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Generalization of Abrams' law for cement mortars

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Abstract

The influence of constituent materials and various mix proportions on compressive and splitting tensile strength of mortar (ranged from high strength to low strength) has been studied experimentally and reported. In this study, cement/sand ratios of 1:2, 1:2.5, and 1:3 are used. At each cement/sand ratio, the water/cement ratio varied from 0.30 to 0.65 at a constant increment of 0.05. For all the mortar mixes, the compressive strength of mortar was determined at ages of 3, 7, and 28 days. From the experimental test results, it has been observed that the important parameter affecting the strength of mortar was the water/cement ratio and Abrams' generalization law is applicable to mortar both in compression and splitting tension. Generalizing Abrams' law for mortars with different mix proportions has been proposed for mortars falling in the range between very low strength to very high strength. It has been observed that the Abrams' generalized law is applicable to mortars with water/cement ratio greater than 0.40. Empirical model expressions have been proposed to predict the compressive strength of mortar using water/cement ratio.

Keywords: Abrams' law; Compressive strength; Mortar; Splitting tensile strength; Water/cement ratio

1. Introduction

Investigations on various physical properties of concrete can be traced back to the work of Feret who first recognized the importance of water/cement ratio on the strength of concrete. This was about two decades before pronouncement of well-known Abrams' law on the effect of water/cement ratio on concrete strength [5–7]. According to Abrams' generalization law, the compressive strength of concrete varies inversely with the water/cement ratio for concrete. However, from time to time, the water/cement ratio law has been criticized as not being a fundamental law. Information on the influence of water/cement ratio on the strength of mortar is very limited. From an extensive experimental study by Curie and Sinha [3], it has been observed that the important factor affecting the compressive strength of mortar appeared to be the water/cement ratio. It has also been revealed from the investigations that the relationship between compressive strength and water/cement ratio was unaffected by the use of different sands and sand gradings. It has been observed that the Bolomey expression relating strength of concrete to water/cement ratio can be applicable in normal hardened cement pastes with water/cement ratio >0.15 [10]. It has been noticed that Bolomey parameters depend on the degree of hydration. Quite significant information has been reported on high-strength concrete and its behavior under various loading and environmental conditions [1]. Portland cement paste is the most active component between aggregates in mortar and concrete [2]. Generally, it has been observed that the physical properties of cement-based materials are primarily affected by the water/cement ratio, the chemical composition, micro-structure and pore geometry of the cementitious materials, properties of aggregates, cement/aggregate ratio, and properties of cement/aggregate interfacial zone [1,4,8]. In this study, an effort has been made to understand the Abrams' law for cement mortars made with ordinary Portland cement conforming to IS: 8112-1989.

2. Abrams' water/cement ratio law

Abrams' water/cement ratio law, pronounced during 1918, has been described as the most useful and significant advancement in the history of cementitious materials technology, in general, and in the concrete technology, in particular. Abrams' performed and published extensive experimental research

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Table 1
Physical properties of cement and recommendations for 43 grade cement

Property	Result	IS: 8112-1989 recommendations
Fineness, % (residue on 90 μm sieve)	2.70	< 10
Standard consistency, %	27.50	–
Setting times		
(a) Initial (min)	120	≥ 30
(b) Final (min)	210	≤ 600
Specific gravity	3.10	–
Compressive strength, MPa at:		
(a) 3 days	24.00	≥ 23.00
(b) 7 days	34.50	≥ 33.00
(c) 28 days	45.00	≥ 43.00

work on the relationship between concrete compressive strength and water/cement ratio. The most important observation was the inverse proportionality between water/cement ratio and the strength of concrete. Among all other factors influencing the strength of concrete, it was strongly believed that concrete strength was dependent on only one factor, the water/cement ratio, as a first approximation. The mathematical relationship between concrete strength and water/cement ratio, according to Abrams', is [Eq. (1)]:

$$\sigma_C = A/B^{W/C} \tag{1}$$

Where σ_C = compressive strength of concrete, W/C = water/cement ratio, A and B are constants for a given material, age and test conditions.

