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Sphericity, shape factor, and convexity measurement of coarse aggregate for concrete using digital image processing

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Abstract

A method of measuring the sphericity, shape factor, and convexity of coarse aggregate for concrete using digital image processing (DIP) is developed. Unlike other DIP methods, this method is capable of estimating the thickness and volume of the particles, and can thus be used to measure shape parameters dependent on thickness and to evaluate weighted mean values of the shape parameters of the individual particles in an aggregate sample. A total of 46 rock aggregate samples obtained from five different sources derived from three different types of rock have been analyzed by the method and the shape parameters so measured are correlated to the traditional measure of angularity with the hope of identifying shape parameters that may be used as direct measures of angularity. Several shape parameters are found to have good correlation with the traditional measure of angularity but among them, only the convexity ratio and fullness ratio may be used as measures of angularity. Lastly, it is advocated that the traditional measure of angularity in terms of packing density should be abandoned; packing density by itself is an important indicator of aggregate performance but is not a good measure of angularity.

Keywords: Aggregate; Coarse aggregate for concrete; Digital image processing; Packing density; Particle shape analysis

1. Introduction

Digital video technology has advanced so rapidly that it is now much more affordable and easier to use than before. From a video camera, a scene can be captured electronically producing video signals, which are first digitized and then stored as an array of pixels. Subsequently, pictorial information about the scene may be extracted from the pixel array by the use of a technique called digital image processing (DIP). Using the DIP technique [1], objects in the scene can be discriminated from the background and then analyzed or measured. The geometric parameters that can be measured include particle count, area fraction, size distribution, shape characteristics and spatial distribution, etc. This method of measurement has the major advantages that it is almost automatic, quick, not prone to human errors and capable of performing sophisticated measurements.

As part of a research program to explore the possible applications of DIP to concrete technology, the authors are investigating viable means of applying DIP to the size and shape analysis of aggregate particles. This is a topic of practical importance because a concrete mix is constituted largely of aggregate and its quality is hence dependent on the grading, size, and shape of the aggregate used. Applications of the DIP technique to particle size and shape analysis have been attempted by Barksdale et al. [2], Li et al. [3], Yue and Morin [4], and Kuo et al. [5]. Many useful results have been obtained. However, there are a number of problems associated with the application of DIP to particle size and shape analysis.

One major problem with the DIP technique is that only the two dimensional projection of the particles is captured and measured. Consequently, the third dimension, i.e., thickness of the particles is not directly obtainable from the DIP results. Due to this problem [4], the DIP results have to be expressed in terms of area fractions rather than mass fractions and are thus more difficult to interpret, as most people are more used to measuring quantity by mass.

Another problem is that many traditional measures of particle size and shape, e.g., the sieve size, flakiness index, and elongation index, which were originally developed to suit mechanical sieving and manual methods of measure-

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ment, are not suitable for use by the DIP technique. In fact, the sieve size of a particle is neither the length, breadth nor thickness of the particle and thus, cannot be measured directly by DIP [6]. Furthermore, since the flakiness and elongation indexes are measured based on the mean sieve sizes of the particles, they are also not easily obtained by DIP [7]. Hence, when applying the DIP technique, new parameters more suitable for direct measurement from the geometry of the particles may have to be employed.

There is also the problem that different researchers are using different shape indexes to describe the same shape attribute and even different definitions for the same shape index [2,5]. It may take many more years before agreement on the use of standardized indexes to describe shape characteristics could be reached.

This paper is the third of a series by the authors on applications of DIP to aggregate analysis. In the first paper [6], the authors have applied DIP to size distribution analysis of coarse aggregate and developed a simple method of converting area fractions to mass fractions so that grading results obtained by DIP can be interpreted more easily. In the second paper [7], DIP was applied to evaluate the flakiness and elongation of coarse aggregate particles. Although thickness is not directly measurable by DIP, the authors have successfully developed a method of estimating the mean thickness/breadth ratio of an aggregate by supplementing the DIP results with the weight of the aggregate sample analyzed. In this paper, the DIP method is applied to sphericity, shape factor, and convexity measurement of coarse aggregate and the shape parameters obtained by DIP are correlated to the traditional measure of angularity so as to identify those having good correlation with the traditional measure of angularity. Angularity is very difficult to be measured directly from the shape of the particles and the traditional method is to measure angularity indirectly from the packing density of the aggregate particles. It is hoped that among those shape parameters having good correlation with the traditional measure of angularity, some may be used as direct measures of angularity.

