



# Tensile basic creep of early-age concrete under constant load

Lennart Østergaard<sup>a,\*</sup>, David A. Lange<sup>b</sup>, Salah A. Altoubat<sup>b</sup>, Henrik Stang<sup>a</sup>

<sup>a</sup>*Department of Structural Engineering and Materials, Building 118, Technical University of Denmark, Brovej, DK-2800 Kgs. Lyngby, Denmark*

<sup>b</sup>*Department of Civil Engineering, University of Illinois at Urbana-Champaign, 2122 Newmark CE Lab,  
MC-250, 205 North Mathews, Urbana, IL 61801-2352, USA*

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## Abstract

Viscoelastic behavior of early-age normal and high-strength concrete has been investigated. The study shows that concrete exhibits high tensile creep strain if loaded at an age less than or equal to 1 day. The investigations furthermore show that the creep strain is not proportional to the stress level in the specimen when loading occurs at 1 day. Creep experiments were also carried out on concretes with different w/c ratios and some qualitative comments are made. Finally, an approach for mathematical modeling of early-age creep for normal concrete was explored. © 2001 Elsevier Science Inc. All rights reserved.

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## 1. Introduction and objective

The early-age basic tensile creep response of concrete has been investigated only scarcely in the literature. The majority of past work on creep of concrete has been concerned with the compression creep behavior of mature concrete. However, detailed knowledge about the tensile creep behavior at early ages is important for estimating the possibility of cracking due to shrinkage and thermal stresses.

Former work includes the results obtained by Gutch and Rostásy [1]. Their tensile creep tests showed pronounced viscoelasticity when the load was applied at early age. Moreover, they concluded that the initial stress/strength ratio does not exert a very significant influence on creep. Umehara and Uehara [2] demonstrated the influence of temperature on early-age tensile creep. Bissonnette and Pigeon [3] identified the w/c ratio and the age at loading as significant parameters for tensile creep at early age. Their experiments showed a higher specific creep when the w/c ratio increases. Westman [4] determined the early-age creep in compression and his results indicated a strong age dependency when the sample was loaded within the first 24 h after set. Altoubat [5] and Altoubat and Lange [6] found that the initial rate of basic creep is very sensitive to the age at loading in the first 2 days.

It was also concluded that the basic creep model based on solidification theory satisfactorily describes the tensile creep behavior at early ages. Hauggaard et al. [11–13] found that temperature significantly influences the creep properties at early age. It was also found that compressive and tensile creep are comparative. Specific creep response was found to be linear below a stress/strength ratio of 60%.

In this paper, the early-age basic creep behavior of ordinary Portland cement concrete is investigated. The experiments deal with the influence of age at loading, initial stress/strength ratio and w/c ratio. It will be investigated whether existing creep models can be applied to determine creep in early age. Presumably, such models underestimate the actual creep in early age. It will also be investigated whether the creep response is linear for low stress/strength ratios in the first days after mixing. If these hypotheses are correct, the stresses calculated in early age by existing models will be too high. Thus, a very high risk of cracking will be predicted.

## 2. Experiments

### 2.1. Materials

Experiments were performed with three different concrete mixes. Table 1 shows the compositions. ASTM Type 1

\* Corresponding author. Tel.: +45-45-25-1861; fax: +45-45-88-3282.  
E-mail address: los@byg.dtu.dk (L. Østergaard).

Table 1  
Composition of concrete mixes

Concrete	A	B	C
w/c ratio	0.50	0.40	0.39
w/b ratio <sup>a</sup>	0.50	0.40	0.32
Cement content C (kg/m <sup>3</sup> )	421	480	533
Superplasticizer (ml/m <sup>3</sup> )	0	562	949
C/CA/FA/SF	1/2.20/1.76/0	1/1.93/1.55/0	1/1.83/1.17/0.22

CA: coarse aggregates,  $d_{\max} = 25.4$  mm; FA: fine aggregates; SF: silica fume.

<sup>a</sup> Efficiency factor for the silica fume has been set to 1.0.

cement was used throughout the experiments. The mix designs and the materials are identical to those used by Altoubat [5], and this article is a continuation of his work.