For high-performance concretes (HPC), supplementary cementitious materials or pozzolanic materials such as fly ash, ground granulated blast furnace slag, silica fume, etc., are generally incorporated as partial replacement of cement. It has been observed that Abrams' water/cement ratio law is not directly applicable in such concretes [7]. Alternative augmented water-cementitious materials ratio law can be proposed for designing concrete mixes with the incorporation of cement replacement materials. In such cases, the Abrams' law can be proposed using the concept of "equivalent cement" as [Eq. (2)]:

$$\sigma_C = A/B^{W/C_{eq}} \tag{2}$$

Where C_{eq} = equivalent cementitious material.

Bolomey's formula relates strength to water/cement ratio [9]. Bolomey's formula is often used to predict the strength of concrete materials. It is of the form given by [Eq. (3)]:

$$\sigma_C = k_1(1/(W/C) - k_2) \tag{3}$$

The first k parameter depends on concrete composition and degree of hydration, while the second k parameter is a constant.

3. Experimental program

3.1. Materials

3.1.1. Cement

Ordinary Portland cement namely 43 grade cement conforming to IS: 8112-1989 and similar to ASTM Type III (C150-95) was used for the present experimental investigation. Various physical properties of the cement are shown in Table 1.

3.1.2. Fine aggregate

Natural river sand with sand fraction passing through 4.75-mm sieve and retained on 600-μm sieve was used for this program. Care has been taken to avoid the presence of inorganic and silt particles in the adopted sand fraction. The fineness modulus of sand was 2.81 with a specific gravity of 2.65. The bulk density of the sand was 1584 kg/m^3.

3.1.3. Water

Potable tap water available in the laboratory with pH value of 7.85 ± 0.1 was used for mixing mortar and curing the mortar specimens as well.

3.2. Proportioning of mortar mixes

In order to study the influence of water/cement ratio on strength of mortar, mortar mixes were designed with water/cement ratio varying from 0.30 to 0.65 at a constant increment of 0.05. To study the effect of mix proportioning, three proportions of cement/sand ratios, i.e., 1:2, 1:2.5, and 1:3, were adopted. Tables 2, 3, and 4 show the mix proportions in HSM-I, HSM-II, and HSM-III mortar mixes, respectively.

Table 2
Mix proportions of HSM-I mortar mixes

S. no.	Mix designation	Mix proportions Cement/water/sand
1	HSM-I-1	1:0.30:2.0
2	HSM-I-2	1:0.35:2.0
3	HSM-I-3	1:0.40:2.0
4	HSM-I-4	1:0.45:2.0
5	HSM-I-5	1:0.50:2.0
6	HSM-I-6	1:0.55:2.0
7	HSM-I-7	1:0.60:2.0
8	HSM-I-8	1:0.65:2.0

Table 3
Mix proportions of HSM-II mortar mixes

S. no.	Mix designation	Mix proportions Cement/water/sand
1	HSM-II-1	1:0.30:2.5
2	HSM-II-2	1:0.35:2.5
3	HSM-II-3	1:0.40:2.5
4	HSM-II-4	1:0.45:2.5
5	HSM-II-5	1:0.50:2.5
6	HSM-II-6	1:0.55:2.5
7	HSM-II-7	1:0.60:2.5
8	HSM-II-8	1:0.65:2.5

Table 4
Mix proportions of HSM-III mortar mixes

S. no.	Mix designation	Mix proportions Cement/water/sand
1	HSM-III-1	1:0.30:3.0
2	HSM-III-2	1:0.35:3.0
3	HSM-III-3	1:0.40:3.0
4	HSM-III-4	1:0.45:3.0
5	HSM-III-5	1:0.50:3.0
6	HSM-III-6	1:0.55:3.0
7	HSM-III-7	1:0.60:3.0
8	HSM-III-8	1:0.65:3.0

3.3. Preparation of test specimens

3.3.1. Compressive strength

Standard metallic cube moulds (100-mm) were used for preparation of the mortar specimens for compressive strength. A table vibrator was used for compaction of the mortar filled cubes. The specimens were demoulded after 24 h and subsequently immersed in water till the time of testing. Three cube specimens were used for the determination of average compressive strength.

