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Effect of fracture path on the fracture energy of high-strength concrete

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Abstract

The paper indicates that there may not exist a direct relationship between fracture energy and compressive strength. The fractal theory is used to quantitatively study the fracture surface and the relationship between fracture energy and fracture characteristics. The test results show that fractal dimension increases with the increase of maximum aggregate size. For the concrete with higher water–binder ratio, fractal dimension increases more rapidly than that for the concrete with lower water–binder ratio. For the same water–binder ratio, fracture energy of concrete increases with the increase of fractal dimension. Fracture energy for lower water–binder ratio increases with fractal dimension more rapidly than that for higher water–binder ratio. The ductility index is used to assess the brittleness of concrete. There exists a good lineal correlation between ductility index and fractal dimension for all test series. Thus, fractal dimension can be used as a parameter to characterize the brittleness of concrete.

Keywords: Fracture energy; Compressive strength; Fractal dimension

1. Introduction

Since fracture energy was proposed as a parameter of fictitious crack model [1], many mechanical parameters that could be simply measured have been investigated to evaluate the variation of fracture energy. The compressive strength is the most widely used parameter, and its relationships with fracture energy have been established [2–4]. Due to the convenient measurement, the compressive strength was used as a variable to determine the fracture energy of concrete in Mode Code 90 by CEB-FIP [5]. Similar to other mechanical parameters, however, fracture energy is influenced by many factors. Recent results showed that, fracture energy for high-performance concrete (HPC), especially with active additives, may not increase with the increase of compressive strength [6–8]. Normally, most aggregates are stronger than the cement-based matrix, so a crack runs around the inclusions in normal concrete. In contrast to the energy consumed for crack formation in normal concrete, a crack in high-strength concrete runs through the inclusions and forms an approximate plane as observed on fine mortars and pure hardened cement paste. In this case, the mechanical interaction between inclusions and matrix cannot be activated. This leads to a more brittle behavior for high-strength concrete. Thus, some study indicated that the fracture energy is greatly influenced by fracture process path [7,8]. However, these conclusions were only based on the photos of fracture surface of concrete, and there is still lack of quantitative measurement on fracture surface. On the other hand, as concrete is a quasibrittle material, the irregularity of fracture surface could be described using fractal geometry [9,10].

This research attempts to investigate the intrinsic factors that influence the fracture energy through studying the characteristics of fracture surface. For this purpose, the experimental instrument has been designed here and used to measure fractal dimension of fracture surface of concrete. It allows to establish the relationship between fracture energy and fracture process path for further understanding of fracture process and providing the useful information for designing ductile concrete using postmortem method. Meanwhile, the brittleness of HSC is assessed using a ductility index so that a relationship between the ductility index and fractal dimension can be proposed.

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2. Experimental procedure

2.1. Concrete mixes

2.1.1. Binder

Ordinary Portland cement corresponding to ASTM Type I was used in all the mixes. Ultrafine slag powder produced by grinding ground granulated blast furnace slag together with activator was also used as binder.

2.1.2. Aggregates

Natural river sand with a fineness modulus of 2.85 was used as the fine aggregate. Crushed gravel with maximum sizes of 5, 10, 16 and 20 mm, respectively, was used as coarse aggregates, with a crush index of 8.3%.

2.1.3. Superplasticizer

Sulphonated naphthalene formaldehyde superplasticizer was used.

Only two main factors that influence the fracture energy of concrete were considered, i.e. strength and maximum aggregate size. As shown in Table 1, the mixes include two different water–binder ratios and four different maximum aggregate sizes, classified as HPC-*X*-*Y* (*X* and *Y* refer to water–binder ratio and maximum aggregate size, respectively). For each mix, six cubes of $100 \times 100 \times 100$ mm were used for measuring compressive strength f_c and splitting tensile strength f_{st}. Three beams of $515 \times 100 \times 100$ mm were used for measuring the fracture energy. The specimens were demoulded 1 day after casting and cured in the curing room at 20 ± 2 °C and r.h. of 90% for further 27 days.

2.2. Measurement of mechanical properties

The compressive strength and splitting tensile strength were tested in conventional testing machine. The three-point bending tests were conducted on notched beam specimens in an INSTRON 8501 servo hydraulic machine under closed-loop position control. A displacement rate of 0.025 mm/min was applied and other experimental procedures were based on the recommendation of RILEM.