2. Measurement of shape parameters by DIP

The DIP system used is a Quantimet Q600 manufactured by Leica Cambridge. It incorporates a 3-chip CCD video camera having a resolution of 736 × 574 pixels, a frame grabber with three A/D converters each of 8-bit resolution, a photographic stand fitted with light sources, a computer and a set of software for image analysis.

To conduct image analysis of an aggregate sample, an image of the aggregate particles is first acquired by placing the particles on a sample tray and putting the sample tray underneath the video camera mounted on the photographic stand. Details of the image acquisition process and the calibration procedures have been reported previously [6]. Having acquired an image of the aggregate particles, DIP is performed to discriminate the aggregate particles from the background. This involves increasing the contrast between the particles and the background and finding the boundary of each particle. Once the particle boundaries are located, their geometry is analyzed to measure the dimensions and shape characteristics of the particles. Upon completion, the measured results are saved in a spreadsheet file for statistical analysis and post-processing.

2.1. Estimating thickness and volume of particles from DIP results

Since only the two dimensional projection of the particles is captured for image analysis, the thickness and volume of the particles are not directly obtainable from the DIP results. Nevertheless, a method of estimating the thickness and volume of the particles has been developed previously [6]. It is based on the assumption that aggregate particles from the same source should have more or less the same shape characteristics. Using this assumption, the mean thickness of a particle may be estimated from the breadth of the particle as:

$$\text{mean thickness} = \lambda \times \text{breadth} \tag{1}$$

in which λ is a parameter dependent on the flakiness of the aggregate. From this equation, the volume of the particle may be estimated as:

$$\text{volume} = \text{mean thickness} \times \text{area} = \lambda \times \text{breadth} \times \text{area}. \tag{2}$$

Adding the volume of all particles and multiplying by the density ρ, the equation for the total mass of the aggregate sample M is derived as Eq. (3):

$$M = \rho \times \lambda \times \sum_{i=1}^{n} (\text{breadth} \times \text{area}) \tag{3}$$

where n is the total number of particles. Solving the above equation, λ is determined as:

$$\lambda = \frac{M}{\rho \times \sum_{i=1}^{n} (\text{breadth} \times \text{area})}. \tag{4}$$

This value of λ is actually the mean thickness/breadth ratio of the aggregate sample. Substituting the value of λ into Eqs. (1) and (2), the thickness and volume of each particle may then be obtained.

2.2. Arithmetic and weighted mean values of shape parameters

An aggregate sample consists of many particles. To determine the shape parameter of an aggregate, it is first necessary to measure the shape parameter of each particle.

After measuring the shape parameters of all the particles, there is the question of how to calculate the mean value of the shape parameter for the aggregate sample. There are at least two ways of calculating the mean value. The first is to calculate the mean value as the arithmetic mean of the shape parameters of the particles, as given in the following formula [Eq. (5)]:

$$\text{arithmetic mean of shape parameter} = \frac{1}{n}\sum_{i=1}^{n}(\text{shape parameter}) \tag{5}$$

in which each value of shape parameter derived from a particle is given equal weight regardless of the size of the particle. However, since the larger particles generally have greater effects on the overall performance of the aggregate, a better alternative is to take the mean value as the weighted mean of the shape parameters of the particles, as Eq. (6):

$$\text{weighted mean of shape parameter} = \frac{\sum_{i=1}^{n}(\text{volume} \times \text{shape parameter})}{\sum_{i=1}^{n}(\text{volume})}. \tag{6}$$

Both the arithmetic and weighted mean values are calculated and used in the present study.