3.3.2. Splitting tensile strength

Standard metallic cylindrical moulds measuring 100 × 200 mm were used for the preparation of the test specimens for split tensile strength. The compacted mortar test specimens were demoulded after 24 h and immersed in water till the time of testing. The procedure outlined in ASTM C496-90 has been followed to determine the split tensile strength. Average of three specimens was used to determine the splitting tensile strength of every mortar mix.

3.4. Testing of specimens

The cube specimens were tested for compressive strength at the age of 3, 7, and 28 days — using 200-tonnes capacity compression-testing machine. Three specimens were used to obtain the average strength of every mortar mix at each water/cement ratio. The cylindrical specimens were tested using a splitting frame under 40-tonne capacity UTM under load control. The split tensile strength was calculated as $\sigma_t = 2P/\pi LD$.

4. Test results and discussion

4.1. Variation of compressive strength

HSM-I, HSM-II, and HSM-III are the three mortar mix groups with cement/sand ratios of 1: 2, 1:2.5, and 1:3, respectively. From the test results, it has been observed that HSM-I mortar mix group performed better in compression than those in the other two groups, i.e., HSM-II and HSM-III. Tables 5–7 show the comparison of experimental and predicted values of compressive strength at different ages in HSH-I, HSM-II and HSM-III respectively. It has been generally observed that the compressive strength at the ages of 3, 7, and 28 days increases as the water/cement ratio decreases. However, at a water/cement ratio of 0.30 in series HSM-I and HSM-II, and at 0.30 and 0.35 in series HSM-III mortar mixes, the trend of strength development has been deviated from the generally observed trend. At W/C = 0.30/0.35, the mortar was supplied with insufficient amount of water required for hydration of cement particles and the aggregate surface was completely wet as the specific surface area is generally high. As a result of the above fact, the workability of mortar appeared to be very difficult at low water/cement ratio, which led to incomplete compaction. As a result of this, the strength of mortar did not follow the general trend, as it should have with the decrease in the water/cement ratio. In other words, Abrams' generalized water/cement ratio law is valid for high-strength mortars when the water/cement ratio is greater than 0.40.

The optimum gradation of fine aggregate for high-strength concrete is determined more by its effect on water requirement than on physical packing. Sand with a fineness modulus below 2.5 results in a sticky consistency, while sand with fineness modulus above 3.0 produces mortar with good workability. The water requirement of mortar of standard consistency increases with the fineness modulus of sand [3]. Increase in cement content beyond a certain point may not always increase compressive strength. Curie and Sinha [3] reported that the water requirement, and hence, the strength of mortar, is influenced by the shape and texture of the aggregate and these factors are expressed indirectly by the packing of the

Table 5
Results from the experimental and predicted equation and comparison in HSM-I mortar mixes

Mix designation	Compressive strength, MPa					Splitting tensile strength, MPa		
	$\sigma_{C3,\ ext}$	$\sigma_{C7,\ ext}$	$\sigma_{C28,\ ext}$	$\sigma_{C28,\ eq}$	% Difference	$\sigma_{t28,\ ext}$	$\sigma_{t28,\ eq}$	% Difference
HSM-I-1	22.67	25.83	40.00	–	–	–	–	–
HSM-I-2	26.00	34.83	46.37	46.97	– 1.29	2.66	2.70	– 1.50
HSM-I-3	25.83	34.00	43.67	43.97	– 0.069	2.55	2.52	1.18
HSM-I-4	20.67	30.00	42.33	41.49	1.98	2.35	2.37	– 0.85
HSM-I-5	20.50	28.33	40.00	39.39	1.52	2.27	2.25	0.88
HSM-I-6	16.00	20.83	38.16	37.58	1.52	2.20	2.14	2.73
HSM-I-7	12.33	17.16	35.50	36.00	– 1.41	2.07	2.05	0.97
HSM-I-8	11.00	16.00	34.16	34.61	– 1.32	1.91	1.97	– 3.14

Table 6
Results from the experimental and predicted equation and comparison in HSM-II mortar mixes

