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Effective Young's modulus estimation of concrete

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Abstract

A two-step analytical procedure is proposed to evaluate the quantitative influence of the maximum aggregate size and aggregate gradation on the effective Young's modulus of concrete. In the first step, the effective Young's modulus of a specified "basic element," which is composed of an aggregate coated with interfacial transition zone and again covered with cement paste, is obtained based on a proposed four-phase sphere model. The theory of elasticity and Eshelby's equivalent medium theory are used to achieve the goal. In the second step, the rule of mixture method is used to estimate the effective Young's modulus of concrete. Following the two-step procedure, the maximum aggregate size and aggregate gradation are included in the formulations for the effective Young's modulus of concrete. The calculated results are compared with experimental results from the literature. The comparison results show a reasonable agreement when isostrain is assumed for every basic element in the second step. Parameters influencing the effective Young's modulus of concrete are discussed via calculated results

Keywords: Micromechanics; Concrete; Elastic moduli; Interfacial transition zone; Particle size distribution

1. Introduction

In recent years, concrete has been assumed to be a three-phase composite material [1]. The influence of the Young's modulus and volume fraction of each phase—cement paste phase, interfacial transition zone (ITZ), and aggregate phase on the effective Young's modulus of the three-phase concrete is well established. Yang [2] has given an in-depth review of the theoretical and experimental studies on the effective Young's modulus of concrete. It was found experimentally that the effective Young's modulus of concrete decreases as the volume fraction of ITZ increases [3]. Because the volume fraction of ITZ is determined by the surface area of aggregate, while the surface area of aggregate is dependent on the maximum aggregate size and aggregate gradation, the volume fraction of ITZ and the corresponding effective Young's modulus of concrete should be closely related to the maximum aggregate size and aggregate gradation. Qualitatively, experiments have shown that the effective Young's modulus of concrete increases as the maximum aggregate size increases [4] and densely graded concrete has higher effective Young's modulus [5]. The same trend is also observed in other types of particulate-filled composite materials [6,7]. However, very little is known about the quantitative influence of the maximum aggregate size and aggregate gradation on the effective Young's modulus of concrete. Theoretical studies are thus aid in understanding the quantitative influence.

For common particulate-filled composite materials, theoretical methods estimating the effective elastic moduli mainly include the rule of mixture method and the method based on the theory of elasticity. Each method can be subdivided into several specified approaches. The merit of the rule of mixture method is that it can deal with composite materials containing multiple phases, but it fails to involve the geometrical parameters of each phase, such as the size of the particles. In contrast, the merit of the method based on the theory of elasticity is that it can consider the geometrical parameters of each phase, but significant difficulties are involved when tackling multiphase composite materials [8]. Therefore, it is inferred that if the two methods were combined properly, the effect of the maximum particle size and particle size distribution on the effective Young's modulus of composite materials could be considered.

According to the literature, concrete is usually assumed to be a three-phase composite material. However, it is assumed to be a special multiphase composite material in the present paper in order to use the two methods mentioned above. Taking an arbitrary size aggregate coated with ITZ

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and covered with cement paste as a "basic element" (shown schematically in Fig. 1), concrete is thus made up of a large number of such basic elements due to aggregate size distributions. Assuming each basic element as a phase, then concrete is a special multiphase composite material. Based on this treatment, the effective Young's modulus estimation of concrete is arranged as a two-step procedure in this paper. In the first step, the effective Young's modulus of a basic element with arbitrary size is obtained based on the theory of elasticity and Eshelby's equivalent medium theory. A four-phase sphere model will be proposed to achieve this goal. Following the first step, the aggregate size will be included in the formulations of the effective Young's modulus of the basic element. In the second step, concrete is assumed to be a special multiphase composite material composed of all the basic elements. Because the effective Young's modulus of a basic element with arbitrary size is obtained in the first step, the rule of mixture method can be used to estimate the effective Young's modulus of concrete containing all basic elements. This method uses an averaged summation procedure with the volume fraction corresponding to every specified basic element as its weighting factor. The volume fraction with respect to every specified basic element is determined according to the aggregate gradation. As a result, the influence of the maximum aggregate size and aggregate gradation on the effective Young's modulus of concrete can be considered.

The objective of this paper is (1) to establish a four-phase sphere model to estimate the effective Young's modulus of a basic element with an arbitrary size, (2) to use the rule of mixture method to estimate the effective Young's modulus of concrete, and (3) to compare the calculated results with experimental results in the literature and discuss the influential parameters.

