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Service life prediction of reinforced concrete structures

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Abstract

This paper is focused on the estimation of durability and service life of reinforced concrete structures. Assuming that the chloride ion in concrete can be absorbed on tricalcium aluminate, calcium silicate hydrate, and by other constituents of hardened cement paste, hydrated or not, the exact analytical solution of the governing partial differential equation together with its boundary and initial conditions can be obtained through nondimensional parameters and Laplace's transform. When the results of an exact analytical solution using suitable parameters were compared with the results of previous experimental work, the differences were found to be very small. This suggests that the absorption model is of considerable value. The exact analytical solution with the saturation parameter and time and diffusion coefficients under different effective electrical potential could be used to predict both the experimental results and the service life of reinforced concrete structures.

Keywords: Diffusion; Calcium-silicate-hydrate (C-S-H); Electrochemical properties; Chloride; Concrete

1. Introduction

Chloride diffusivity is a very important property of concrete that affects the durability and service life of a reinforced concrete structure. The correct choice of chloride diffusivity is usually provided as a reference for designing cover thickness of a reinforced concrete structure. Deterioration of reinforced concrete structures due to chloride ingress followed by reinforcement corrosion is a growing problem all over the world. Service life predictions [1–3] are of great importance for existing and planned structures. Service life may be variously defined, but in the present paper, it is used to mean the time from construction until the chloride content at the depth of reinforcement is high enough to initiate corrosion.

Many different mathematical methods for service life prediction have been presented. Generally speaking, such mathematical methods are usually a theoretical method combined with an empirical approach. However, from a practical point of view, most of these methods are very difficult to apply because there are often too many unknown parameters.

The mathematical model expressed in this study is based on Fick's second law [4,5] of diffusion, combined with previous laboratory tests and empirical results from existing reinforced concrete structures. The model can be used to predict service life in practical situations. Furthermore, the refinement of input data in the future should lead to even better predictions.

One of the principal challenges in developing a mathematical model for service life prediction is that many parameters, including material characteristics, climatic environment, and construction method, will all affect service life. At present, it is difficult to specify many of these parameters with sufficient precision for them to be used in a mathematical model. Nevertheless, the object of this investigation is to establish a usable, practical, and reliable mathematical model that can accommodate new reinforced concrete parameters as the information becomes available.

The service life of a reinforced concrete structure can be divided into two stages, the initiation period and the propagation period. The initiation period is defined as the time from exposure until chlorides have penetrated the concrete cover and the chloride content of the concrete structure in contact with the reinforcing steel is high enough to start corrosion. The propagation period is considered to be the time from when the reinforcing steel starts to corrode until a critical limit of a material property (such as mechanical strength) has been reached. In the present work, service life is used to mean the initiation period only, which is to say that only the time taken for the chlorides to penetrate is considered.

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The main purpose of this paper is to present a practical model for predicting service life in existing reinforced concrete structures. To this end, a mathematical model is established for predicting the service life of existing reinforced concrete structures. Evaluation of service life for an existing reinforced concrete structure is based on detection and testing of the actual structure. This means that the unknown parameters such as the concentration and diffusion coefficient of chloride ion are included in the test result.

2. Theoretical formulation

Fick's first law [4–11] is shown in Eq. (1).

$$J_{ix} = -D_i \frac{\partial C_i}{\partial x} \tag{1}$$

where J_{ix} is the flux of chloride ion, D_i is the diffusion coefficient of free ion, and C_i is the concentration of free ions of species i(mol/L).

For the free chloride ion [12–15] Eq. (1) becomes Eq. (2).

$$J_{ix} = -D_e \frac{\partial C_f}{\partial x} \tag{2}$$

where D_e is the effective diffusion coefficient for the free chloride ion and C_f is the concentration of the free chloride ion.

From Tang and Nilson [4] and a consideration of the influence of added potential for accelerated diffusion, one knows [see Eq. (3)]:

$$J_{ix} = -D_e\left(\frac{\partial C_f}{\partial x} - \frac{eFE}{RT}C_f\right) \tag{3}$$

where e is the valency of the ion, R is the ideal gas constant (8.31 J/mol K), T is the absolute temperature (K), F is the Faraday constant (96500 coul/mol), and E is the electrical field density (volt/m). Eq. (3) is Fick's second law with an extra term for accelerated diffusion.