2.3. Determination of fracture surface

The fracture surface of notched specimens after three-point bending tests could be measured using a new technique based on laser triangulation [11]. The previous results showed that fracture surface of concrete was irregular and fractal character existed [12,13]. Based on the classical concept of covering dimension originally used by Mandelbrot et al. [14], the box-counting method was used to calculate the fractal dimension of lacunar and invasive fractal sets. The fracture surface was assumed to be an invasive self-affine fractal in a statistical sense. This means that a three-dimensional representation of the surfaces, $f(x,y,z)$, would be statistically similar to $f(rx,ry,r^H z)$, where r was the scaling factor and H was the Hurst exponent index due to the anisotropy in the scaling procedure. The fractal dimension could be evaluated from the increase rate of number N of prisms necessary to cover the entire surface, as the size d of elementary prisms with the volume of $V = d \times d \times d^H$ decreased. The following equation holds (Eq. (1)):

$$\Delta_{\mathrm{box}} = \lim_{d\to 0} \frac{\log \mathrm{N}}{\log(1/\mathrm{d})} \tag{1}$$

Practically, the fractal dimension could be evaluated from the slope of the linear regression line (log N vs. log $1/d$). While the box-counting algorithm estimated the fractal dimension from the vanishing rate of the overall covering volume with the increase of the resolution, the patchwork method could approximately estimate the fractal domain. The fractal dimension was evaluated from the rate of increase of the apparent area A as the size of the surface elements decreased. In other words, the patchwork method aimed at evaluating the same limit value. The patchwork dimension could be computed as (Eq. (2)):

$$\Delta_{\mathrm{patch}} = 2 - \lim_{r\to 0} \frac{\log A(r)}{\log r} \tag{2}$$

where A is the total area of microunits that have been shown and calculated by Lange et al. [15]. A typical relationship between log A and log r is shown in Fig. 1. It can be seen that a good linear correlation exists between log A and log r, which indicates that fractal character can be used to assess

Table 1
Concrete mixes

Series	Cement (kg/m^3)	Slag (kg/m^3)	Gravel (kg/m^3)	Sand (kg/m^3)	Water–binder ratio	Max. aggregate size (mm)	Water–binder ratio
HPC-44-5	267	115	1110	740	0.44	5	0.44
HPC-44-10	267	115	1110	740	0.44	10	0.44
HPC-44-16	267	115	1110	740	0.44	16	0.44
HPC-44-20	267	115	1110	740	0.44	20	0.44
HPC-26-5	472	202	948	632	0.26	5	0.26
HPC-26-10	472	202	948	632	0.26	10	0.26
HPC-26-16	472	202	948	632	0.26	16	0.26
HPC-26-20	472	202	948	632	0.26	20	0.26

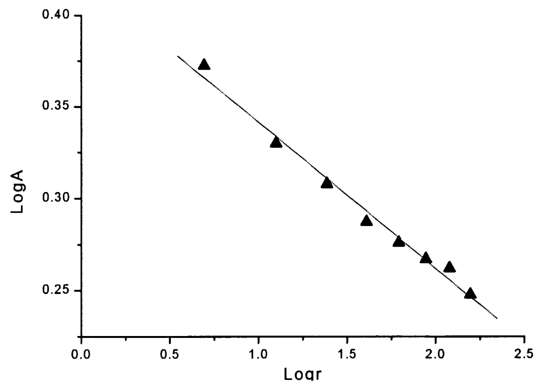

Fig. 1. Logarithmic area log A versus logarithmic scaling factor log r.

the fracture surface of concrete. The fractal dimension is obtained by adding the absolute slope value of the regression line (log A vs. log r) to 2.

3. Experimental results and discussion

Table 2 presents mechanical properties and fractal dimension for each mix. It can be seen that for the same maximum aggregate size, the compressive strength for HPC-44-16 is lower than that for HPC-26-16, but the fracture energy for HPC-44-16 is higher. According to the equation of Mode Code 90 [5], the fracture energy G_F can be written as a function of f_c:

$$G_F = \alpha_d f_c^{0.7} \quad (3)$$

where α_d is a coefficient according to the maximum aggregate size. If Eq. (3) is used to predict fracture energy, then G_F for HPC-44-16 will be lower than that for HPC-26-16. The prediction is controversial to the experimental results. Furthermore, Ref. [6] indicated that, for different-type aggregates, compressive strength could not reflect fracture energy. Refs. [7,8] presented the strength and fracture energy with or without silica fume for the maximum aggregate sizes of 10 and 20 mm (see Table 3). It is shown that fracture energy of higher strength concrete with silica fume is lower than that of lower strength concrete without silica fume. This can further confirm the results in this study, i.e. a higher concrete strength does not necessarily correspond to a higher fracture energy.