2.3. Type of shape parameters measured

2.3.1. Flakiness ratio

The flakiness ratio is defined as the thickness to breadth ratio. It is also called flatness ratio [2]. Although the thickness is not directly obtainable by DIP, the value of λ determined according to Eq. (4) is actually a weighted mean value of the mean thickness/breadth ratio of the aggregate sample. Hence, λ is used as the flakiness ratio in the present study.

2.3.2. Elongation ratio

The elongation ratio is defined as the length to breadth ratio. It is obtained directly from the DIP results.

2.3.3. Sphericity

Sphericity is usually defined as the ratio of the surface area of a sphere having the same volume as the particle to the actual surface area of the particle [2]. However, since the surface area has to be evaluated by three-dimensional shape analysis, this ratio cannot be determined by the present or any other two-dimensional DIP methods. Therefore, an alternative definition of sphericity proposed by Krumbein [8] is used. It is given by Eq. (7):

$$\text{sphericity} = \sqrt[3]{\frac{\text{thickness} \times \text{breadth}}{\text{length}^2}}. \tag{7}$$

Substituting the value of thickness as given by Eq. (1) into the above equation, the sphericity can be expressed as Eq. (8):

$$\text{sphericity} = \sqrt[3]{\lambda\left(\frac{\text{breadth}}{\text{length}}\right)^2} \tag{8}$$

which can then be evaluated from the DIP results.

2.3.4. Shape factor

Shape factor is a commonly used index but different researchers adopt different definitions for it to describe different aspects of shape [2,4,5]. In the present study, the definition used follows that adopted by Kuo et al. [5]. It is defined as Eq. (9):

$$\text{shape factor} = \frac{\text{thickness}}{\sqrt{\text{breadth} \times \text{length}}}. \tag{9}$$

Replacing the thickness by $\lambda \times$ breadth, the above equation becomes Eq. (10):

$$\text{shape factor} = \lambda \times \sqrt{\frac{\text{breadth}}{\text{length}}}. \tag{10}$$

2.3.5. Convexity ratio

Convexity ratio is a measure of convexity [8]. Due to the difficulty of conducting three dimensional shape analysis, the convex ratio is evaluated from the two dimensional projection of the particle, as illustrated in Fig. 1. It is defined in Eq. (11) as the area to convex area ratio, i.e.,

$$\text{convexity ratio} = \frac{\text{area}}{\text{convex area}} \tag{11}$$

where the convex area is the area of the minimum convex boundary circumscribing the particle.

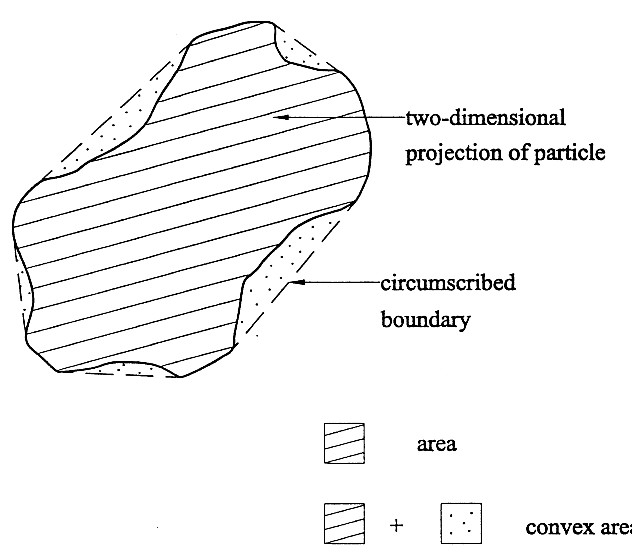

Fig. 1. Evaluation of area and convex area.