## 2.2. Equipment

The tensile creep tests were performed in an experimental setup developed by Altoubat [5] based on a setup by Kovler [7]. The setup operates with two dog bone-shaped concrete specimens, one of which is unloaded while the other is loaded. Two linear voltage displacement transducers (LVDT) are recording the axial deformation of both specimens. The creep strain is obtained as the difference between the deformations of the specimens. The gauge length is 626.25 mm and the side length of the quadratic cross-section is 76.20 mm.

The setup was developed to investigate the early-age creep behavior of fully restrained concrete subject to drying. In this paper, the creep response due to a constant load is investigated. The mix designs are identical to those used by Altoubat [5] on the drying creep behavior of fully restrained concrete.

## 2.3. Procedure

The mixing procedure started with mixing of the dry materials for 2 min, gradual addition of water during the next 1 min, addition of the plasticizer if any and continued mixing for another 2 min.

The concrete specimens were cast immediately after mixing and sealed for 12–14 h. The initial sealing consisted of four layers of plastic foil snugly fitted around the moulds. After this initial sealing period, a wet cover was placed onto the specimens to obtain a wet curing condition. The cover

was made of four layers of burlap plus four layers of cotton cloth. The cover was rewetted every 4–8 h. Altoubat [5] showed that this type of covering is sufficient to suppress deformation from drying shrinkage.

Creep tests were performed at ages  $t' = 0.67, 1, 3$  and 5 days with Mix A. All specimens were loaded to the same initial stress/strength ratio of approximately 0.45, as shown in Table 2. The experiments also included one specimen loaded at  $t' = 1$  day, but at a lower stress/strength ratio equal to 0.25. The development of tensile strength of the mixes has been determined by Altoubat [5]. Those experiments were performed with the same setup in which the creep experiments of this work were done. Since the mix designs and the materials also were identical, Altoubat's determination of the tensile strength development has been adopted.

All specimens were cured and tested in an environmentally controlled chamber at constant temperature (23 °C) and at a constant humidity (50% RH). Therefore, maturity of the specimens is proportional to age.

The reproducibility of the test method has been established previously by Altoubat [5], who conducted several replicate tests with the test setup that was also used in this paper. Altoubat did not obtain inconsistent results and it was concluded that the test setup gives reliable and reproducible results.

## 3. Results

Fig. 1 shows the development of creep strain as a function of age at loading for tests where the applied load was 45% of tensile strength at the respective ages. The tests were run long enough to establish early-age trends. None of the tests caused failure of the concrete. The test at age of 3 days was truncated prematurely due to a lab power failure, but even the shortened test was found useful.

The results reveal a high rate of creep within the first day after casting. This corresponds to the findings by Westman [4] concerning compressive creep. This response diminished as the concrete aged. For example, the 5-day specimen exhibited a far stiffer behavior than the 1-day specimen.

The influence of the initial stress/strength ratio on the creep compliance is shown in Fig. 2. Specific creep com-

Table 2  
Experimental matrix

Test no.	Mix design	Age ( $t'$ ) at loading (days)	Tensile strength at time of loading (MPa) [5]	Applied tensile stress (MPa)	Initial stress/strength ratio
1	A	0.67	0.77	0.34	0.45
2	A	1	1.36	0.60	0.45
3	A	3	2.11	0.95	0.45
4	A	5	2.47	1.12	0.45
5	A	1	1.36	0.34	0.25
6	B	1	2.00	0.60	0.30
7	C	1	3.16	0.60	0.19

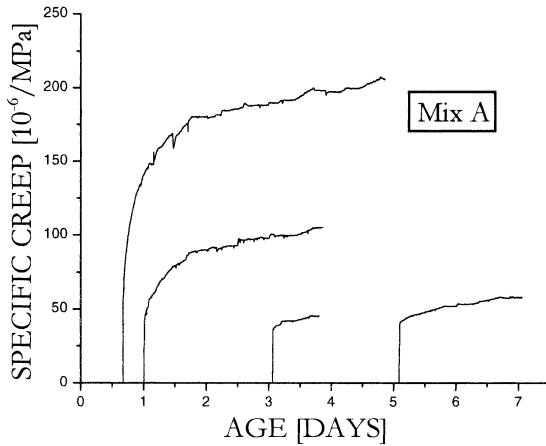


Fig. 1. Results from tension creep tests of young concrete (Tests 1–4).

pliance was measured for specimens with the same age at loading but subject to different load levels and for specimens with different ages at loading but subject to the same load level.