Table 3
Mechanical properties in Refs. [7,8]

Series	NC10	SC10	NC20	SC20
Compressive strength (MPa)	72.7	87.5	72	84.5
Splitting tensile strength (MPa)	4.58	5.42	3.45	4.03
Fracture energy (N/m)	106	87	142	87

From the experimental results in this study and the listed references, it can be seen that compressive strength may not be a good parameter that relates fracture energy of high-strength concrete. With the development of concrete techniques, more additives have been used to obtain low water–binder ratio for high strength and durability. As shown in Fig. 2, with the decrease of water–binder ratio and the contents of active additives, the improved bond strength between hardened cement paste and aggregate led to the transition of fracture mode from the passing-around the coarse aggregate to the transgranular type. In Fig. 2, mode (a) represents that a crack passes through the aggregate directly, while mode (b) shows that a crack runs around the aggregate.

From Fig. 3, obviously, the fracture surface of HPC-26-20 series was smoother than that of HPC-44-20. This phenomenon can be attributed to the fact that the possibility of fracture path for Mode (a) became larger duo to the lower water–binder ratio. Fig. 3 also shows that fractal dimension of HPC-44-20 was larger than that of HPC-44-5 for the same water–binder ratio. This indicated that the increase of maximum aggregate size also resulted in the increase of fractal dimension. From the viewpoint of fracture, the higher the fractal dimension, the more energy would be consumed. Therefore, it is possible to use fractal dimension to reflect the fracture energy of concrete. The strength of concrete is an important factor that influences the fracture energy and was mainly determined by water–binder ratio.

The influence of maximum aggregate size on the fractal dimension was investigated for the same water–binder ratio in this study. Fig. 4 shows an appreciable increase in the fractal dimension caused by the increase of maximum aggregate size. The increase of fractal dimension for HPC-44 series was greater than that for HPC-26.

Concrete is considered as a quasibrittle material, so fracture energy in Hillerborg's fictitious crack model can properly characterize the intrinsic properties of concrete. It is

Table 2
Mechanical properties and fractal dimension of fracture surface

Series	Compressive f_c (MPa)	Splitting tensile strength f_{st} (MPa)	Fracture energy G_F (N/m)	Ductility index (mm)	Fractal dimension
HPC-44-5	31.9	2.68	71.6	0.247	2.051
HPC-44-10	60.3	5.11	160.0	0.315	2.087
HPC-44-16	68.1	5.33	200.2	0.348	2.121
HPC-44-20	59.1	4.23	212.0	0.429	2.126
HPC-26-5	59.1	4.33	111.9	0.223	2.048
HPC-26-10	81.7	6.89	172.2	0.266	2.074
HPC-26-16	84.6	7.47	193.9	0.280	2.079
HPC-26-20	75.7	6.32	205.3	0.281	2.082

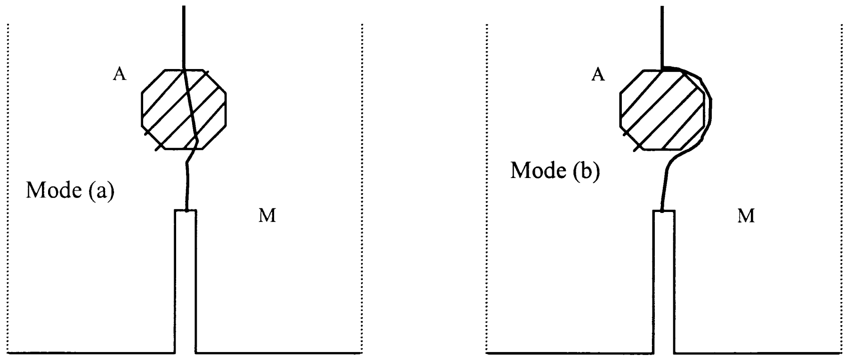

Fig. 2. Two possible fracture paths.

noted in the previous study that strength also plays an important role in influencing fracture energy. If water–binder ratio is constant, strength of matrix can be considered can be uniquely determined. The values of fracture energy for the series studied herein are listed in Table 2. For the same water–binder ratio, the relationship between fracture energy G_F and the increment of fractal dimension $(D-2)$ is approximately linear and can be expressed as follow (Fig. 5):

$$G_F = -12.48 + 1802(D-2) \quad \text{for W/B} = 0.44$$

$$G_F = -21.00 + 2730(D-2) \quad \text{for W/B} = 0.26 \qquad (4)$$

with the linear regression coefficient of r=.986 for both cases.