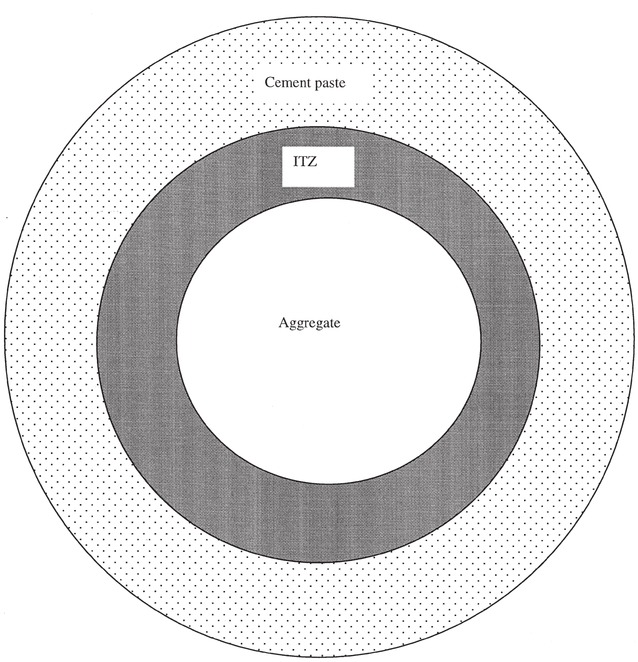

Fig. 1. A basic element.

2. Formulation development

2.1. Four-phase sphere model

Based on the theory of elasticity and Eshelby's equivalent medium theory, Christensen and Lo [9] developed a three-phase sphere model to estimate the effective shear modulus of a particulate-filled two-phase composite material. This model was formed by embedding a matrix-coated spherical particle into an infinite equivalent composite medium. A merit of this model is that it considers the stress and strain field interactions between the inclusion and the surrounding equivalent composite medium. In other words, when solving for the effective Young's modulus of the inclusion, this model does not treat the inclusion independently. Instead, the inclusion is constrained by the infinite surrounding equivalent composite medium so that the inclusion is in a stress condition similar to reality. By overall evaluations, Christensen [10] concluded that this three-phase sphere model was more reasonable and reliable than other generally used models, such as the differential scheme and the Mori-Tanaka model [11]. Following Christensen and Lo's procedure for two-phase composite materials, a four-phase sphere model is proposed by embedding a basic element with an arbitrary size into an infinite equivalent concrete medium, shown in Fig. 2. For simplicity, the specified basic element in Fig. 2 is assumed to be a three-layer sphere. The assumption of a spherical inclusion was also used in Christensen and Lo's model. This four-phase sphere model is used in this paper to estimate the effective Young's modulus of the specified basic element.

In Fig. 2, letter a denotes the radius of the aggregate, b–a the thickness of ITZ layer, and c–b the thickness of cement paste layer. p is the radial boundary stress at infinite applied as an external force; p_0 is the radial stress at $r = c$; p_1 is the radial stress at $r = b$; p_2 is the radial stress at $r = a$; and r is the spherical polar coordinate. u_{0c} and u_{1c} are radial displacements of the equivalent concrete medium and cement paste at the interface $r = c$, respectively. u_{1b} and u_{2b} are the radial displacements of the cement paste and ITZ at the interface $r = b$, respectively. u_{2a} and u_{3a} are the radial displacements of the ITZ and aggregate at the interface of $r = a$, respectively. For elastic parameters, denoting E_i and ν_i (i = 1, 2, 3) are the Young's modulus and Poisson's ratio of the cement paste ($i = 1$), ITZ ($i = 2$), and aggregate ($i = 3$), respectively. $E_0(a)$ and $\nu_0(a)$ are the effective Young's modulus and Poisson's ratio of the equivalent concrete medium, respectively. It is noted that $E_0(a)$ and $\nu_0(a)$ are not constants; instead, they are dependent on the basic element. When the basic element is changed, $E_0(a)$ and $\nu_0(a)$ will change accordingly.

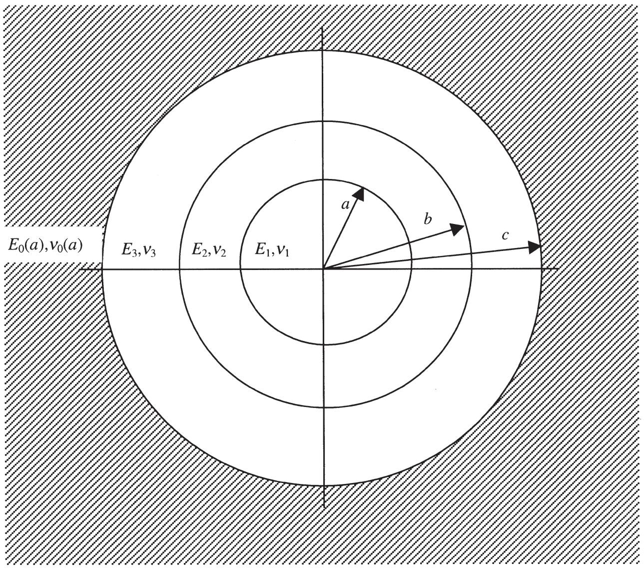

Fig. 2. Four-phase sphere model.