Referring to Fig. 1, C_t is the total concentration of chloride ion in $\delta V = A \cdot \delta x$, and is given by Eq. (4):

$$C_t = C_b + \varepsilon d C_f \tag{4}$$

where C_b is the bound chloride ion (mol/L), ε is the total porosity of concrete, and d is the degree of saturation by the pore water solution.

The equation of continuity for the chloride ion is shown in Eq. (5):

$$(\delta C_t)A \cdot \delta x = J(x) \cdot A \cdot \delta t - J(x+\delta x)A \cdot \delta t \tag{5}$$

Expanding Taylor's series, this becomes Eq. (6):

$$= -\frac{\partial J}{\partial x}A \cdot \delta x \cdot \delta t \tag{6}$$

The limit of Eq. (6) leads to Eq. (7):

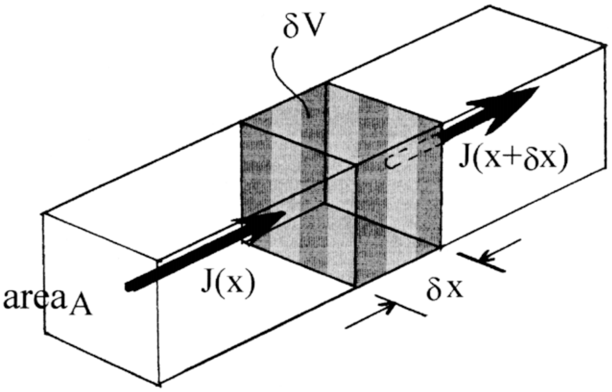

Fig. 1. A scheme for the equation of continuity of chloride ion.

$$\frac{\partial C_t}{\partial t} = -\frac{\partial J}{\partial x} \tag{7}$$

The substitution of Eq. (3) into Eq. (7) yields Eq. (8):

$$\frac{\partial C_t}{\partial t} = \frac{\partial}{\partial x}\left[D_e\left(\frac{\partial C_f}{\partial x} - \frac{eFE}{RT}C_f\right)\right] \tag{8}$$

Substituting Eq. (4) into Eq. (8), one obtains Eq. (9):

$$\frac{\partial(C_b + \varepsilon d C_f)}{\partial t} = \frac{\partial}{\partial x}\left[D_e\left(\frac{\partial C_f}{\partial x} - \frac{eFE}{RT}C_f\right)\right] \tag{9}$$

Assuming that C_f and C_b are parts of a reversible process [16] (i.e., $C_f \leftrightarrow C_b$), the forward and backward absorption rates [17–20] can be respectively written as Eq. (10)

$$r_a = k_a \varepsilon d C_f \tag{10}$$

and Eq. (11).

$$r_b = k_b C_b \tag{11}$$

where k_a and k_b are the forward and backward deposition coefficients, respectively.

The net absorption rate [21] is shown in Eq. (12).

$$\frac{\partial C_b}{\partial t} = r = r_a - r_b = k_a \varepsilon d C_f - k_b C_b \tag{12}$$

From Eqs. (9) and (12), one gets Eq. (13):

$$\frac{\partial(\varepsilon d C_f)}{\partial t} = \frac{\partial}{\partial x}\left[D_e\left(\frac{\partial C_f}{\partial x} - \frac{eFE}{RT}C_f\right)\right] - r \tag{13}$$

If both porosity and the effective diffusion coefficient for mature concrete are independent of space and time, that is, both ε and D_e are constant with respect to x and t, then one obtains Eq. (14).

$$\varepsilon\frac{\partial(C_f)}{\partial t} = D_e\left(\frac{\partial^2}{\partial x^2}C_f - \frac{eFE}{RT}\frac{\partial C_f}{\partial x}\right) - r \tag{14}$$

Consider the following dimensionless parameters [see Eq. (15)]:

$$C = \frac{C_f}{C_{en}},\ \omega = \frac{C_b}{C_{b\cdot sat}},\ z = \frac{x}{L},\ \tau = \frac{D_e}{\varepsilon d L^2}t,$$
$$\lambda = \frac{eFEL}{RT},\ \beta = \frac{C_{b\cdot sat}}{\varepsilon d C_{en}} \tag{15}$$

where C_{en} is the chloride ion concentration at the interface of the concrete surface to the external environment, $C_{b\cdot sat}$ is the maximum concentration that can be absorbed by the concrete, and L is the cover thickness of concrete.