Mix designation	Compressive strength, MPa					Splitting tensile strength, MPa		
	$\sigma_{C3,\ ext}$	$\sigma_{C7,\ ext}$	$\sigma_{C28,\ ext}$	$\sigma_{C28,\ eq}$	% Difference	$\sigma_{t28,\ ext}$	$\sigma_{t28,\ eq}$	% Difference
HSM-II-1	20.00	24.67	32.00	–	–	2.35	–	–
HSM-II-2	22.67	30.33	46.00	49.56	– 7.74	2.51	2.59	– 3.19
HSM-II-3	22.00	28.20	44.50	43.59	2.05	2.42	2.427	– 0.29
HSM-II-4	20.83	27.67	39.83	38.92	2.28	2.32	2.294	1.12
HSM-II-5	20.50	27.00	38.67	35.61	7.91	2.30	2.181	5.17
HSM-II-6	12.00	18.33	32.33	32.08	0.77	2.13	2.083	2.21
HSM-II-7	11.83	17.50	30.00	29.51	1.63	1.97	2.00	– 1.52
HSM-II-8	9.16	12.67	25.00	25.00	0.00	1.85	1.92	– 3.78

aggregate particles, i.e., the percentage of voids in a loose condition. It has been further observed that the specific surface measured by nitrogen adsorption seems to have no relation with the water content of mortar and it has not influenced the strength of mortar. This may be due to the formation of a dense layer of cement paste on the aggregate surface, although this is not usually thought to prevent the aggregate from becoming saturated.

Figs. 1–3 show the variation of compressive strength at 3, 7, and 28 days, respectively, with water/cement ratio in the three series of mortar mixes, i.e., HSM-I, HSM-II, and HSM-III. It has been observed that the test results are exactly obeying Abrams' law on the variation of compressive strength with water/cement ratio. Owing to its pore filling effect of cement, the HSM-I mortar mixes exhibit decrease in the porosity in the interfacial zone. As a result of the above mechanism, the mortar mixes with higher cement quantities exhibit higher strength and the fractured surfaces seemed to be smooth with very significant transgranular type of failure. The variations of compressive strength at the age of 28 days for the three mortar mix groups are shown in Fig. 3. From the variations, the generalized Abrams' law relating compressive strength with water/cement ratio has been perfectly observed. The mortar mix groups, HSM-I and HSM-II, exhibited better performance than the other one, i.e., HSM-III. Generalized correlations have been derived to predict the 28 days compressive strength of mortar as a function of water/cement ratio for all the three mortar mix groups. It has been reported [3] that the important factor affecting the compressive strength of mortar is the water/cement ratio. The type of sand and its grading has no effect on the strength of mortar. Therefore, it is quite judicious to state that the water/cement ratio solely controls the strength of mortar. For the HSM I mortar mixes, an empirical expression, Eq. (4), has been derived to predict the compressive strength of mortar at 28 days with water/cement ratio. The regression coefficient is .99.

$$\sigma_{C28} = 27.98(W/C)^{-0.4934} \tag{4}$$

For the HSM-II mortar mixes, Eq. (5) shows the relation between strength and water/cement ratio with a regression coefficient of .96.

$$\sigma_{C28} = 18.049(W/C)^{-0.9622} \tag{5}$$

In case of the HSM III mortar mixes, the empirical equation (Eq. (6)) with a regression cefficient of .901 is given by:

$$\sigma_{C28} = 17.228(W/C)^{-0.8568} \tag{6}$$

From the above relationship, between compressive strength of mortar and water/cement ratio, it would be possible to estimate the design strength of mortar required for any practical purposes. The percentage difference between the experimental results and empirical equations is very less. Almost the values from experiments and expressions are exactly matching. Hence, it is suggested for predicting the strength in both compression and splitting

Table 7
Results from the experimental and predicted equation and comparison in HSM-III mortar mixes

Mix designation	Compressive strength, MPa					Splitting tensile strength, MPa		
	$\sigma_{C3,\ ext}$	$\sigma_{C7,\ ext}$	$\sigma_{C28,\ ext}$	$\sigma_{C28,\ eq}$	% Difference	$\sigma_{t28,\ ext}$	$\sigma_{t28,\ eq}$	% Difference
HSM-III-1	8.50	9.10	13.50	–	–	–	–	–
HSM-III-2	9.50	15.00	18.00	–	–	–	–	–
HSM-III-3	17.15	21.33	35.00	37.77	7.33	2.26	2.365	– 4.65
HSM-III-4	17.65	23.67	37.50	34.15	8.93	2.32	2.20	5.17
HSM-III-5	14.25	21.33	30.33	31.20	– 2.87	2.10	2.063	1.76
HSM-III-6	11.67	18.00	29.67	28.75	3.10	1.94	1.946	– 0.31
HSM-III-7	11.00	17.25	28.33	26.69	5.79	1.81	1.84	– 1.66
HSM-III-8	8.67	12.50	23.00	24.92	– 8.35	1.75	1.756	– 0.34