2.2. Effective Young's modulus estimation of a specified basic element

In addition to general assumptions for elastic bodies (isotropic, linear elasticity, etc.), assuming continuous contact at the interface of $r = a$, $r = b$, and $r = c$, and taking p as uniformly distributed, Fig. 2 describes a spherical symmetry problem. Using the theory of elasticity [12], the displacements at the interface $r = a$, $r = b$, and $r = c$ are obtained [see Eq. (1), Eq. (2), Eq. (3), Eq. (4), Eq. (5), and Eq. (6)]

$$u_{0c} = A_0 c(B_1 p - B_2 p_0) \quad (1)$$

$$u_{1c} = A_1 c(B_3 p_0 - B_4 p_1) \quad (2)$$

$$u_{1b} = A_1 b(B_5 p_0 - B_6 p_1) \quad (3)$$

$$u_{2b} = A_2 b(B_7 p_1 - B_8 p_2) \quad (4)$$

$$u_{2a} = A_2 a(B_9 p_1 - B_{10} p_2) \quad (5)$$

$$u_{3a} = A_3 a p_2 \quad (6)$$

where [see Eq. (7)]

$$A_0 = \frac{1+\nu_0(a)}{E_0(a)};\ A_1 = \frac{1+\nu_1}{E_1};\ A_2 = \frac{1+\nu_2}{E_2};\ A_3 = \frac{1-2\nu_3}{E_3}$$

$$B_1 = 0.5 + \xi;\ B_2 = 0.5;$$

$$B_3 = \frac{0.5n+\delta}{1-n};\ B_4 = \frac{0.5+\delta}{n^{-1}-1};\ B_5 = \frac{0.5+\delta}{1-n}$$

$$B_6 = \frac{0.5n^{-1}+\delta}{n^{-1}-1};\ B_7 = \frac{0.5k+\eta}{1-k};$$

$$B_8 = \frac{0.5+\eta}{k^{-1}-1};\ B_9 = \frac{0.5+\eta}{1-k};\ B_{10} = \frac{0.5k^{-1}+\eta}{k^{-1}-1}$$

$$\xi = \frac{1-2\nu_0(a)}{1+\nu_0(a)};\ \delta = \frac{1-2\nu_1}{1+\nu_1};\ \eta = \frac{1-2\nu_2}{1+\nu_2};$$

$$n = \frac{b^3}{c^3};\ k = \frac{a^3}{b^3} \quad (7)$$

According to the continuous conditions at the interfaces that the radial displacements are equal, the relations shown in Eq. (8), Eq. (9), and Eq. (10) hold.

$$u_{0c} = u_{1c} \quad (8)$$

$$u_{1b} = u_{2b} \quad (9)$$

$$u_{2a} = u_{3a} \quad (10)$$

Simultaneously solving Eqs. (1) through (10), the interfacial radial stresses are obtained as shown in Eq. (11), Eq. (12), and Eq. (13):

$$p_1 = A/B \quad (11)$$

$$p_2 = \frac{A_2 B_9 p_1}{A_3 + A_2 B_{10}} \quad (12)$$

$$p_0 = \frac{A_0 B_1 p + A_1 B_4 p_1}{A_0 B_2 + A_1 B_3} \quad (13)$$

where [see Eq. (14)]

$$A = \frac{A_0 A_1 B_1 B_5 p}{A_0 B_2 + A_1 B_3}$$

$$B = A_2 B_7 + A_1 B_6 - \frac{A_1^2 B_4 B_5}{A_0 B_2 + A_1 B_3} - \frac{A_2^2 B_8 B_9}{A_3 + A_2 B_{10}} \quad (14)$$

To determine the effective Young's modulus of the specified basic element in Fig. 2, Eshelby's equivalent medium theory is used. According to Eshelby [13], the strain energy, U, for a homogeneous medium containing an inclusion under applied traction conditions is determined by Eq. (15)

$$U = U_0 + \frac{1}{2}\int_{S_i} (\sigma_i^0 u_i - \sigma_i u_i^0)\,ds \quad (15)$$

where S_i is the surface of the inclusion, U_0 is the strain energy in the same medium when it contains no inclusion, σ_i^0 and u_i^0 are the tractions and displacements in the same medium when it contains no inclusion and σ_i and u_i are the corresponding quantities at the same point in the medium when it does contain the inclusion.

Christensen and Lo [9] proved that [see Eq. (16)]

$$U = U_0 \quad (16)$$

Substituting Eq. (16) into Eq. (15), we have Eq. (17)

$$\int_{S_i} (\sigma_i^0 u_i - \sigma_i u_i^0)\,ds = 0 \quad (17)$$

For the problem described in Fig. 2, the inclusion refers to the basic element, the inclusion surface S_i is a spherical surface with radius of $r = c$, and the infinite equivalent concrete medium is the corresponding homogeneous medium.

According to the definition of σ_i and u_i in Eq. (15), they are expressed as shown in Eq. (18) and Eq. (19)

$$\sigma_i = p_0 \tag{18}$$

$$u_i = u_{0c} \tag{19}$$

where p_0 and u_{0c} are shown in Eqs. (13) and (1), respectively.