From Eqs. (12), (14), and (15), one gets the nondimensional Eq. (16) and Eq. (17):

$$\frac{\partial C}{\partial \tau} = \frac{\partial^2 C}{\partial z^2} - \lambda\frac{\partial C}{\partial z} - \beta\frac{\partial \omega}{\partial \tau} \tag{16}$$

and

$$\beta\frac{\partial \omega}{\partial \tau} = aC - \mu\omega \tag{17}$$

where [see Eq. (18) and Eq. (19)]:

$$a = \frac{k_a \varepsilon L^2}{D_e} \tag{18}$$

$$\mu = \frac{k_b L^2 C_{b\cdot sat}}{D_e C_{en}} \tag{19}$$

In the simultaneous Eq. (16) and Eq. (17), which were derived from the diffusion of accelerated chloride ions in an electric field [4–6,10] and the absorption in concrete pores, α, β, λ, and μ are all constant with respect to z and τ.

If $C_b \to 0$, then $\omega \to 0$. Thus, Eq. (17) should be modified as Eq. (20):

$$\beta\frac{\partial \omega}{\partial \tau} = \alpha C \tag{20}$$

Substituting Eq. (20) into Eq. (16), one gets the governing Eq. (21):

$$\frac{\partial C}{\partial \tau} = \frac{\partial^2 C}{\partial z^2} - \lambda\frac{\partial C}{\partial z} - \alpha C \tag{21}$$

This is also an expression of Fick's second law with two extra terms. It has an initial condition shown in Eq. (22).

$$C(z,0) = 0 \tag{22}$$

and boundary conditions shown in Eq. (23) and Eq. (24).

$$C(z,\tau) = 1,\ \text{when } z = 0,\ \tau = \text{any value} \tag{23}$$

$$C(z,\tau) = 0,\ \text{when } z \to \infty,\ (0 < < \tau = \tau_M < \infty) \tag{24}$$

Define the Laplace transform of $C(z,\tau)$ as shown in Eq. (25).

$$\Omega(z,s) = L[C(z,\tau)] = \int_0^\infty C\cdot e^{-s\tau}d\tau \tag{25}$$

Applying the Laplace transform to Eq. (21) (the aging of concrete is not considered), one gets Eq. (26):

$$s\Omega - C(z,0) = \Omega'' - \lambda\Omega' - \alpha\Omega \tag{26}$$

Putting Eq. (22) into Eq. (26) and rearranging, one obtains Eq. (27):

$$\Omega'' - \lambda\Omega' - (\alpha + s)\Omega = 0 \tag{27}$$

Applying the Laplace transform to boundary conditions in Eq. (23) and Eq. (24), one gets Eq. (28) and Eq. (29):

$$\Omega(0,s) = \frac{1}{s} \tag{28}$$

$$\Omega(\infty,s_M) = 0 \tag{29}$$

Using Eqs. (28) and (29) and solving the ordinary differential equation (27), one obtains Eq. (30).

$$\Omega = \frac{1}{s}\exp[(\frac{\lambda}{2} - \sqrt{\frac{\lambda^2}{4} + \alpha + s})z] \tag{30}$$

The following equation can be derived from the inverse Laplace transform [see Eq. (31)] (see Appendix):

$$L^{-1}\frac{e^{-x\sqrt{s+A}}}{s} = \frac{1}{2}[e^{-x\sqrt{A}}erfc(\frac{x}{2\sqrt{t}} - \sqrt{At})$$
$$+ e^{x\sqrt{A}}erfc(\frac{x}{2\sqrt{t}} + \sqrt{At})] \tag{31}$$

where A is constant and is independent of x, s, and t.

Subjecting Eq. (30) to the inverse Laplace transform as expressed in Eq. (31), one arrives at the exact analytic solution [see Eq. (32)]

$$C(z,\tau) = \frac{1}{2}\exp[\{\frac{\lambda}{2} - \sqrt{\frac{\lambda^2}{4} + \alpha}\}z]\cdot$$
$$\{erfc[\frac{z}{2\sqrt{\tau}} - \sqrt{\left(\alpha + \frac{\lambda^2}{4}\right)\tau}] + \exp(z\sqrt{4\alpha + \lambda^2})\cdot$$
$$erfc[\frac{z}{2\sqrt{\tau}} + \sqrt{\left(\alpha + \frac{\lambda^2}{4}\right)\tau}]\} \tag{32}$$

where $erfc(x)$ is called the complementary error function and is defined as Eq. (33)

$$erfc(x) = 1 - erf(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}du \tag{33}$$

A sensitivity analysis (data not shown) suggested that all of the terms in Eq. (32) are significant (i.e., that none of them can be omitted without introducing an appreciable error).