For each water–binder ratio, fracture energy has a corresponding linear relationship with increment of fractal dimension. Each slope corresponds to the respective water–binder ratio. These relationships are more appropriate than those between fracture energy and strength of concrete. If the water–binder ratio of concrete is available, the fracture energy of the concrete can be predicted through the above relationship. Compared with concrete without silica fume, concrete with silica fume has higher strength and lower fracture energy. If the empirical Eq. (4) is used to predict fracture energy, lower fracture energy will be obtained corresponding to the smaller increment of fractal dimension duo to the higher rupture possibility of coarse aggregate. Thus, the increment of fractal dimension is an intrinsic

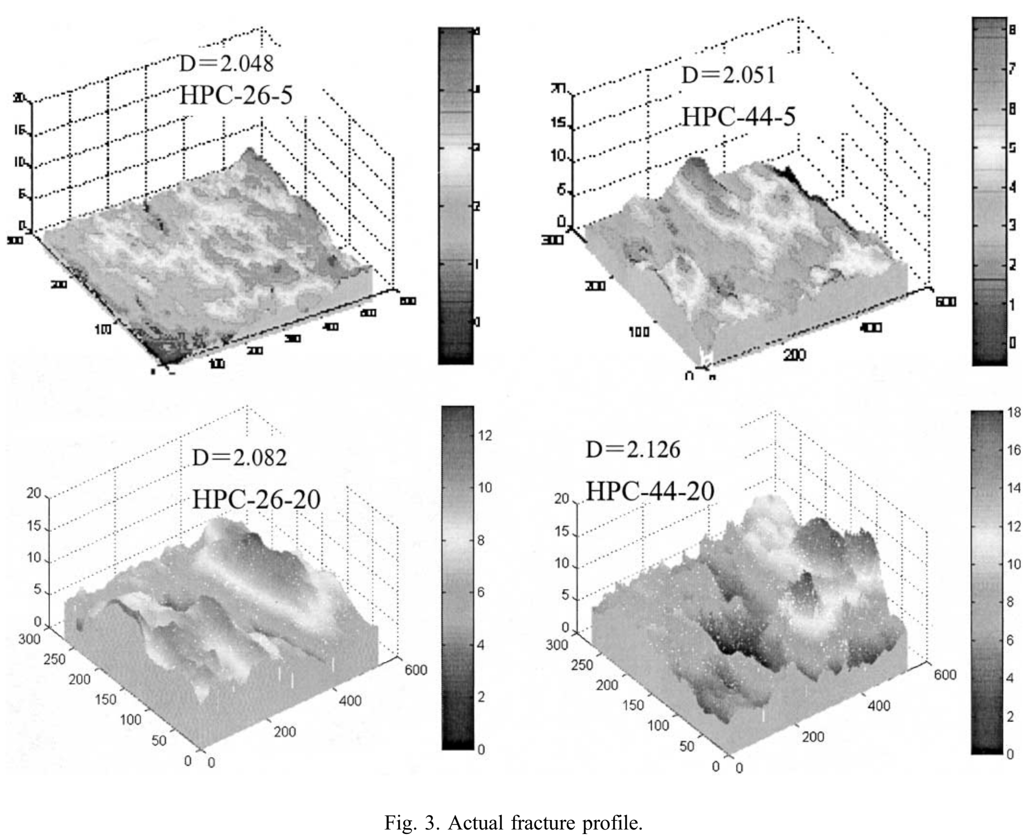

Fig. 3. Actual fracture profile.

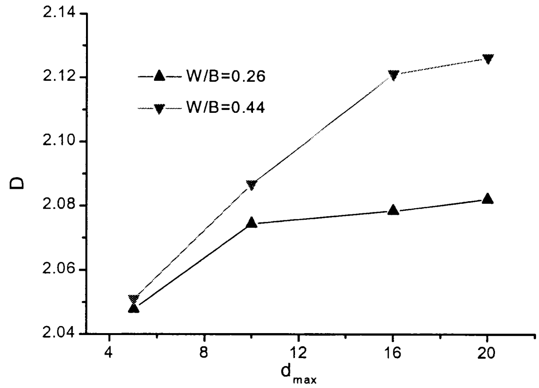

Fig. 4. Fractal dimension D versus maximum aggregate size d_{max}.