According to the definition of σ_i^0 and u_i^0 in Eq. (15), the space occupied by the basic element in Fig. 2 should be replaced by the same volume-equivalent concrete medium. Therefore, the effective Young's modulus and Poisson's ratio in the space of $r < c$ is the same as that in the space of $r > c$. This means that when the basic element is treated as an equivalent homogeneous material, its effective Young's modulus and Poisson's ratio are equal to the effective Young's modulus and Poisson's ratio of the surrounding equivalent concrete medium, i.e., $E_0(a)$ and $\nu_0(a)$, respectively. Using the theory of elasticity, σ_i^0 and u_i^0 are obtained as shown in Eq. (20) and Eq. (21)

$$\sigma_i^0 = p \tag{20}$$

$$u_i^0 = \frac{1-2\nu_0(a)}{E_0(a)}pc \tag{21}$$

Substituting Eqs. (18) through (21), (13), and (1) into Eq. (17), the effective Young's modulus of the basic element, $E_0(a)$, and Poisson's ratio, $\nu_0(a)$, are correlated by Eq. (22)

$$D_1\left[\frac{1+\nu_0(a)}{E_0(a)}\right]^2 + D_2\left[\frac{1+\nu_0(a)}{E_0(a)}\right] + D_3 = 0 \tag{22}$$

where [see Eq. (23)]

$$\begin{aligned} D_1 &= B_2C_2 \\ D_2 &= C_1 + C_3\xi + A_1B_3C_2\xi - A_1B_2B_3C_2 \\ D_3 &= -A_1B_3C_3 - A_1^2B_3^2C_2 \end{aligned} \tag{23}$$

in which [see Eq. (24)]

$$\begin{aligned} C_1 &= A_1^2B_1B_4B_5(A_3 + A_2B_{10}) \\ C_2 &= (A_2B_7 + A_1B_6)(A_3 + A_2B_{10}) - A_2^2B_8B_9 \\ C_3 &= -A_1^2B_4B_5(A_3 + A_2B_{10}) \end{aligned} \tag{24}$$

It is noted in Eq. (22) that the effective Young's modulus and Poisson's ratio are coupled. The Poisson's ratio will be estimated by using the rule of mixture method in the next section.

It can be seen from Eq. (22) that the effective Young's modulus of the basic element depends not only on the elastic properties of the aggregate, ITZ, and cement paste, but also on the aggregate size, a, ITZ layer thickness, b–a, and cement paste layer thickness, c–b. For a given concrete, the elastic properties of each phase, the maximum aggregate size, and the aggregate gradation are known, which means E_i and ν_i ($i = 1, 2, 3$) and a are known parameters. However, it is necessary to determine the parameters, b and c, before Eq. (22) can be used to estimate the effective Young's modulus of the basic element.

It has been shown that in a typical concrete, although the thickness of the ITZ layer depends on factors such as the water/cement ratio, it seems to be independent of the size of the inclusion [14]. Therefore, it is reasonable to assume the thickness of the ITZ layer (b–a) to be constant, regardless of the size distribution of aggregates. Based on this assumption, the following relationship can be derived [see Eq. (25)]:

$$b - a = f_2 \Big/ \left(3f_3\sum_{i=1}^{N}\frac{K_i}{r_i}\right) \tag{25}$$

where r_i ($i = 1, 2, \ldots, N - 1$) is the average size of the No. i sieve and No. ($i + 1$) sieve when aggregates are divided into N grades by sieving; K_i the residue of aggregates on No. ($i + 1$) sieve; f_2 and f_3 are the volume fractions of the ITZ and aggregate, respectively.

In determining the parameter c, Christensen and Lo's assumption is used. According to Christensen and Lo [9], the following relationship holds [see Eq. (26)]:

$$c = b/(f_2 + f_3)^{1/3} \tag{26}$$

Before Eq. (25) and Eq. (26) can be used to determine b and c, f_2 and f_3 should be determined. For a given concrete, f_3 and ($f_2 + f_1$) are known, where f_1 is the volume fraction of the cement paste, and the following relation holds [see Eq. (27)]:

$$f_1 + f_2 + f_3 = 1 \tag{27}$$

According to experimental results, the average thickness of the ITZ layer in a typical concrete is about 0.05 mm [14,15], thus the term $b - a$ in Eq. (25) is about 0.05 mm. For a given concrete, the aggregate gradation is also known; thus the volume fraction of the ITZ is obtained from Eq. (25) as shown in Eq. (28):

$$f_2 = 3(b-a)f_3\sum_{i=1}^{N}\frac{K_i}{r_i} \tag{28}$$

Once the volume fraction of ITZ, f_2, is determined from Eq. (28), the volume fraction of the cement paste, f_1, can be derived from Eq. (27).