Jastrzebski [22] pointed out that 30–40% of the volume of a porous concrete consists of pores; that is, the porosity is 30–40%. The diameter of the pores ranges from 0.002 to 10 μm, and the area of porous surface per unit gram is approximately 200–400 m^2 [22]. Thus the porosity of the concrete can obviously have a significant effect on the absorption of

Table 1
Comparison of theoretical diffusion coefficient of chloride ion with experimental results

		10 V				20 V				30 V			
No.	f_c' (Mpa)	Time (h)	D[a] ($\times 10^{-13}$)	D_P[b] ($\times 10^{-13}$)	Error (%)	Time (h)	D ($\times 10^{-13}$)	D_P ($\times 10^{-13}$)	Error (%)	Time (h)	D ($\times 10^{-13}$)	D_P ($\times 10^{-13}$)	Error (%)
A06X	60.0	95	24.58	24.95	1.5	45	26.24	28.1	7.1	21	35.86	40.7	13.5
A07X	62.5	120	21.74	19.75	9.1	41	26.51	30.5	15.0	22	35.69	38.8	8.7
B07X	63.1	132	17.93	17.9	0.17	60	17.71	21.0	18.6	28	27.08	30.5	12.6
A08Z	65.1	145	16.39	16.3	0.55	80	14.99	15.7	4.7	38	21.44	22.5	4.9
A08Y	69.2	192	12.23	12.3	0.57	89	15.08	14.1	6.5	53	15.34	16.3	6.2
A08X	82.9	300	7.83	7.9	0.89	108	11.57	11.6	0.26	64	13.33	13.45	0.90
A09X	87.7	399	5.96	5.9	1.0	190	6.97	6.6	5.3	102	8.19	8.4	2.6
A10X	91.1	422	5.45	5.6	2.7	200	6.14	6.3	2.6	105	7.60	8.2	7.9

[a] D is the diffusion coefficient in m^2/s obtained by Yang [6].

[b] D_P is the porous diffusion coefficient in m^2/s as determined by exact analytical solution. Forward deposition coefficient $k_a = 1 \times 10^{-8}$ L/s, T = 300°K (27°C), pore water saturation $d = 1.0$, the ratio of chloride ion concentration between the two sides of concrete sample C = 0.003.

chloride ions. If, the porous diffusion coefficient (D_p) is defined as shown in Eq. (34):

$$D_p = \frac{D_e}{\varepsilon} \tag{34}$$

then Eqs. (21) and (22) become Eq. (35).

$$\tau = \frac{D_p}{dL^2}t, \ \alpha = \frac{K_a L^2}{D_p} \tag{35}$$

3. Analysis of results

In order to verify the accuracy of the present theory, the results of the exact analytical solution are compared with experimental results previously obtained by Yang [6], who used a 20 × 20 × 25 cm (length × width × height) chloride migration apparatus with accelerated electrical potential [5] to measure the diffusion coefficient in cross sections (10 cm diameter × 2 cm thickness) cut from cylindrical samples of concrete that had been cured in CaO solution for 91 days.