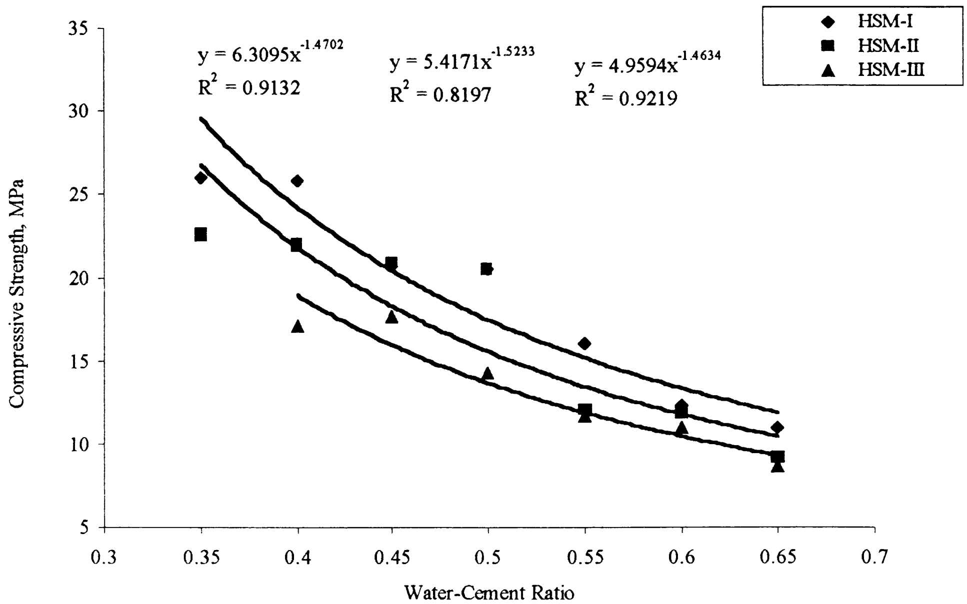

Fig. 1. Variation of compressive strength at 3 days with water/cement ratio in mortars.

tension for practical design purposes. Figs. 1 and 2 show the variation of compressive strength at 3 and 7 days with water/cement ratios for different types of mortar mixes.

Bolomey's formula relates strength to water/cement ratio. The scope of the present work is to evaluate these relations with a more general expression for the mortar mixes. Fig. 4 shows the relation between compressive strength at 28 days of mortar with 1/(W/C) for high-strength mortars. It has been clearly observed that the relation is linear. The compressive strength of mortar increases with the cement/water ratio. The variations of compressive strength with cement/water ratio are given in Eqs. (7)–(9) for series HSM-I, HSM-II, and HSM-III mortar mixes, respectively. From these expressions, it can be seen that variation of compressive strength with cement/water ratio seems to be well coincided with the Bolomey type of expression.

$$\sigma_{C28} = 9.1586(1/(W/C) + 2.28) \quad (7)$$

$$\sigma_{C28} = 15.678(1/(W/C) + 0.25) \quad (8)$$

$$\sigma_{C28} = 12.66(1/(W/C) + 0.46) \quad (9)$$

The regression coefficients for Eqs. (7), (8), and (9) are .98, .96, and .89, respectively. The *k* parameters for HSM-I