2.3.6. Fullness ratio

Fullness ratio is another measure of convexity that is also evaluated from the two dimensional projection of the particle. It is defined by Eq. (12) [5]:

$$\text{fullness ratio} = \sqrt{\frac{\text{area}}{\text{convex area}}} \tag{12}$$

3. Measurement of angularity

Angularity is a measure of the sharpness of the edges and corners of a particle. It is the opposite of roundness. The British Standard BS812: Part 1: 1975 classifies particles as angular if they possess well-defined edges formed at the intersection of roughly planar faces and rounded if they are fully water-worn or completely shaped by attrition. A commonly used scale for describing the degree of angularity or roundness is given below:

angular	little evidence of wear
sub-angular	some wear but faces untouched
sub-rounded	considerable wear, faces reduced in area
rounded	faces almost gone
well-rounded	no original faces left

As commented by BS812: Part 1, the angularity of an aggregate is a morphological property of importance because it affects the ease of handling of a mixture of aggregate and binder (e.g., the workability of concrete) and the stability of mixtures that rely on the interlocking of the particles. Furthermore, it would also affect the packing of the aggregate particles or the amount of voids in the aggregate. Roughly speaking, the least angular (most rounded) aggregates can achieve a packing density in terms of solid ratio of about 67% while relatively more angular (less rounded) aggregates would only achieve a lower packing density.

3.1. Traditional method of angularity measurement

There is, so far, no precise definition of angularity given in terms of the geometry of the particle boundary that would allow measurement of angularity directly from the shape of the aggregate particles. Nevertheless, based on the assumption that the packing density of a single-sized aggregate is dependent solely on the angularity of the aggregate, an indirect measure of angularity may be obtained from the packing densities of single-sized fractions of the aggregate sample. This methodology is followed by BS812: Part 1, which introduces the "angularity number" as a measure of angularity. The angularity number is defined as the amount by which the percentage of solid volume measured during a packing density test falls below 67 (or the amount by which the percentage of voids exceeds 33). It is determined by first separating the aggregate sample into different size fractions each falling within a very narrow size range (so that each size fraction is basically a single-sized aggregate), as listed in Table 1, and then, measuring the packing density of each size fraction. The packing density of a size fraction is measured by filling up a steel cylinder with the aggregate particles, subjecting the particles to prescribed tamping and weighing the amount of particles in the cylinder. Since single-sized spherical particles have a packing density of 67%, their angularity number is equal to zero. Practical aggregates have angularity numbers ranging from 0, for very rounded aggregates to 12, for very angular aggregates.

3.2. Problems with angularity measurement

Strictly speaking, angularity is a morphological property and should be measured directly from the shape of the aggregate particles. This has compelled researchers to develop methods of measuring angularity in terms of the shape characteristics of the particles. However, the different methods developed employ different definitions of angularity and yield different shape parameters which are not related to each other [9]. For instance, Yudhbir and Abedinzadeh [10] quantify the angularity of a particle as the number of tangents on the particle boundary (a measure of the number of protrusions) without taking into account the actual shape of the protrusions. Verspui et al. [11] measure the angularity of a particle as the mean angle of the convex corners (convex corners and protrusions are synonymous) on the particle boundary by taking the angle of a corner as the angle between the tangents at the begin and end points of the corner. Antoine and Courard [12] also measure the angularity of a bump (a bump is the same as a protrusion) as the angle of the bump but employ a geometric construction technique to measure the angle. Palasamudram and Bahadur [13], on the other hand, measure the angularity of a particle as a function of the sharpness of the corners (the sharpness of a corner is taken as inversely proportional to the angle of the corner) and the probabilities of the corners being contacted by other bodies.

The above differences show the difficulty of coming to a consensus on the definition of the angularity of a particle. As a result, every researcher has to define what angularity means before embarking on angularity measurement. From the various studies conducted so far, it appears that the angularity of a particle involves at least

Table 1
Sieve size ranges for measurement of angularity number

Upper limit (mm)		Lower limit (mm)
20.0	to	14.0
14.0	to	10.0
10.0	to	6.3
6.3	to	5.0

the following: (1) the angles of the corners, (2) the radii of curvature of the corner tips, (3) the height of the corners, and (4) the convexity/concavity of the corners which affects their probabilities of interacting with other bodies. Measurement of angularity is by no means simple. Up to now, there is no commonly accepted direct measure of angularity.