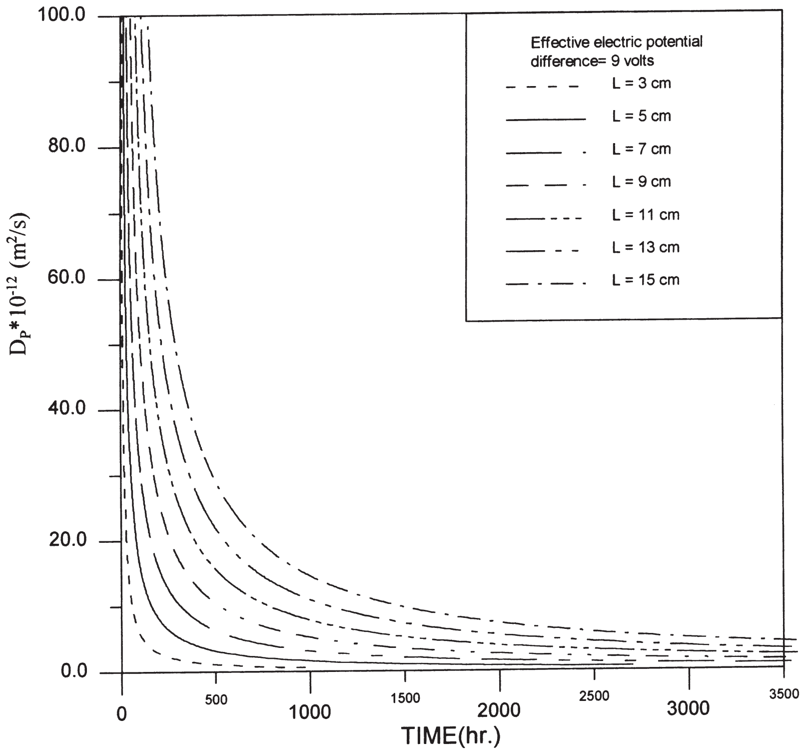

Fig. 2. Porous diffusion coefficient at effective electric potential difference 9 V vs. time based on Eq. (32) ($d = 1.0$).

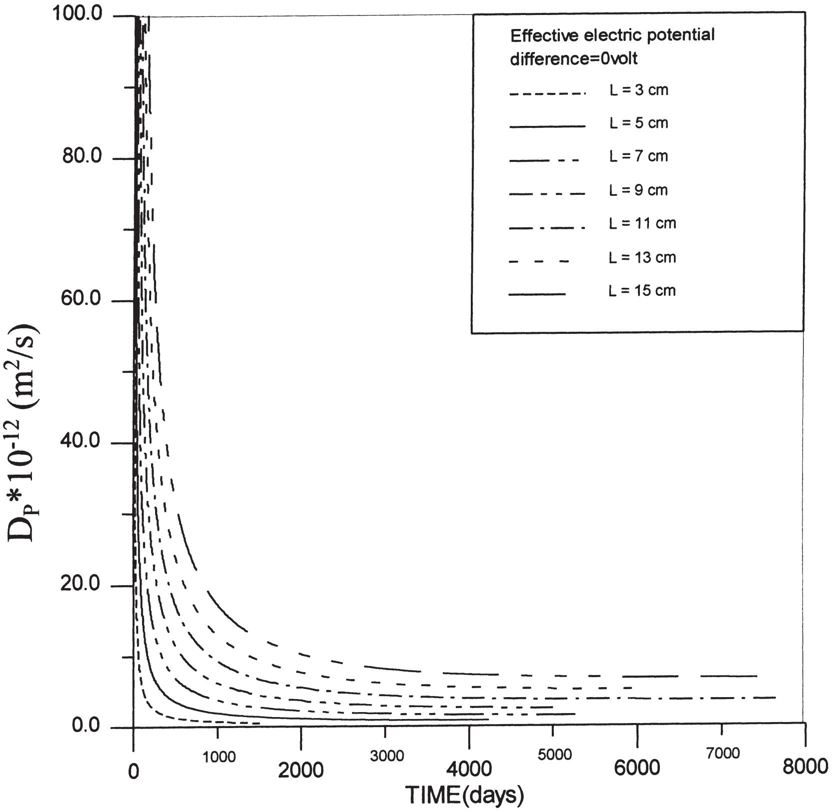

Fig. 3. Porous diffusion coefficient at natural state without electric potential difference versus time based on Eq. (32) ($d = 1.0$); the unit of D_p is m^2/s.

Table 1 compares the present theory and Yang's experiment. The values of D are close to D_p at low potential. However, the difference between D and D_p increases at comparatively large voltages. Generally speaking, it is convenient to use an electric potential difference to increase the rate of diffusion of the chloride ion and thereby decrease the testing time in approximately inverse proportion because theoretically the porous diffusion coefficient should be almost completely unaffected. According to the model used here [i.e., Eq. (32)], even at voltages as high as 30 V in the calculated porous diffusion coefficient will still be accurate to within ±1%. In practice, however, the discrepancies between calculated and measured values become quite appreciable even at 20 V (Table 1). This may be because at higher electrical potentials, there is a greater voltage drop in the solutions and electrolyte chambers of the test apparatus. This can be due to temperature effect, polarization, etc.

