introduced by making reference to Fig. 1. In particular, it is required to determine the softening function $\sigma = \sigma(w)$ (Fig. 1a), where σ is the tensile stress and w is the fictitious crack opening for an experimentally obtained load-displacement (or $P - \delta$) curve (Fig. 1b) from a three-point bending test. We also note that f_t and w_c in Fig. 1a are the familiar tensile strength and critical fictitious crack opening displacement, respectively, and that the area under the $\sigma - w$ curve represents the RILEM specific fracture energy G_F. In this work, we assume that f_t is known. Our proposed method uses the tail of the $P - \delta$ curve, that is, points such as (P_f, δ_f) (Fig. 1b). A loose (acceptable and often visual) criterion for choosing such points is the condition $\delta_f \geqslant \delta_e$, where δ_e is the deflection for a point E on the $P - \delta$ curve at which the slope starts to flatten out or, if it exists, beyond the point of contraflexure of the $P - \delta$ curve. The reason for using only that portion of the experimentally obtained $P - \delta$ is because the formulae which we will derive are based on the crucial assumption that the fracture is near its "terminal" stages. That is, the beam is about to break into two pieces. We need also emphasize that point (P_f, δ_f) is not a *specific* point on the curve but rather a *generic* point; it represents the set of *all* points beyond the point of contraflexure.

Crucial in the procedure is the calculation of the partial fracture energy G_f, shown shaded in Fig. 1a. This quantity, referred to henceforth as a "quasi-fracture" energy, is obviously defined mathematically by Eq. (1).

$$G_f = \int_0^{w_f} \sigma \, dw \tag{1}$$

Physically, we interpret this energy as the specific fracture energy or work needed to bring the two faces of a unit surface of craze apart by a distance of w_f. Clearly, if $w_f = w_c$, then we recover the RILEM fracture energy, or $G_f = G_F$. In order to calculate G_f, we first need to be more specific regarding the relationship between w_f and δ_f. We define the deflection δ_f as being the beam central deflection when the maximum crack opening just reaches w_f. We can now calculate G_f as the sum of the unrecoverable work done for the beam to deform until δ_f is reached and the specific work required to propagate a crack of width w_f at the potential crack interface.

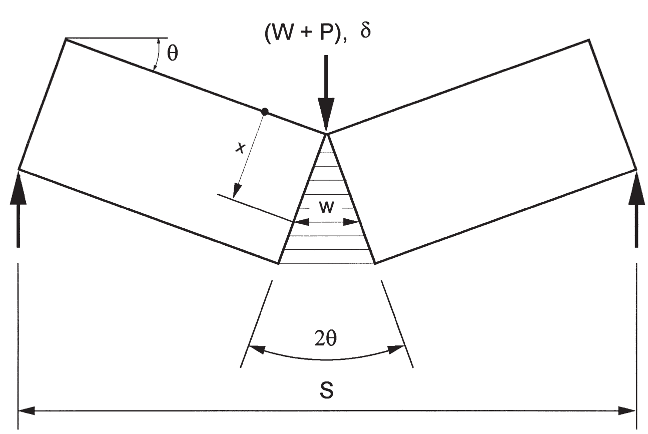

Fig. 2. Rigid body kinematics near the end of the test.

Since we are dealing with a condition in which the craze is fully developed, a point such as (P_f, δ_f) corresponds to the last phase of the three-point bending test which can then be modeled following the rigid-body kinematics used by Petersson [10]. This, as pointed out earlier, is an important assumption and thus enables the ensuing formulae to be applicable only in the terminal phase of the fracture process — in our case, identified by the set of points (P_f, δ_f). In Petersson's approach, as shown in Fig. 2, it is assumed that the beam is divided into two rigid rectangular pieces which are connected only through a process zone, with the compression zone assumed to be concentrated on a point at the top of the beam. This idealization will allow us to calculate the aforementioned specific work required to propagate a crack of constant width w_f throughout the depth of the beam, until the beam "fails".

We first concentrate on the unrecoverable work U_1 required to produce a central deflection of δ_f. When this deflection is reached, the corresponding work W_0 done by an imposed load P, and the corresponding work W_1 done by the weight m_1g of the beam itself and the weight m_2g of any testing equipment (assumed to act at midspan) not fixed to the testing machine are, respectively, given by Eqs. (2) and (3):

$$W_0 = \int_0^{\delta_f} P\, d\delta \tag{2}$$

$$W_1 = \left(\frac{m_1 g}{2} + m_2 g\right)\delta_f = W\delta_f. \tag{3}$$

The corresponding recoverable elastic deformation energy of the beam can be written as Eq. (4),