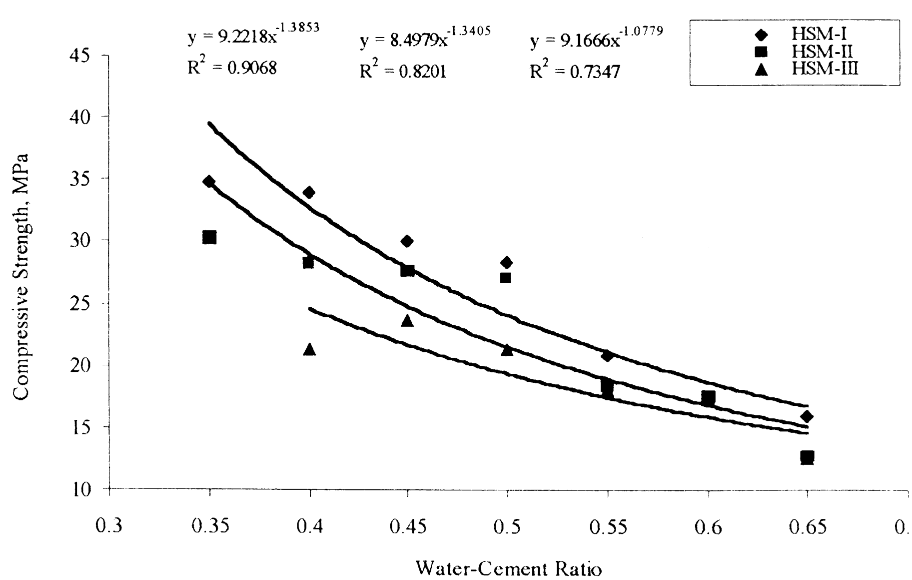

Fig. 2. Variation of compressive strength at 7 days with water/cement ratio in mortars.

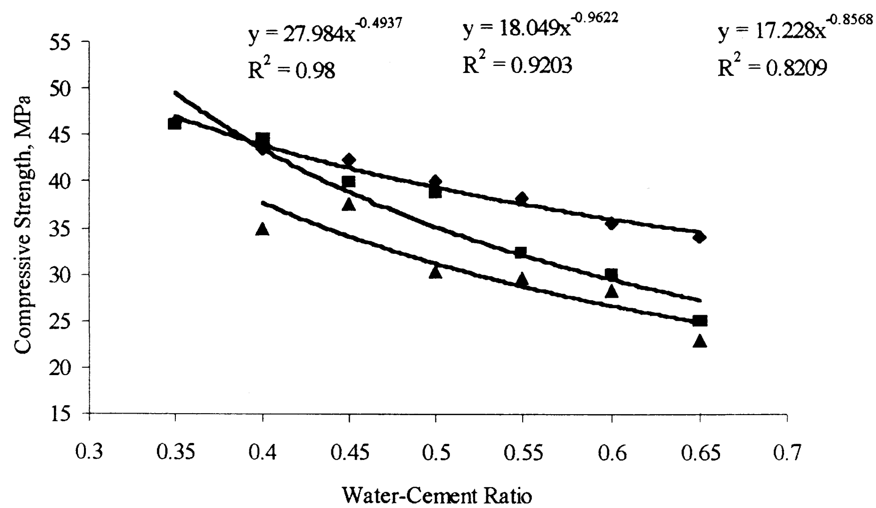

Fig. 3. Variation of compressive strength of high-strength cement mortar at 28 days with water/cement ratio.

mortar mixes are $k_1 = 9.16$ and $k_2 = 2.28$; for HSM-II mortar mixes are $k_1 = 15.70$ and $k_2 = 0.25$; and for HSM-III mortar mixes are $k_1 = 12.70$ and $k_2 = 0.46$. From the experimental data, a general expression has been derived to predict the compressive strength of mortar as a function of cement/water ratio. It is shown in Eq. (10) with a regression coefficient of .827.

$$\sigma_{C28} = 13.43(1/(W/C) + 0.63) \quad (10)$$

The k parameters for this expression are $k_1 = 13.43$ and $k_2 = 0.63$.