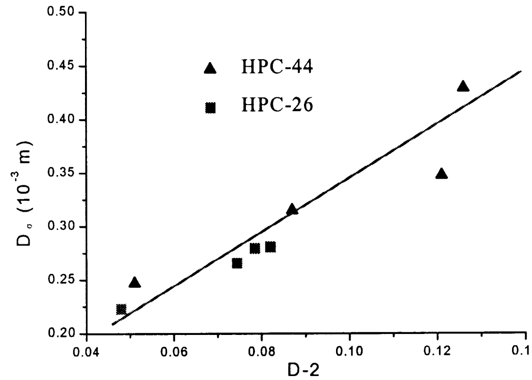

Fig. 6. Ductility index D_σ versus the increment of fractal dimension $(D-2)$.

factor for determining fracture energy and a vital important parameter for analyzing the microstructure of concrete. For example, the previous paper by the authors [16] showed that metallic aggregate, which is stronger and more resistant against wearing and impacts, could increase the increment of fractal dimension of fracture surface because a higher fractal dimension corresponded to a higher fracture energy.

The ductility index D_u was defined as G_F/P_u to characterize the brittleness of concrete materials in Ref. [17], where P_u is the ultimate load of complete load–deflection curve. The smaller the value of the ductility index, the more brittle concrete materials are. Since P_u varied with the size and shape of specimen, the nominal stress defined by Bažant [18] in his size effect model is used to represent the strength of concrete. In this study, the ratio of fracture energy to nominal stress is defined as ductility index as follows (Eq. (5)):

$$D_\sigma = \frac{G_F}{\sigma_n} \tag{5}$$

where σ_n is the nominal stress and $\sigma_n = c_n P_u/bd$, where c_n is an arbitrary constant (here $c_n = 1$), d is the depth of specimen, and b is the breath of specimen. In this study, all specimens had the same sizes, so b and d are constants. Fig. 6 shows that the ductility index D_σ linearly increased with the increment of fractal dimension on fracture surface. The previous paper indicates that fracture energy alone could not distinguish ductility from brittleness [19]. As shown in Fig. 6, a linear relationship of the ductility index D_σ with the increment of fractal dimension $(D-2)$ can be established as:

$$D_\sigma = 0.118\times10^{-3} + 2.16\times10^{-3}(D-2) \tag{6}$$

with r=.941. Within the range in the study, if fractal dimension is known, the ductility index of concrete can be predicted using Eq. (6), and the degree of brittleness can also be estimated from the increment of fractal dimension on fracture surface.

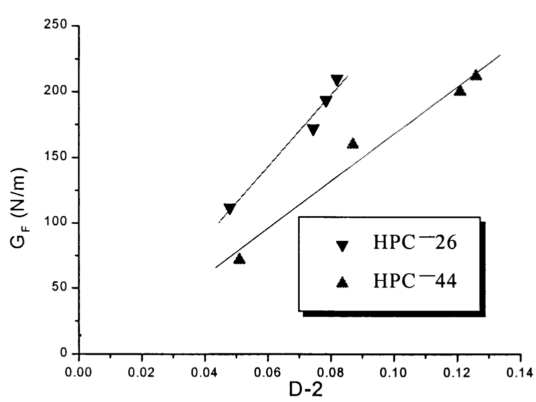

Fig. 5. Fracture energy G_F versus increment of fractal dimension $(D-2)$.

4. Conclusions

Based on the findings in this study, the following conclusions can be drawn.

(1) For high-strength concrete, the variation of strength cannot comprehensively reflect that of fracture energy as lower water–binder ratio improves the bond strength between matrix and coarse aggregate and leads to a smoother fracture surface, which consumes less energy. Fractal dimension of fracture surface is used as a parameter to study fracture energy, and there exists fractal character on the fracture surface.

(2) For the same strength of matrix, fractal dimension increases with the increase of maximum aggregate size. The fractal dimension of HPC-44 series is larger than that of HPC-26 series for the same maximum aggregate size.

(3) For the same strength of matrix, fracture energy linearly increases with increasing fractal dimension. For the same increment of fractal dimension, a higher matrix strength (HPC-26) leads to a higher fracture energy.

(4) The ductility index was defined to characterize the brittleness of concrete. The ductility index D_σ linearly increases with the increase of fractal dimension.

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