Based on the above formulations, Eq. (22) can be used to estimate the effective Young's modulus of the basic element. As expected, the effective Young's modulus of the basic element is related to the aggregate size, a.

2.3. Estimation of the effective Young's modulus of concrete

It is found from Eq. (22) that $E_0(a)$ and $\nu_0(a)$ are coupled. To get $E_0(a)$ from Eq. (22), $\nu_0(a)$ should be determined first. Compared with the effective Young's modulus, the Poisson's ratio of concrete varies slightly. For simplicity, assuming that the Poisson's ratio for every basic element is equal to the effective Poisson's ratio of concrete; that is, $\nu_0(a)$ is equal to ν, the effective Poisson's ratio of concrete. Thus $\nu_0(a)$ is independent of aggregate size, a.

In order to estimate the effective Poisson's ratio of concrete, assumptions are needed. Assuming that concrete is composed of three phases—cement paste, ITZ, and aggregate. These three phases are assumed to be either isostrain or isostress in axial direction. Using the volume fraction of each phase as the weighting factor, the effective Poisson's ratio can be estimated as follows.

Using isostrain assumption, Li et al. [16] developed the following expression [see Eq. (29)]

$$\nu_0(a) = \nu = \nu_1 f_1 + \nu_2 f_2 + \nu_3 f_3 \quad (29)$$

Under isostress assumption, Ahmed and Jones [6] proposed the expression seen in Eq. (30).

$$\nu_0(a) = \nu = \frac{\nu_1 f_1 E_1 + \nu_2 f_2 E_2 + \nu_3 f_3 E_3}{f_1 E_1 + f_2 E_2 + f_3 E_3} \quad (30)$$

Because concrete is assumed to be a special multiphase composite material with each basic element as a phase, there is no easy way of determining the stress or strain relations between each phase, such as the mixed isostrain and isostress relations used by Zhou et al. [17]. For simplicity, the isostrain or isostress relation between every phase is assumed in the present paper. Their applicability will be determined by a comparison with experimental results found in the literature. Based on these assumptions, the effective Young's modulus of concrete can be obtained as follows.

With isostrain assumption [see Eq. (31)]

$$E = \int_{a_{min}}^{a_{max}} E_0(a) dS(a) \quad (31)$$

where a_{min} and a_{max} are the minimum and maximum aggregate sizes, respectively. $S(a)$ is the aggregate gradation.

For densely graded aggregates, $S(a)$ can take the form shown in Eq. (32):

$$S(a) = \left(\frac{a - a_{min}}{a_{max} - a_{min}}\right)^m \quad (32)$$

where m is an exponent determining the shape of the gradation curve. When $a_{max} = 20$ mm and $a_{mim} = 0.02$ mm, two aggregate gradations are shown in Fig. 3 with respect to $m = 0.4$ and $m = 0.5$, respectively.

With isostress assumption [see Eq. (33)]

$$\frac{1}{E} = \int_{a_{min}}^{a_{max}} \frac{1}{E_0(a)} dS(a) \quad (33)$$

Integrating Eq. (31) and Eq. (33) is very complicated. For simplicity, the integration can be replaced by a summation as follows.

With isostrain assumption [see Eq. (34)]

$$E = \sum_{i=1}^{N} E_0(a) K_i \quad (34)$$

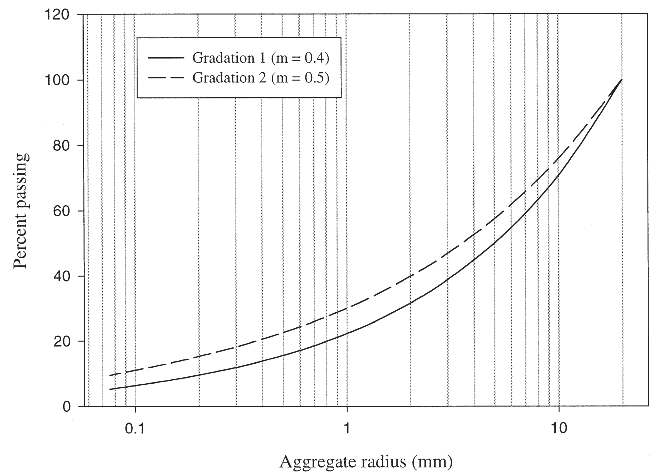

Fig. 3. Aggregate gradations.

With isostress assumption [see Eq. (35)]

$$\frac{1}{E} = \sum_{i=1}^{N} \frac{K_i}{E_0(a)} \quad (35)$$

where K_i is the same as in Eq. (25).

Eq. (34) and Eq. (35) are used to calculate the effective Young's modulus of concrete in the present paper, where the step length for the aggregate radius, Δa, is assumed as $\Delta a = 0.001\, a_{max}$.