4.2. Generalized Abrams' law for mortars

Using the experimental data, a more generalized variation and expression has been presented for easy verification of Abrams' generalized law for mortar mixes ranging from very-low-strength to very-high-strength mortars. In the report by Curie and Sinha [3], results are provided for very-low-strength mortars with different sands with different gradings. Using the data in Ref. [3] and from the present study, a general variation of compressive strength with water/cement ratio has been proposed, for the designing of mortar mixes ranging from lean mortar mixes to very strong mortar mixes as a function of the single largest factor affecting the strength of mortar; the water/cement ratio. Fig. 5 shows the general variation of strength with water/cement ratio. The mathematical relationship [Eq. (11)] is in the general form, with regression coefficient of .977.

$$\sigma_{C28} = 11.30(W/C)^{-1.713} \quad (11)$$

Using the same data, the relation between the compressive strength of mortar and cement/water ratio has been derived. The form of the expression is similar to Bolomey's formula for design of concrete materials. Eq. (12) shows the mathematical relationship between compressive strength and cement/water ratio for mortar mixes, in general, with a regression coefficient (R) of .978. Fig. 6 shows the general

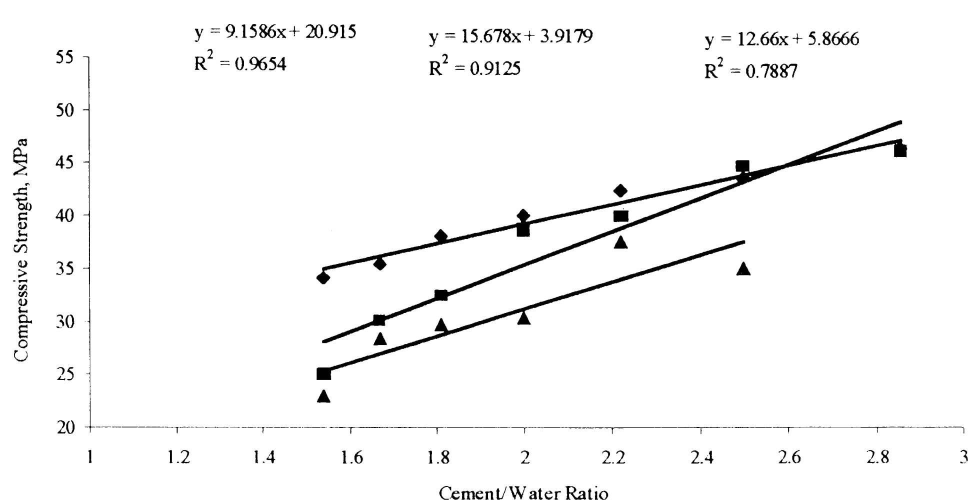

Fig. 4. Variation of compressive strength of high-strength mortars with cement/water ratio.

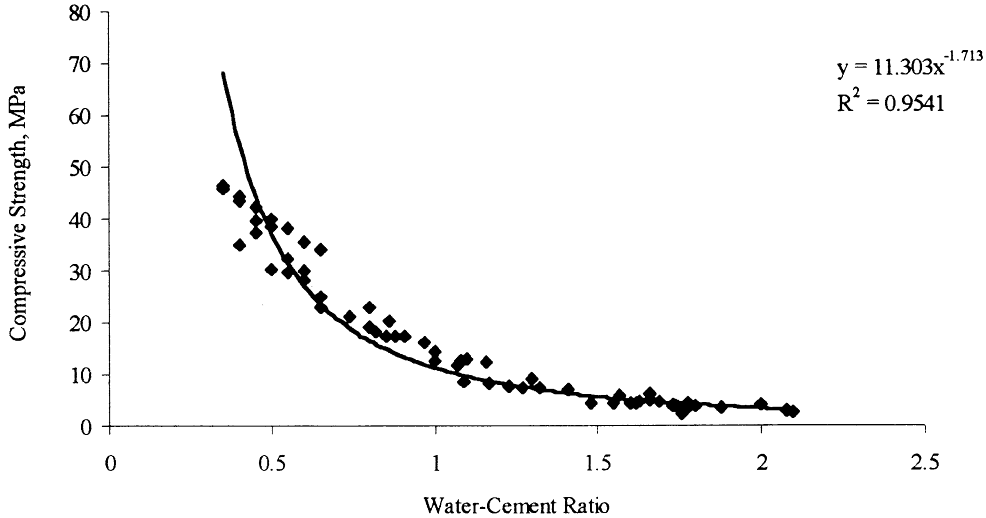

Fig. 5. Variation of compressive strength with water/cement ratio in cement mortars.