$$V = \frac{P_f}{2k} \tag{4}$$

where $k = 48EI/S^3$ (neglecting shear deformations), S is the beam span and EI is the initial rigidity of the beam. We neglect the energy dissipation outside the fracture process zone, U_1, which is obviously given by Eq. (5):

$$U_1 = W_0 + W_1 - V. \tag{5}$$

Also, the central bending moment M_f corresponding to δ_f is clearly given (Fig. 2) by

$$M_f = \frac{S}{4}(W + P_f). \tag{6}$$

We now proceed with the calculation of U_2, the work required to propagate a crack of constant width w_f throughout the depth of the beam until the beam "fails". For simplicity, we assume that the beam is unnotched. For a cohesive law $\sigma = \sigma(w)$, the central bending moment M_x provided only by the quasi-fracture process zone of length x and corresponding maximum crack width $w = 2x\sin\theta$ (Fig. 2) is

$$M_x = \int_0^x \sigma(w)\, Bx\cos\theta\, dx = \frac{B\cos\theta}{4\sin^2\theta}\int_0^w \sigma(w)\, dw \tag{7}$$

where B is the beam thickness. Hence, for a beam of depth H, when $\theta = \theta_f$ and the maximum crack width at $x = H$ is w_f, the total moment M_f supplied by the entire process zone is

$$M_f = \frac{B\cos\theta_f}{4\sin^2\theta_f} G_f \xi \tag{8}$$

noting, from Fig. 1a, that

$$\int_0^{w_f} \sigma(w)\, w\, dw = G_f \xi,$$

where ξ is the centroidal distance of G_f from the origin. We can combine Eqs. (6) and (8) to give

$$G_f \xi = \frac{(W + P_f)S\sin^2\theta_f}{B\cos\theta_f} \tag{9}$$

and hence the total moment, from Eqs. (7) and (9), can be written, after some simplifications, as a function of θ as

$$M = \frac{(W + P_f)S\sin^2\theta_f}{4\cos\theta_f}\frac{\cos\theta}{\sin^2\theta} \tag{10}$$

Finally, the dissipation U_2 in the quasi-fracture process zone—when a crack of width w_f propagates throughout the depth of the beam, which corresponds to an increase in rotation from $\theta = \theta_f$ to $\theta = \pi/2$ (the beam "breaks")—can be written as Eq. (11)

$$U_2 = 2\int_{\theta_f}^{\frac{\pi}{2}} M\, d\theta = 2\int_{\theta_f}^{\frac{\pi}{2}} \frac{(W + P_f)S\sin^2\theta_f}{4\cos\theta_f}\frac{\cos\theta}{\sin^2\theta} d\theta \tag{11}$$

after substitution of Eq. (10). When θ_f is small ($\tan\theta_f \approx \sin\theta_f \approx \theta_f$, and θ_f^2 can be neglected compared to θ_f), the following is obtained:

$$U_2 = (W + P_f)\delta_f \tag{12}$$

since $\delta_f = S\theta_f/2$.

According to the definition of the quasi-fracture energy G_f, it can be calculated from $(U_1 + U_2)/BH$; and using Eqs. (5) and (12), it can be implicitly given as a function of w_f as

$$G_f(w_f) = \frac{1}{BH}\left(\int_0^{\delta_f} P\, d\delta + 2W\delta_f + P_f\delta_f - \frac{P_f^2}{2k}\right) \tag{13}$$

where we also note that

$$w_f = \frac{4H}{S}\delta_f. \tag{14}$$

3. Determination of the cohesive law

We are now in a position to determine the tail part of the softening function from use of the experimentally obtained set of (P_f, δ_f) points. From Eqs. (1) and (13) and some straightforward manipulations, we can obtain an expression for $\sigma_f(w_f)$ as follows:

$$\sigma_f(w_f) = \frac{dG_f}{dw_f} = \frac{S}{4BH^2}\left[2(W + P_f) + \left(\delta_f - \frac{P_f}{k}\right)\frac{dP}{d\delta}\bigg|_{\delta = \delta_f}\right]. \tag{15}$$

Clearly, for a series of experimentally recorded points (P_f, δ_f), Eqs. (14) and (15) can be used directly, by simple

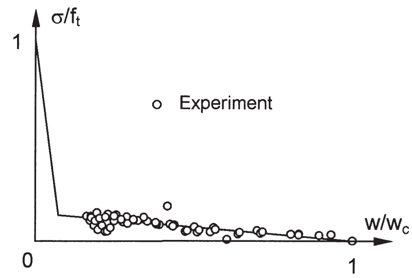

Fig. 3. Identified bilinear law.