4. Aggregate samples analyzed

Aggregate samples obtained from five different sources have been analyzed by the DIP method for their flakiness ratios, elongation ratios, sphericity values, shape factors, convexity ratios and fullness ratios, and by the traditional method (in accordance with BS812: Part 1) for their angularity numbers. The five different sources of aggregates are: (1) crushed granitic rock from a quarry in Hong Kong; (2) crushed granitic rock from a quarry in Mainland China; (3) crushed volcanic rock from a quarry in Hong Kong; (4) crushed volcanic rock from a quarry in Mainland China; and (5) gravel obtained from Vancouver, Canada. The maximum particle sizes of the aggregate samples vary from 10 to 40 mm.

The gravel aggregates are very rounded while the crushed rock aggregates are quite angular. Thus, the aggregate samples analyzed cover both ends of the angularity–roundness scale. In order to produce intermediate angularity values, some of the crushed rock aggregates have been subjected to artificial attrition using the Los Angeles testing machine to reduce their angularity. The number of machine revolutions applied

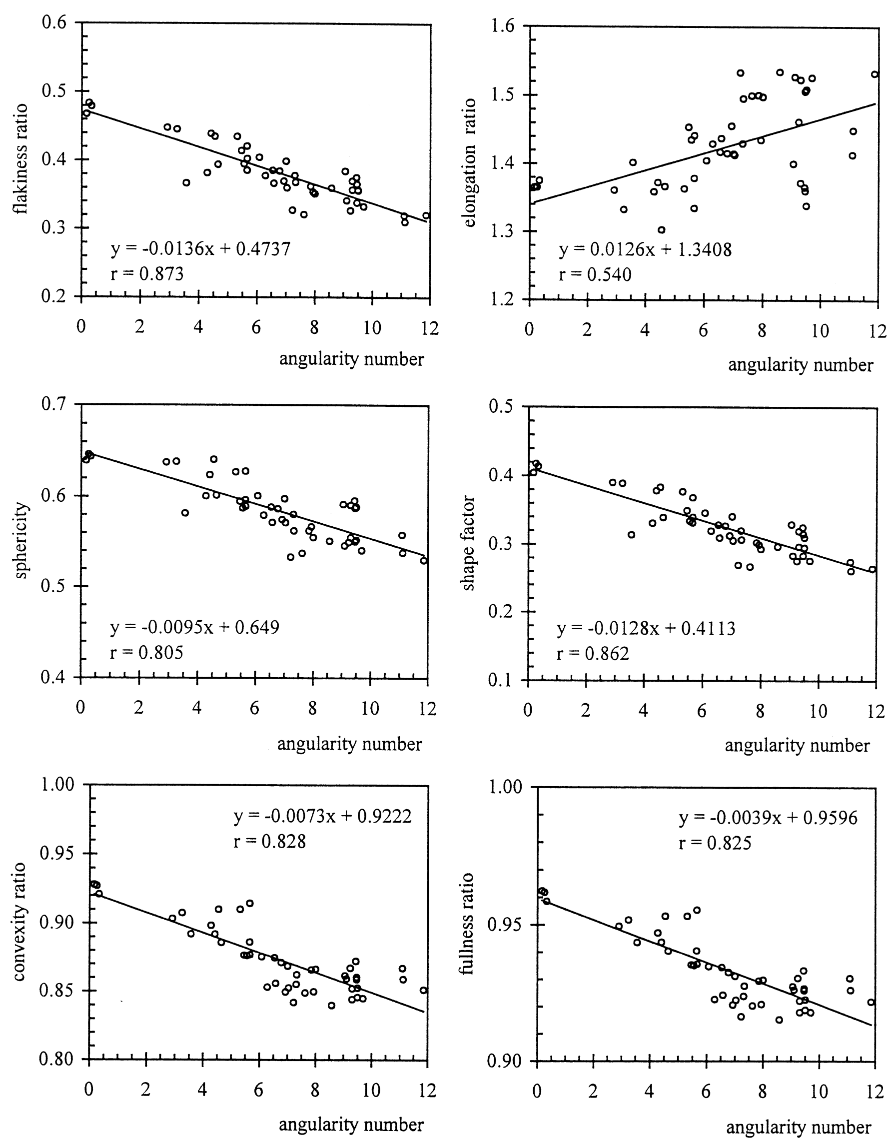

Fig. 2. Correlation of various shape parameters measured by DIP to angularity number measured by traditional method (r = correlation coefficient).

to the aggregate samples varied from 400 to 1600 so as to produce aggregates with different degrees of roundness. All together, a total of 46 aggregate samples have been analyzed.