The results on Fig. 2 reveal some interesting features about early-age tensile creep. First, the strong aging of the material in the first 24 h results in a dramatic change of the creep behavior. Therefore, when the same load (2 kN) is applied at different ages (0.67 and 1 day), the specific creep strain is strongly evolving toward stiffer response. Furthermore, the rate of creep is significantly different when these creep curves, of Tests 1 and 5, are compared.

It is also seen that the creep response at 1 day, for Test 2s and 5, are not proportional to each other. Higher specific creep is observed when the specimen is loaded to 45% instead of 25% of the tensile strength. The rate of creep for Test 2 in the range from age 2 days to end of experiment was approximately  $8 \times 10^{-6}/(\text{day MPa})$ , while Test 5 exhibited a creep rate of  $2 \times 10^{-6}/(\text{day MPa})$  in the same period.

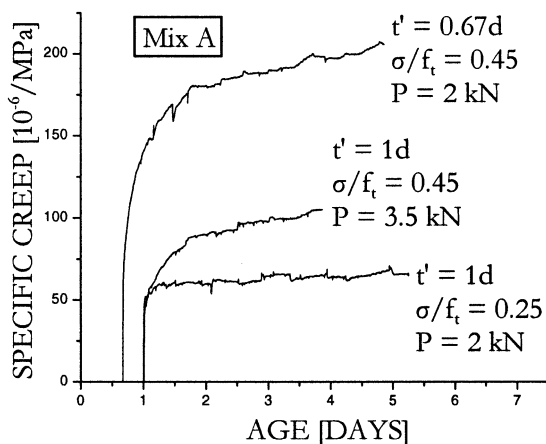


Fig. 2. Results from tension creep tests (Tests 1, 2 and 5).

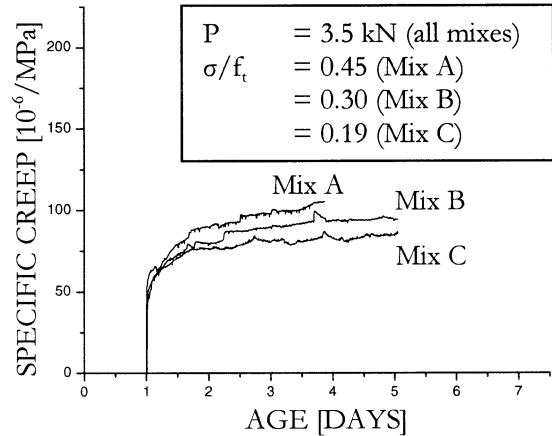


Fig. 3. Results from tension creep tests with different w/c ratios (Tests 2, 6 and 7). Note that the stress/strength ratio for Mixes 2 and 3 only represents approximations obtained from the tensile strength development under drying condition, estimated by Altoubat [5].

Fig. 3 shows the specific creep curves for Mixes A–C (Tests 2, 6 and 7) loaded at 1 day to the same load,  $P = 3.5$  kN.

The results show the known dependency of magnitude of creep with w/c ratio. However, due to the difference in the strength development and thereby the different stress/strength ratios, the mechanisms underlying the results are difficult to isolate. This is due to the nonlinear relation between stress/strength ratio and creep at early age. Thus, the difference in behavior cannot be ascribed only to the different w/c ratios. The contribution from this effect might partly explain the difference in creep rate.

#### 4. Analysis

The solidification theory by Bazant [8] and Bazant and Prasannan [9,10] provides a good basis for the description of concrete creep behavior for concrete with  $w/c \geq 0.40$  [14]. The theory is based on the idea that the aging aspect is due to growth of the volume fraction of load-bearing portion of solidified matter (i.e., hydrated cement), the properties of which are age independent. The model also satisfies the condition of nondivergence of the creep curves for different ages at loading. Mathematically, the solidification theory found the basis for Bazant's structural creep law:

$$\frac{\varepsilon(t)}{\sigma} = q_1 + q_2 Q(t, t') + q_3 \ln[1 + (t - t')^{0.1}] + q_4 \ln \frac{t}{t'} \quad (1)$$

in which  $q_1$ ,  $q_2$ ,  $q_3$  and  $q_4$  are empirical constants.  $q_1$  represents the instantaneous compliance. The term with the quotient  $q_2$  is the aging viscoelastic compliance,  $q_3$  the nonaging viscoelastic compliance and  $q_4$  the viscous compliance.  $\varepsilon(t)$  and  $\sigma$  represent the strain and stress, respectively. Time is denoted by  $t$  ( $t=0$  when water is added