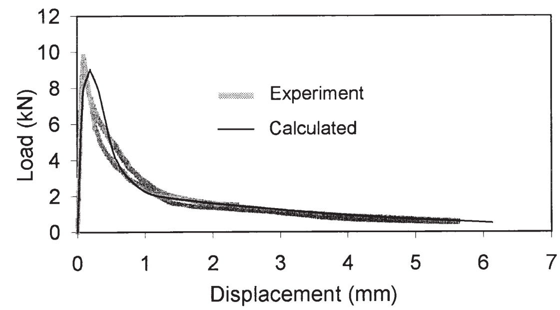

Fig. 4. Comparison of predicted with experimental load-deflection responses.

substitutions, to furnish corresponding values of (σ_f, w_f). A curve fit (regression) on the series of points so obtained will give an estimate of w_c, and hence of G_F from Eq. (13) by letting $w_f = w_c$. Finally, to identify the initial portion of the softening curve, we make use of the known point $\sigma = f_t$, $w = 0$ and the estimated value of G_F to achieve an appropriate fit. In summary, assuming that a bilinear law is required, the steps for obtaining it are:

- Use experimental points (P_f, δ_f) with Eqs. (14) and (15) to obtain (σ_f, w_f).
- Estimate G_F from Eq. (13).
- Use a linear regression analysis to fit the best straight line to the set of points (σ_f, w_f).
- Use G_F (representing the area under the entire $\sigma - w$ curve), f_t and the obtained tail of the $\sigma - w$ curve to obtain the appropriate initial linear branch of the bilinear softening law. In this case, the initial linear branch is obviously unique.

For *nonlinear* laws, this procedure, it must be noted, may not necessarily yield the correct shape of that initial part of the softening function (since the fitting procedure may not be unique). However, reasonable results are expected in practice when a long tail has already been fitted.

To validate the method, we have applied it to identify the softening function for short fiber reinforced concrete using experimental data of unnotched three-point bending specimens obtained by the first author. Some details of specimen geometry and material properties are: $S = 360$ mm; $H = 100$ mm, $B = 100$ mm; fiber length $= 12.5$ mm; volume percentage of fiber $= 0.5\%$; Young's modules $E = 18.1$ GPa; Poisson's ratio $= 0.237$; uniaxial tensile strength $f_t = 2.20$ MPa. Further details of materials and experimental procedures can be found in [11,12]. Three sets of $P - \delta$ curves have been used.

For simplicity, we have approximated the softening function as a bilinear model. As outlined earlier, the tail was obtained by a linear regression applied to (σ_f, w_f) points obtained from Eqs. (14) and (15). Then the initial linear branch was identified (uniquely in this linear case) through making the area enclosed by the two softening lines and axes equal to the fracture energy. The results are shown in Fig. 3. It is also interesting to note that linear regression gave a value of w_c close to half the fiber length — a commonly quoted figure for fiber reinforced concrete.

4. Verification of cohesive law obtained

It is common to "verify" the identified cohesive law by using it in an *analysis* code founded on the discrete crack model to assess how close the computed $P - \delta$ response is to the experimentally recorded one. We have therefore carried this out with the obtained bilinear softening law (Fig. 3). A discrete crack quasibrittle fracture boundary element based code [13] to predict the $P - \delta$ response of the experimentally tested specimens was used as the analysis tool. Predicted versus experimental results are compared in Fig. 4. The agreement between experimental results and numerical prediction is very good, considering the difficulty of predicting fracture parameters. The suitability of the identified softening law appears to be comparable to that obtained by so-called "sophisticated" methods such as those reported in [8, 9]. Although this observation is not sufficient in *fully* demonstrating the capabilities of the present proposed technique, we must add that such comparison with sophisticated and generally accepted (albeit computationally demanding) schemes does give considerable merit to our simple procedure.

5. Concluding remarks

A simple and novel procedure is proposed in this paper to determine the softening function of quasibrittle materials. It uses information on the tensile strength of the material and the tail of a load-displacement curve as obtained from a standard three-point bending test. The method rests on evaluation of the so-called "quasi-fracture" energy.

We have tested the fitting procedure using experimental load-deflection data of three-point bending tests to obtain an idealized bilinear softening law for fiber reinforced concrete. The predictive accuracy of the identified softening law has then been verified through a discrete crack analysis; good agreement between predicted and measured load-deflection responses validates the softening model obtained and, to a large extent, the simple method we are proposing to obtain it. However, more extensive comparisons between experimental results and numerical prediction need to be carried out, especially on specimens of different geometry and size, to further verify the identification procedure developed. We, however, believe that the procedure is simple enough—especially as all formulae are given, for anyone with load-deflection data to apply it to the particular situation concerned.

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