5. Results and discussions

The weighted mean values of the various shape parameters measured by the DIP method are plotted against the angularity numbers of the aggregate samples determined by the traditional method in Fig. 2. To show the overall trend of the correlation, best-fitted straight lines are drawn on the graphs along with the data points plotted. The different degrees of scattering of the data points indicate that the various shape parameters measured by DIP have different correlation with the angularity number.

In the determination of the shape parameters of the aggregate samples, both the arithmetic mean and weighted mean values of the shape parameters of the individual particles have been calculated. The arithmetic and weighted mean values are not the same and they have slightly different correlation with the angularity number, as depicted in Table 2, where the correlation coefficients of the correlation between the various shape parameters and the angularity number are listed. These results reveal that, in general, the weighted mean values yield slightly better correlation results. Detailed analysis of the correlation results is presented below.

Among the two form factors (i.e., flakiness ratio and elongation ratio) measured, the flakiness ratio is found to have a strong correlation with the angularity number. However, the flakiness ratio is not a measure of angularity. Visual inspection of the aggregates reveals that the flaky particles are generally quite angular and the rounded particles are seldom flaky. Perhaps, the various factors affecting flakiness, such as the way that the particles are produced, the weathering and attrition processes that shaped the particles and the hardness of the rock, also affect angularity in a similar way. Because of the common factors affecting flakiness and angularity, there is a natural correlation between the flakiness ratio and the angularity number. Another possible reason for the strong correlation is that the flakiness ratio by itself might have significant effect on the packing density of the aggregate particles, which is reflected in the angularity number results. On the other hand, the correlation between the elongation ratio and the angularity number is much weaker. Visual inspection of the aggregates reveals that there is little correlation between elongation and angularity, as elongated particles can either be angular or rounded. The apparent correlation between the elongation ratio and the angularity number is probably due to the effect of elongation on packing density.

Table 2
Correlation of various shape parameters to angularity number

	Correlation coefficient of the correlation between the shape parameter and the angularity number	
Shape parameter measured by DIP	Arithmetic mean of shape parameter used	Weighted mean of shape parameter used
Flakiness ratio	–	0.873
Elongation ratio	0.535	0.540
Sphericity	0.797	0.805
Shape factor	0.855	0.862
Convexity ratio	0.818	0.828
Fullness ratio	0.809	0.825

No arithmetic mean of the flakiness ratio is available because the flakiness ratios of the individual particles have not been measured.

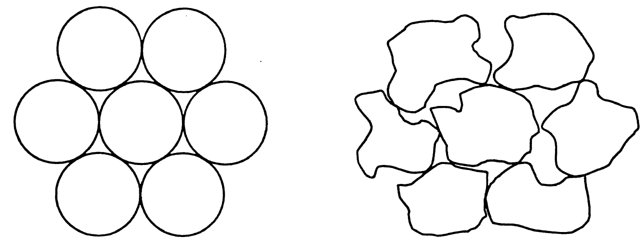

Fig. 3. Effect of convexity on packing.

Both the sphericity and the shape factor are functions of the form factors. The correlation coefficients of their correlation to the angularity number are 0.805 and 0.862, respectively, which are quite high. It is believed that their high correlation to the angularity number is due partly to the natural correlation between flakiness and angularity and partly to the effects of the form factors on packing density.