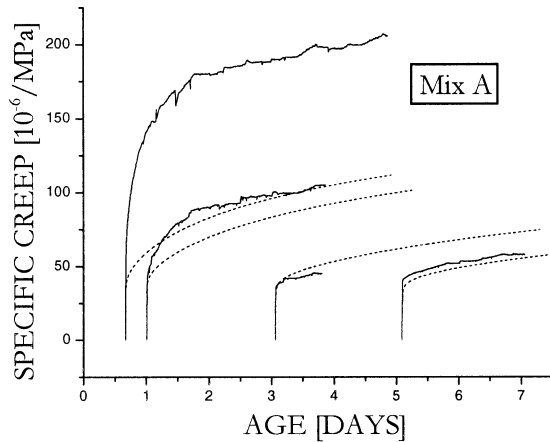


Fig. 4. Experimental results (solid lines) compared with creep model (lines).

to the dry materials), while  $t'$  denotes the time at loading. The function  $Q(t, t')$  is given by [8]:

$$Q(t, t') = Q_f(t') \left[ 1 + \left( \frac{Q_f(t')}{Z(t, t')} \right)^r \right]^{-1/r} \quad (2)$$

where  $f_u$  is the initial tensile strength of the material and where  $Q_f(t')$ ,  $Z(t, t')$  and  $r$  are given by [8]:

$$Q_f(t') = [0.086(t')^{2/9} + 1.21(t')^{4/9}]^{-1}$$

$$Z(t, t') = (t')^{-1/2} \ln[1 + (t - t')^{0.1}]$$

$$r = 1.7(t')^{0.12} + 8 \quad (3)$$

where time  $t$  and  $t'$  must be entered in days. The model as presented by Bazant does not fit the results of the present study, which is understandable since the model was developed for concrete at later ages. The discrepancies are seen in Fig. 4, which shows the results of Tests 1–4 with a set of predicted responses by the unmodified model (Eqs. (1)–(3)). The correlation is poor at early ages and better at later ages.

Fig. 4 shows the difficulty of fitting the high specific creep in early ages (0.67–1 day) to the creep model (Eqs. (1)–(3)). Results from fitting the single curves to the creep model reveal a strong age dependency of the material parameter  $q_2$ . All other parameters can be kept relatively constant. Since parameter  $q_2$  is associated with the aging viscoelastic compliance of the creep model, it seems that the very early-age highly viscoelastic behavior is not seized by the creep model. However, the observed behavior may be incorporated into the model by redefining the parameter  $q_2$  as follows:

$$q_2 = q_5 \frac{t'}{t' - q_5} \quad (4)$$

where  $q_5$  is a new material parameter,  $q_5 > 0$ , introduced as a fitting parameter to capture very high early-age creep. It should be observed that this relation leads to a prediction of

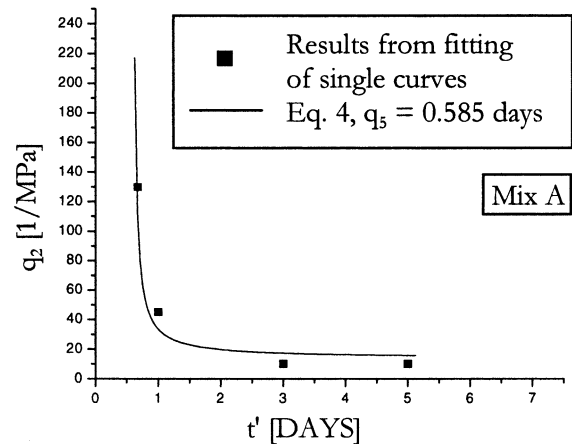


Fig. 5. Fitting of Eq. (4) against parameter  $q_2$  (values obtained using original creep law fitted against single creep curves).