3. Results and discussions

3.1. Comparison with experimental results

In order to compare the calculated results with the experimental results in a comparatively wide range, the test results by Stock et al. [18] are used, except for the Poisson's ratio and Young's modulus of the ITZ. In order to compare the test results with the predicted results by the present model, these parameters should be assumed. According to Simeonov and Ahmad [3], the Poisson's ratio for each phase is taken as $\nu_1 = 0.25$, $\nu_2 = 0.3$, and $\nu_3 = 0.15$. According to the results calculated by Lutz et al. [19], the Young's modulus of the ITZ is assumed as $E_2 = 0.4E_1$, where E_1 is the Young's modulus of the cement paste.

Fig. 4 illustrates the test results by Stock et al. [18], the predicted results by Eq. (34) and Eq. (35), and the predicted results by Li et al. [16]. It is found from Fig. 4 that when the aggregate content is in the range of a typical concrete ($f_3 = 0.6$–0.8), the predicted results from Eq. (34) are close to the test results, while the predicted results form Eq. (35) show a significant divergence from the test data. This means the isostrain assumption for Eq. (34) is more suitable than the isostress assumption for Eq. (35). The only exception is when $f_3 = 0.4$. The test datum shows a large difference from that by Eq. (34) and also a lower value than that by Eq. (35). This may be caused by different external and internal constraints between the model and the test. First, the boundary condition or the external constrain is different. The compression specimen used by Stock et al. is in a one-dimensional stress condition, while the four-phase sphere model in Fig. 2 is in a three-dimensional stress condition because a hydraulic pressure p is applied as the boundary stress. The uniaxial compressive test leads to higher deformation than that when lateral constraints are applied, such as in the proposed model. Consequently, the higher deformation in the test results in lower effective Young's modulus. Second, the internal constraint is different. A specified basic element is constrained by an infinite number of other basic elements in the proposed model, while only a limited number of basic elements interact with a specified basic element in the test due to the limited size of the specimens. The difference becomes larger when f_3 is lower because a specified basic element is constrained by a reduced number of other basic elements in the test. As a result, the difference between the test results and the predicted results by Eq. (34) is small when f_3 is comparatively large ($f_3 \geqslant 0.6$) and becomes large when f_3 is comparatively small ($f_3 = 0.4$).

In a previous study, the authors [16] have obtained the effective bulk modulus of concrete based on the formulations by Christensen and Lo. A key step in this previous study is to transform a four-phase sphere model to an equivalent three-phase sphere model and an equivalent two-phase sphere model. This current study is a refined study compared to the previous one. The effective Young's modulus is obtained directly based on the proposed concepts of a basic element and a special multiphase composite material. In Fig. 4, it is seen that Eq. (34) is closer to the test results than our previous study when the aggregate concentration is within the commonly used range (f_3 = 0.6–0.8). This suggests that the current model is more suitable than the previous one. However, the current formulations are more complicated than those in the previous study. Therefore, as an initial estimation, the previous study is still valuable.

3.2. Parameters influencing the effective Young's modulus of concrete

To evaluate the parameters influencing the effective Young's modulus of concrete, the values used in the calculations are specified as follow: $E_1 = 15$ GPa, $E_2 = 7.5$ GPa, $E_3 = 70$ GPa, $f_3 = 0.6$, $\nu_1 = 0.25$, $\nu_2 = 0.3$, $\nu_3 = 0.15$, $a_{max} =$ 20 mm, and $a_{min} = 0.02$ mm. Gradation 1 in Fig. 3 is used as the aggregate gradation. The ITZ thickness is taken as 0.05 mm. Eq. (34) is used in calculations.

In Fig. 5, two aggregate gradations shown in Fig. 3 are used. It is found from Fig. 5 that the effective Young's modulus of concrete increases as the maximum aggregate size increases. The effect is large when the maximum aggregate size is comparatively small and becomes small when the maximum aggregate size is comparatively large. This trend is qualitatively supported experimentally [4]. The possible reason for this result is that the total aggregate surface area decreases as the coarser aggregates increase, which will reduce the volume fraction of the ITZ. Because the Young's modulus of the ITZ is low, the decrease of the volume fraction of the ITZ makes the effective Young's modulus of concrete increase. From Fig. 3, gradation 2 contains a larger amount of coarser aggregates than gradation 1; thus concrete with gradation 2 should have a higher effective Young's modulus than that with gradation 1. The result in Fig. 5 is in agreement with this analysis.

In Fig. 6, the effective Young's modulus of concrete increases as the Young's modulus of each phase increases. However, their influence on the effective Young's modulus of concrete is phase-dependent. It is found from Fig. 6 that the cement paste phase has a significant effect, while the effect of the ITZ and aggregate is comparatively small. This is

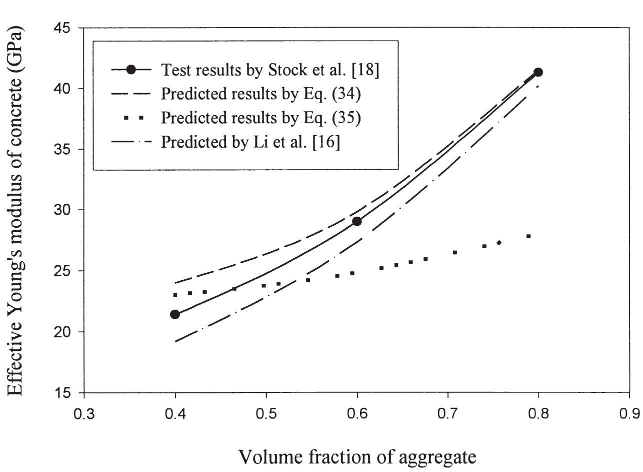

Fig. 4. Comparison of test results with predicted data.