variation of compressive strength of mortar with cement/water ratio. The expression for compressive strength with cement/water ratio is given by:

$$\sigma_{C28} = 20.80(1/(W/C) - 0.35) \qquad (12)$$

4.3. Variation of splitting tensile strength

Fig. 7 shows the variation of 28 days splitting tensile strength of mortar mixes as a function of water/cement ratio. The splitting tensile strength decreases as the water/cement ratio increases. From the fractured surfaces, the type of failure was clearly observed in tension than in compression. Moreover, in stronger mortar mixes, i.e., HSM-I mortar mixes, no pulling out of aggregate from the cement paste has been observed. But in HSM-III mortar mixes, pulling out of aggregate particles was encountered significantly. In HSM-II mortar mixes, two types of failures were observed. From this study, it was clearly understood that the cement/aggregate interface has been stronger in HSM-I mortar mixes than that of the other two. Empirical equations have been derived to predict the splitting tensile strength of mortar with water/cement ratio in all the three mortar mixes. For HSM-I mortar mixes, R=.986 [Eq. (13)]

$$\sigma_{t28} = 1.58(W/C) - 0.5097 \qquad (13)$$

For HSM-II mortar mixes, R=.958 [Eq. (14)]

$$\sigma_{t28} = 1.563(W/C)^{-0.4806} \qquad (14)$$

For HSM-III mortar mixes, R=.956 [Eq. (15)]

$$\sigma_{t28} = 1.348(W/C)^{-0.6137} \qquad (15)$$

The splitting tensile strength at 28 days can be estimated as a function of compressive strength of mortar at 28 days. It

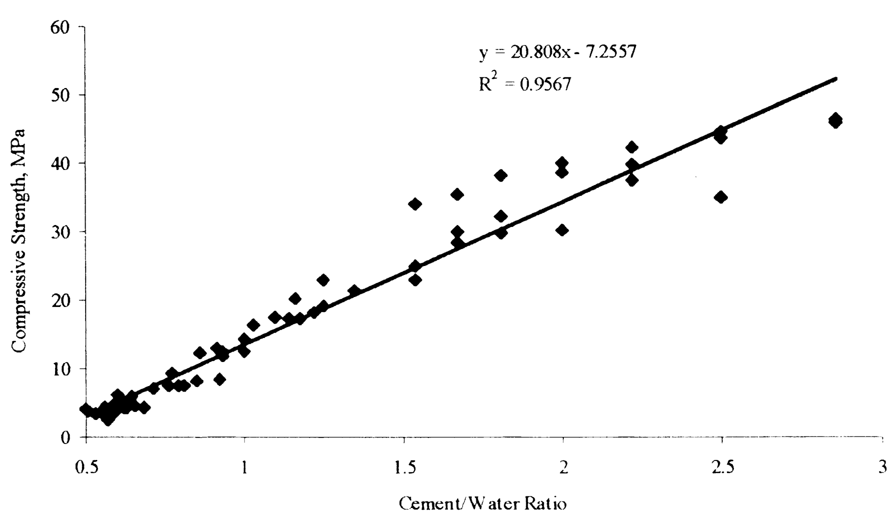

Fig. 6. Variation of compressive strength of mortar with cement/water ratio with strength ranging from very-low-strength to very-high-strength mortars.

shows that nonlinear variation has been observed between the two important properties of mortar. The regression coefficient (R) is .94 [Eq. (16)].

$$\sigma_{t28} = 0.30(\sigma_{C28})^{0.55} \quad (16)$$

5. Conclusions

The cement mortars made with HSM I and HSM II mixes show better performance than those of HSM-III. It has been generally observed that relatively brittle failures were noticed in high-strength mortars. The HSM-I mortar mixes seemed to be more brittle than the other mortar mix groups. However, the single largest factor affecting the strength of mortar has been observed as the water/cement ratio. Mathematical model expressions have been proposed for practical considerations for the design of mortar mixes. At the end, a more generalized Abrams' law has been proposed which can predict the compressive strength of mortars ranging from very-low-strength to very-high-strength levels. Abrams' generalized water/cement ratio law is applicable to mortars with water/cement ratio greater than 0.40.

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