4. Application

A concrete specimen with a thickness of 5 cm was bored from a reinforced concrete structure (a road bridge build in 1957) with cover thickness of 15 cm. This specimen was put into a chloride migration apparatus with accelerated electrical potential [5]. After 208 h at an electric potential difference of 9 V, the chloride ion concentration was 0.003 times the critical concentration. From this result, the time needed for chloride ions to diffuse inwards until they reach the steel in the original structure (in other words, the service life) can be calculated. Fig. 2 shows the exact analytical solution; when the curve for the 5-cm thick specimen reaches the abscissa time of 208 h, the corresponding value of D_P can be read off the y axis. This value for D_P (7.8×10^{-12} m^2/s) can then be used in Fig. 3, which shows the relationship between D_p and time in the natural state (i.e., with zero effective electric potential difference). Choosing the curve that corresponds to a cover thickness of 15 cm from Fig. 3, a service life of 3330 days (9.12 years) can be read off the abscissa.

Another RC bridge with a deck of cover thickness of 5 cm is situated near the coast. Assuming that the degree of average porous saturation of bridge deck is 0.5, and that the environmental chloride concentration is 6% of the chloride concentration of seawater, the predicted time for the chloride front to reach the steel surface is to be determined. A core with thickness 3 cm was therefore drilled from the bridge deck and subjected to an accelerated electric field test. Since the environmental chloride concentration is 0.06 times that of seawater, the concentration front of interest evaluates to 0.05 times the environmental chloride concentration. This value was reached after 160 h at an electric potential difference of 6 V. From Fig. 4, this corresponds to a D_p value of 2.9×10^{-12} m^2/s. This value can now be used in Fig. 5 to predict the service life for a cover thickness of 5 cm. The predicted value is 1742 days.

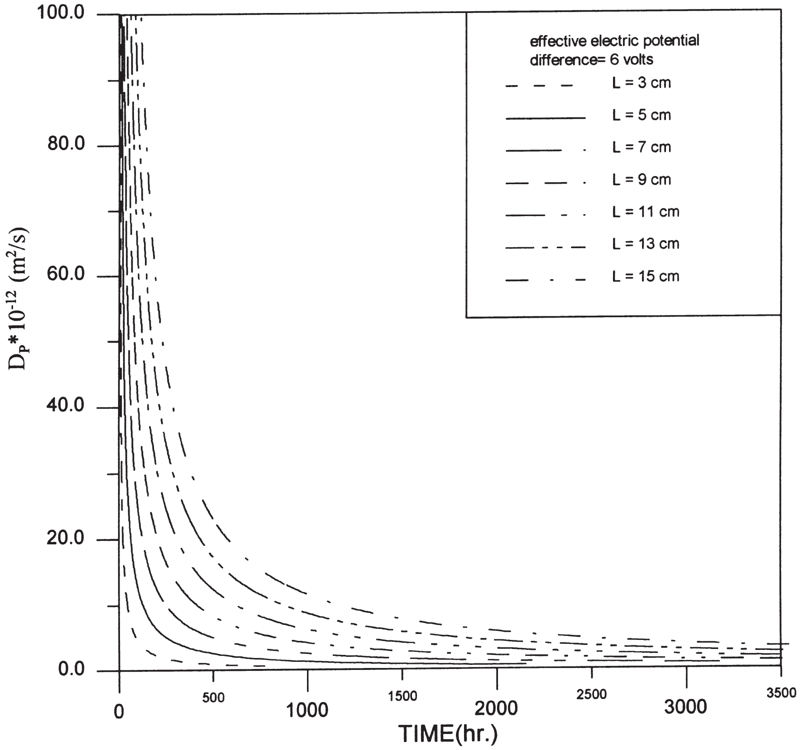

Fig. 4. Porous diffusion coefficient at effective electric potential difference 6 V vs. time based on Eq. (32) (d = 0.5).

Since the service life of reinforced concrete structures can be expressed as Eq. (36)

$$t = t_0 + t_c + t_s \tag{36}$$

where t is service life (or natural life, durability life) in years, t_0 is time in years from construction to the initiation period of concrete, t_c is the initiation period in years, and t_s is propagation period in years, if $t_0 = 40$ years, $t_c = 9.12$ years, and $t_s = 10$ years (assumed), the service life of this road bridge is 59.12 years.