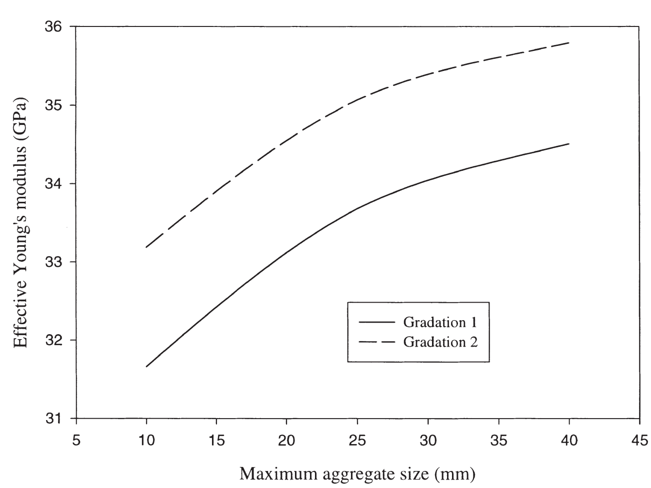

Fig. 5. Effect of maximum aggregate size and aggregate gradation on the effective Young's modulus of concrete.

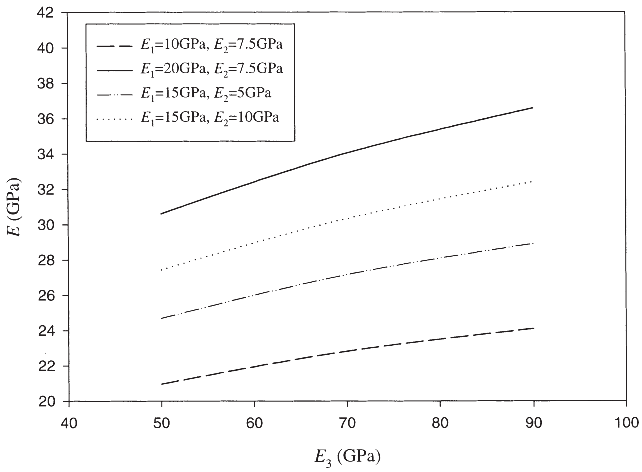

Fig. 6. Effect of Young's modulus of every phase on the effective Young's modulus of concrete.

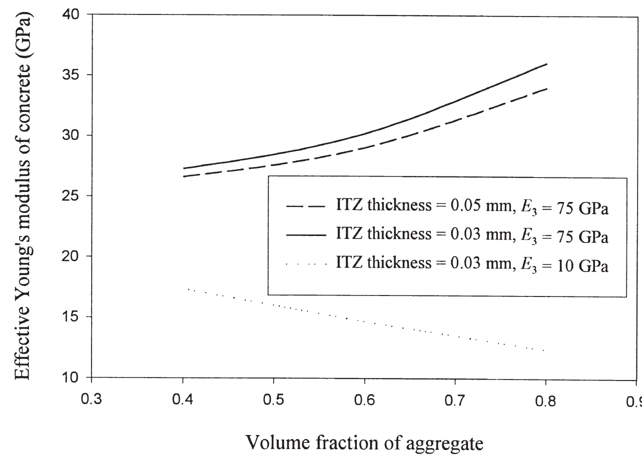

Fig. 7. Effect of aggregate concentration and ITZ thickness on the effective Young's modulus of concrete.

understandable because the cement paste acts as a continuous phase or matrix in concrete. This result means that increasing the Young's modulus of the cement paste through measures such as reducing the water/cement ratio is the most efficient way of increasing the effective Young's modulus of concrete.

Fig. 7 shows the effect of the aggregate concentration and ITZ thickness on the effective Young's modulus of concrete. Two types of aggregates are considered: one is a common aggregate with $E_3 = 70$ GPa, and the other is a lightweight aggregate with $E_3 = 10$ GPa. For concrete composed of common aggregates, the effective Young's modulus of the concrete increases as either the volume fraction of the aggregate increases or the ITZ thickness decreases. This result means that using a densely graded aggregate or reducing the ITZ thickness through measures such as decreasing the water/cement ratio or incorporating silica fume are efficient ways of increasing the effective Young's modulus of concrete. For concrete composed of lightweight aggregates, the effect of the aggregate concentration is the opposite of when common aggregates are used. The effective Young's modulus of the concrete decreases as the volume fraction of the aggregate increases.