Unlike the form factors (flakiness ratio and elongation ratio) and the derivatives of the form factors (sphericity and shape factor), the convexity ratio and the fullness ratio are indexes of convexity. The convexity of a particle is actually a measure of the overall roundness of the particle. A rounded particle should not contain too many concave corners and a particle containing many concave corners is not rounded. Since the concave area is equal to convex area minus area, a high area to convex area ratio implies that the amount of concave area is relatively small and the particle is well rounded, whereas a low area to convex area ratio implies that the amount of concave area is relatively large and the particle has a low roundness. The convexity of the aggregate particles has a direct effect on the packing of the aggregate because the concave areas are generally more difficult to fill up, especially if the aggregate particles are of similar size, as shown in Fig. 3. Hence, the convexity ratio and the fullness ratio should have significant effects on the packing density. This is verified by the high correlation coefficients of 0.828 and 0.825 produced by the correlation between the two convexity indexes and the angularity number.

6. Identifying shape parameters as direct measures of angularity

According to Barret [14], the shape of a rock particle can be expressed in terms of form (overall shape), roundness (large-scale smoothness), and surface texture (small-scale smoothness). Form, roundness, and surface texture are geometrically independent properties of shape although there may be natural correlation between them because of the common physical factors affecting them.

Form is normally measured in terms of form factors which are defined as the ratios of the three major dimensions: length, breadth, and thickness. Form factors and the derivatives of form factors are measures of form and cannot be used for measuring roundness.

Roundness is a property independent of form. It is not a measure of sphericity (sphericity is a measure of form) although roundness is best displayed by a sphere. There are two aspects of roundness, namely, the roundness of the corners and the roundness of the outline of the particle. The roundness of the corners is the opposite of the sharpness of the corners and is more important when considering the abrasive and perforation properties of the particles. On the other hand, the roundness of the outline (also called the overall roundness) is generally measured in terms of convexity and is more important when considering the interlocking ability and packing density of the particles. Roundness and angularity are opposite to each other. Hence, a measure of roundness is also a measure of angularity. However, angularity is sometimes restricted to describe only the sharpness of the corners (opposite of the roundness of the corners) because there is no appropriate term for the opposite of the roundness of the outline. Thus, roundness often appears to be a more general concept than angularity. Perhaps, the angularity concept can be made more general by referring to the opposite of the roundness of the corners and the opposite of the roundness of the outline as the angularity of the corners and the angularity of the outline (or the overall angularity), respectively.

Surface texture is a third order property of shape (the first and second order properties are form and roundness). It is independent of form and roundness, and is really a measure of the roughness of the particle boundary. It may be measured in terms of the magnitude and sharpness of the protrusions and indentations on the particle boundary. However, this kind of measurement is not expected to be easy as both a microscope and a high-resolution camera would be required. Most existing DIP systems simply do not have the necessary resolution for such measurements.

Among the various shape parameters measured by DIP, the flakiness ratio, sphericity, shape factor, convexity ratio and fullness ratio are found to have good correlation with the traditional measure of angularity. Unfortunately, the flakiness ratio, sphericity, and shape factor are only measures of form and it would not be logical to take any of these as an alternative measure of angularity. On the other hand, the convexity ratio and fullness ratio, which are indexes of convexity, are more closely related to roundness and angularity. They may be used as measures of angularity. However, since the sharpness of the corners is not considered in their measurement, the convexity ratio and fullness ratio cannot be treated as complete measures of angularity. They are just measures of the angularity of the outline (or the overall angularity). When studying the abrasive and perforation properties of particles, direct measurement of the sharpness of the corners is still needed [11,12].

Finally, it should be pointed out that the so-called angularity number measured by the traditional method is rather misleading. It is actually a measure of packing density. Its validity as a measure of angularity is based on the assumption that the packing density of a single-sized aggregate is dependent solely on the angularity of the aggregate particles, whose truth is yet to be verified. However, the correlation results of the present study indicate that apart from angularity, there may be other shape attributes having significant effects on the packing density. Thus, the packing density is not a reliable measure of angularity. The whole concept of the angularity number should be abandoned. Nevertheless, since the packing density is by itself an important indicator of the performance of an aggregate, the packing density test should be retained but the results should be expressed in terms of packing density rather than angularity number.

7. Conclusions

The DIP method has been applied to measure the flakiness ratio, elongation ratio, sphericity, shape factor, convexity ratio and fullness ratio of aggregate particles. Although some of these shape parameters are dependent on the thickness of the particles, which is not directly obtainable by DIP, the method previously developed by the authors of supplementing the DIP results by the weight of the aggregate sample can be used to estimate the thickness of the particles to allow measurement of shape parameters dependent on thickness.