5. Conclusions

The mathematical model for service life prediction presented here for existing reinforced concrete structures exposed to natural environment incorporated Fick's second law of diffusion and a term for accelerated diffusion, and is also associated with the absorption of chloride ions by concrete. In this paper, the coefficients of chloride diffusion are assumed to be time-dependent constants. The theoretical model can be used to estimate the service life (defined as the initiation period) of reinforced concrete structures. The prediction of service lifetime for existing reinforced concrete structures may be used for planning future maintenance and repair of reinforced concrete structures.

Appendix 1

To derive Eq. (31) from the inverse Laplace transform, we know [see Eq. (A1)]

$$L^{-1}\left[\frac{e^{-x\sqrt{s}}}{s}\right] = erfc\left(\frac{x}{2\sqrt{t}}\right) \tag{A1}$$

Applying the first shifting theorem to Eq. (A1), we get Eq. (A2).

$$\begin{aligned} L^{-1}\left[\frac{e^{-x\sqrt{s}}}{s-A}\right] &= L^{-1}\left[\frac{e^{-x\sqrt{s}}}{s}\left(1+\frac{A}{s-A}\right)\right] \\ &= erfc\frac{x}{2\sqrt{t}} + A\cdot\int_0^t erfc\frac{x}{2\sqrt{t}}\cdot e^{A(t-\tau)}d\tau \end{aligned} \tag{A2}$$

The second term on the right side of Eq. (A2) can be calculated as Eq. (A3).

$$\begin{aligned} A\cdot\int_0^t erfc\frac{x}{2\sqrt{t}}\cdot e^{A(t-\tau)}d\tau &= -erfc\frac{x}{2\sqrt{t}} + \\ \int_0^t \frac{x}{2\tau\sqrt{\pi\tau}}e^{-\frac{x^2}{4\tau}+A(t-\tau)}d\tau \end{aligned} \tag{A3}$$

The substitution of Eq. (A3) into Eq. (A2) becomes [see Eq. (A4)]:

$$\begin{aligned} L^{-1}\left[\frac{e^{-x\sqrt{s}}}{s-A}\right] &= \int_0^t \frac{x}{2\tau\sqrt{\pi\tau}}e^{-\frac{x^2}{4\tau}+A(t-\tau)}d\tau \\ &= \frac{1}{2}e^{At+x\sqrt{A}}\int_0^t \frac{x}{\tau\sqrt{\pi\tau}}e^{-\left(\frac{x}{2\sqrt{\tau}}+\sqrt{A\tau}\right)^2}d\tau \end{aligned} \tag{A4}$$

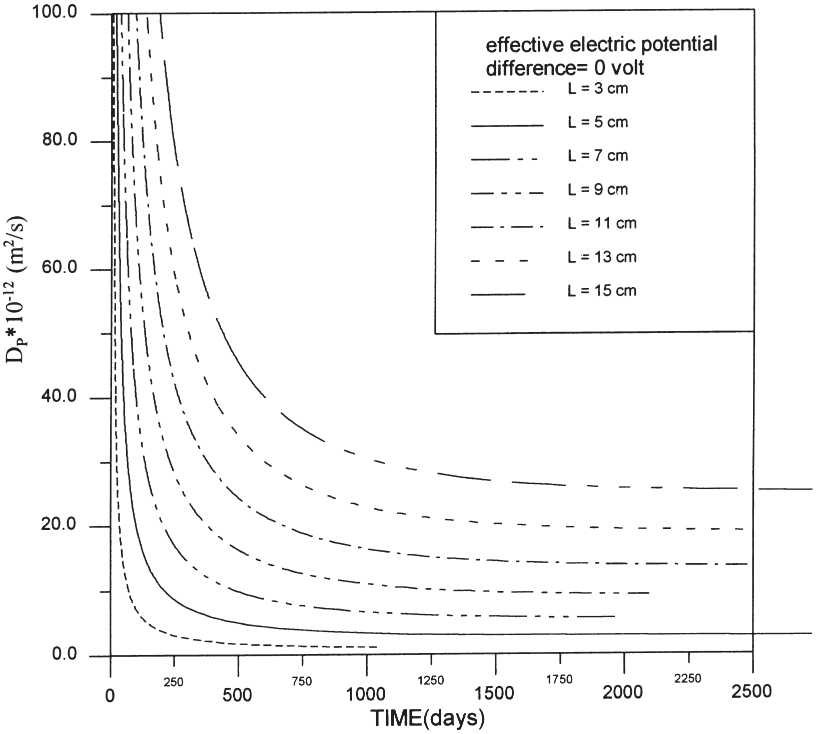

Fig. 5. Porous diffusion coefficient at natural state without electric potential difference versus time based on Eq. (32) ($d = 0.5$); the unit of D_p is m²/s.