4. Conclusions

A way of considering the effect of the maximum aggregate size and aggregate gradation on the effective Young's modulus of concrete is proposed through a two-step analytical procedure. In the first step, a four-phase sphere model is established to predict the effective Young's modulus of a basic element with an arbitrary size. The theory of elasticity and Eshelby's equivalent medium theory are used to develop the formulations. In the second step, the rule of mixture method is used to estimate the effective Young's modulus and Poisson's ratio of concrete. In this step, concrete is assumed to be a special multiphase composite material containing all the basic elements. The calculated results are compared to those found in the literature. A reasonable agreement is found between the experimental results and the predicted results from Eq. (34). The parameters influencing the effective Young's modulus of concrete are evaluated. Based on the calculated results, the following preliminary conclusions are reached:

1. Increasing the maximum aggregate size within a certain extent or using densely graded aggregate leads to increased effective Young's modulus of concrete.
2. Increasing the Young's modulus of the cement paste through measures such as reducing the water/cement ratio is the most efficient way of increasing the effective Young's modulus of concrete. Increasing the Young's modulus of the ITZ or aggregate is not as efficient as increasing the Young's modulus of the cement paste.
3. Increasing the volume fraction of the aggregates or reducing the ITZ thickness through measures such as using densely graded aggregate, incorporating silica fume, and reducing the water/cement ratio are efficient ways of increasing the effective Young's modulus of concrete. For concrete composed of lightweight aggregates, the effective Young's modulus of the concrete decreases as more aggregates are used.
4. Although the predicted results are confirmed by an experimental study found in the literature, further experimental studies are needed to evaluate systematically the maximum aggregate size and aggregate gradation on the effective Young's modulus of concrete.

References

[1] A.U. Nilsen, P.J.M. Monteiro, Concrete: A three phase material, Cem Concr Res 23 (1993) 147–151.

[2] C.C. Yang, Effect of the transition zone on the elastic moduli of mortar, Cem Concr Res 28 (1998) 727–736.

[3] P. Simeonov, S. Ahmad, Effect of transition zone on the elastic behavior of cement-based composites, Cem Concr Res 25 (1995) 165–176.
[4] J. Vilardell, A. Aguado, L. Agullo, R. Gettu, Estimation of the modulus of elasticity for dam concrete, Cem Concr Res 28 (1998) 93–101.
[5] F. Jiang, Roller Compacted Concrete, People's Railroad Publishing House, Beijing, China, 1991 (in Chinese).
[6] S. Ahmed, F.R. Jones, A review of particulate reinforcement theories for polymer composites, J Mater Sci 25 (1990) 4933–4942.
[7] W. Heukelom, A.J.G. Klomp, Road design and dynamic loading, Proc AAPT 33 (1964) 92–125.
[8] R.F. Gibson, Principles of Composite Material Mechanics, McGraw-Hill, Inc., New York, 1994.
[9] R.M Christensen, K.H. Lo, Solutions for effective shear properties in three phase sphere and cylinder models, J Mech Phys Solids 27 (1979) 315–330.
[10] R.M Christensen, A critical evaluation for a class of micro-mechanics models, J Mech Phys Solids 38 (1990) 379–404.
[11] T. Mori, K. Tanaka, Average stress in matrix and average elastic energy of materials with misfitting inclusions, Acta Metallurgica 21 (1973) 571–574.
[12] P.S. Timoshenko, J.N. Goodier, Theory of Elasticity, 3d ed., McGraw-Hill, Inc., New York, 1970.
[13] J.D. Eshelby, The determination of the elastic field of an ellipsoidal inclusion, and related problems, Proc Roy Soc Lond A241 (1957) 376–392.
[14] K.L. Scrivener, KM Nemati, The percolation of pore space in the cement paste/aggregate interfacial zone of concrete, Cem Concr Res 26 (1996) 35–40.
[15] P.K. Mehta, P.J.M. Monteiro, Concrete: Structure, Properties, and Methods, 2d ed., Prentice-Hall, Englewood Cliffs, NJ, 1993.
[16] G. Li, Y. Zhao, S.S. Pang, Four-phase sphere modeling of effective bulk modulus of concrete, Cem Concr Res 29 (1999) 839–845.
[17] F.P. Zhou, F.D. Lydon, B.I.G. Barr, Effect of coarse aggregate on elastic modulus and compressive strength of high performance concrete, Cem Concr Res 25 (1995) 177–186.
[18] A.F. Stock, D.J. Hannant, R.I.T. Williams, The effect of aggregate concentration upon the strength and modulus of elasticity of concrete, Mag Conc Res 31 (1979) 225–234.
[19] M.P. Lutz, P.J.M. Monteiro, R.W. Zimmerman, Inhomogeneous interfacial transition zone model for the bulk modulus of mortar, Cem Concr Res 27 (1997) 1113–1122.