The shape parameter of an aggregate sample may be taken either as the arithmetic mean or weighted mean value of the shape parameters of the individual particles. While evaluation of arithmetic mean is straightforward, evaluation of weighted mean involves the volume of the particles, which again is not directly obtainable by DIP. Nevertheless, using the method previously developed by the authors for estimating the volume of the particles, the weighted mean value may be determined. It is found that the weighted mean values of the shape parameters generally correlate better to the packing density of the particles than the arithmetic mean values.

Correlation of the various shape parameters measured by DIP to the traditional measure of angularity reveals that the flakiness ratio, sphericity, shape factor, convexity ratio and fullness ratio have fairly high correlation with the traditional measure of angularity. Among them, only the convexity ratio and fullness ratio, which are closely related to roundness and angularity, may be taken as measures of angularity. However, as the sharpness of the corners is not considered in their measurement, the convexity ratio and fullness ratio cannot be treated as complete measures of angularity. For a complete picture of angularity, additional measurement on the sharpness of the corners is required.

Lastly, it is advocated that the present concept of angularity number should be abandoned. The angularity number as measured by the traditional method is, in reality, a measure of the packing density of the aggregate particles, which may be dependent also on other shape attributes apart from angularity.

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References

[1] M.A. Sid-Ahmed, Image Processing: Theory, Algorithms and Architectures, McGraw Hill, New York, (1995) 611 pp.

[2] R.D. Barksdale, M.A. Kemp, W.J. Sheffield, J.L. Hubbard, Measurement of aggregate shape, surface area, and roughness, Transportation Research Record 1301, National Research Council, Washington, DC, 1991, pp. 107–116.

[3] L. Li, P. Chan, D.G. Zollinger, R.L. Lytton, Quantitative analysis of aggregate shape based on fractals, ACI Mater J 90 (4) (1993) 357–365.

[4] Z.Q. Yue, I. Morin, Digital image processing for aggregate orientation in asphalt concrete mixtures, Can J Civil Eng 23 (1996) 480–489.

[5] C.Y. Kuo, J.D. Frost, J.S. Lai, L.B. Wang, Three-dimensional image analysis of aggregate particles from orthogonal projections, Transportation Research Record 1526, National Research Council, Washington, DC, 1996, pp. 98–103.

[6] C.F. Mora, A.K.H. Kwan, H.C. Chan, Particle size distribution analysis of coarse aggregate using digital image processing, Cem Concr Res 28 (6) (1998) 921–932.

[7] A.K.H. Kwan, C.F. Mora, H.C. Chan, Particle shape analysis of coarse aggregate using digital image processing, Cement and Concrete Research, Cem Concr Res 29 (9) (1999) 1403–1410.

[8] W.C. Krumbein, Measurement of geological significance of shape and roundness of sedimentary particles, Sediment Petrol 11 (1991) 64–72.

[9] M.E. Fayed, L. Otten (Eds.), Handbook of Powder Science and Technology, 2nd edn. Chapman & Hall, New York, (1997) 898 pp.

[10] Yudhbir, R. Abedinzadeh, Quantification of particle shape and angularity using the image analyzer, ASTM Geotech Test J 14 (3) (1991) 296–308.

[11] M.A. Verspui, P. van der Velden, G. de With, P.J. Slikkerveer, Angularity determination of abrasive powders, Wear 199 (1) (1996) 122–126.

[12] R. Antoine, L. Courard, Perforation strength of geosynthetics and sphericity of coarse grains: a new approach, Geotext Geomembr 14 (1996) 585–600.

[13] S.L. Palasamudram, S. Bahadur, Particle characterization for angularity and the effects of particle size and angularity on erosion in a fluidized bed environment, WEAR 203–204 (1) (1997) 455–463.

[14] P.J. Barret, The shape of rock particles, a critical review, Sedimentology 27 (1) (1980) 15–22.