Let [see Eq. (A5) and Eq. (A6)]:

$$EP = \frac{1}{2}e^{At + x\sqrt{A}} \tag{A5}$$

$$u = \frac{x}{2\sqrt{\tau}} + \sqrt{A\tau} \tag{A6}$$

Eq. (A6) can be expressed as Eq. (A7)

$$d\tau = \frac{4\tau\sqrt{\tau}}{2\tau\sqrt{A} - x}du \tag{A7}$$

Substituting Eqs. (A5), (A6), and (A7) into Eq. (A4), one obtains Eq. (A8):

$$\begin{aligned} L^{-1}\left[\frac{e^{-x\sqrt{s}}}{s-A}\right] &= EP \cdot \frac{2}{\sqrt{\pi}}\int_{\infty}^{u(t)} \frac{2x}{2\tau\sqrt{A} - x}e^{-u^2}du \\ &= EP \cdot \left[\frac{2}{\sqrt{\pi}}\int_0^t \frac{2\tau\sqrt{A} + x}{4\tau\sqrt{\tau}}e^{-\left(\frac{x}{2\sqrt{\tau}} + \sqrt{A\tau}\right)^2}d\tau \right. \\ &\quad \left. + erfc\left(\frac{x}{2\sqrt{t}} + \sqrt{At}\right)\right] \end{aligned} \tag{A8}$$

Let [see Eq. (A9)]

$$\nu = \frac{x}{2\sqrt{\tau}} - \sqrt{A\tau} \tag{A9}$$

Eq. (A9) can be written as Eq. (A10)

$$d\tau = -\frac{4\tau\sqrt{\tau}}{2\tau\sqrt{A} + x}d\nu \tag{A10}$$

Then [see Eq. (A11)]

$$\begin{aligned} &\frac{2}{\sqrt{\pi}}\int_0^t \frac{2\tau\sqrt{A} + x}{4\tau\sqrt{\tau}} \cdot e^{-\left(\frac{x}{2\sqrt{\tau}} + \sqrt{A\tau}\right)^2}d\tau \\ &\quad = e^{-2x\sqrt{A}}erfc\left(\frac{x}{2\sqrt{t}} - \sqrt{At}\right) \end{aligned} \tag{A11}$$

The substitution of Eq. (A11) into Eq. (A8) yields Eq. (A12)

$$\begin{aligned} &L^{-1}\left[\frac{e^{-x\sqrt{s}}}{s-A}\right] = \\ &EP \cdot \left[e^{-2x\sqrt{A}}erfc\left(\frac{x}{2\sqrt{t}} - \sqrt{At}\right) + erfc\left(\frac{x}{2\sqrt{t}} + \sqrt{At}\right)\right] \end{aligned} \tag{A12}$$

Substituting Eq. (A5) into Eq. (A12), one obtains Eq. (A13):

$$\begin{aligned} &L^{-1}\left[\frac{e^{-x\sqrt{s}}}{s-A}\right] = \\ &\quad \frac{1}{2}e^{At}\left[e^{-x\sqrt{A}}erfc\left(\frac{x}{2\sqrt{t}} - \sqrt{At}\right)\right. \\ &\quad \left. + e^{x\sqrt{A}}erfc\left(\frac{x}{2\sqrt{t}} + \sqrt{At}\right)\right] \end{aligned} \tag{A13}$$

Now, applying the first shifting theorem to Eq. (A13) leads to Eq. (31) [or Eq. (A14)]

$$L^{-1}\left[\frac{e^{-x\sqrt{s+A}}}{s}\right] = \frac{1}{2}\left[e^{-x\sqrt{A}}erfc\left(\frac{x}{2\sqrt{t}}-\sqrt{At}\right) + e^{x\sqrt{A}}erfc\left(\frac{x}{2\sqrt{t}}+\sqrt{At}\right)\right] \tag{A14}$$

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