infinite creep strain if age at loading,  $t'$ , equals the fitting parameter,  $q_5$ . However, this singularity at  $t' = q_5$  is reasonable if  $q_5$  is interpreted as a transition time from liquid viscoelastic to solid viscoelastic behavior. That is, the  $q_5$  parameter may be regarded as a structural setting time, the time at which true creep behavior commences. The magnitude of  $q_5$  will always be less than the earliest (valid) physical test. In our study, the earliest physical test occurred at 16 h (0.67 day), and the best fit value of  $q_5$  for the full data set was 14 h. A more precise determination of  $q_5$  could be obtained by conducting more tests at earlier ages.

Eq. (4) is also simpler than the additional functions introduced by Westman [4] in the fitting of very early compression creep results to the triple power law. The correlation of Eq. (4) to the original parameter  $q_2$  obtained by fitting the single curves to the model is shown on Fig. 5.

Using Eq. (4) in the creep model (Eqs. (1)–(3)) results in a model, which fits the experimental results with a significantly better correlation than the original one (see Fig. 6). Comparing Figs. 4 and 6 furthermore shows that the effect

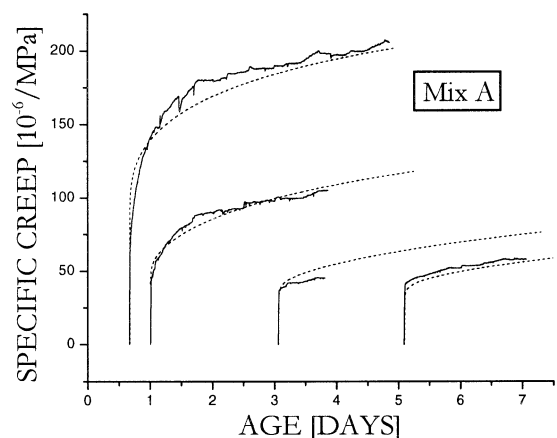


Fig. 6. Results from creep tests (solid lines) compared with modified creep model (dashed lines; Eqs. (1)–(4)).

Table 3  
Fit parameters

Parameter	$q_1$	$q_2$	$q_3$	$q_4$	$q_5$
Value ( $1 \times 10^{-6}$ /MPa)	6.7	16.6	47.1	28.3	0.59

of the proposed correction vanishes after less than 3 days. Based on present results, the creep model may therefore be used without the modification if concrete older than approximately 3 days is considered.

The set of parameters,  $q_1$ – $q_5$ , which gives the best fit of the experimental data, is tabulated in Table 3.

The same procedure has been applied in the interpretation of the nonlinear specific creep vs. initial stress/strength response. It is noted that the parameter  $q_2$  may now be kept constant, while the quotient associated with the flow term,  $q_4$ , change. Perhaps, a modification here could lead to a model that seizes the different magnitudes of the flow at very early ages as a function of stress/strength ratio. However, more experiments must be performed to get a quantitative idea about this trend. Finally, it is noted that also the creep of the concrete mixes B and C can be fitted with the model. This can be achieved by setting the parameter  $q_4 \approx 16$  for Mix C and  $q_4 \approx 21$  for Mix B, respectively. However, since the fit has been carried out at only one age, this does not represent a complete verification of the model for these mixes.

## 5. Conclusions

The study shows that a high creep response is observed when the concrete is loaded within the first 24 h after adding water to the dry materials. Bazant's structural creep model based on solidification theory [8] does not capture this behavior. After an initial aging period of 3 days, the response is far stiffer than the earlier ages. A simple modification to the solidification creep model is proposed. The proposed  $q_5$  parameter may be regarded as a structural setting time.

The study also indicate that the creep response is not proportional to the applied stress when the concrete specimen is loaded at  $t' = 1$  day.

Also, these experiments indicate that the rate of creep after a short initial time after loading is constant regardless of age at loading given constant initial stress/strength ratio.

Finally, it should be noted that this article only represents limited amounts of experimental evidence, and the conclusions should be regarded as preliminary. More experiments are needed for confirmation of the results.

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