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Theoretical elucidation on the empirical formulae for the ultrasonic testing method for concrete structures

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Abstract

The main purpose of this paper was to theoretically elucidate the compressive strength of concrete using the empirical formula with exponential form for the ultrasonic testing method through a generalization of the one-dimensional wave equation. The approximate solutions derived from a generalization of the one-dimensional wave propagation theory using the separation of variables method can be represented as the exponential form. If the viscosity of the concrete materials in stress wave method is neglected, the elastic waves can then be reduced to the ultrasonic waves. Through the membership function concept in the fuzzy set theory, the approximate solution with exponential form can be represented as the empirical formulae for the ultrasonic testing method in predicting the compressive strength of concrete. This present study indicates that both the ultrasonic pulse velocity (UPV) and ultrasonic pulse amplitude (UPA) methods are just a special case of the combined method. The predicted results of the combined method are actually better than the results obtained from either the UPV or UPA method.

Keywords: Nondestructive testing; Pulse velocity; Pulse amplitude; Wave equation

1. Introduction

Reinforced concrete (RC) is an important and widely used material in construction and civil engineering all over the world. The load capacity of a RC structure after construction may be reduced due to corrosion, fatigue and so on. In-situ quality measurement and the actual load capacity of a RC structure are thus very important. Certainly, the suitable and corrective measurement may need to be adopted.

At present, the investigation of nondestructive testing techniques [1] is a very popular subject. The ultrasonic method [2,3] is one of the nondestructive testing techniques and is frequently adopted for evaluating the quality of in-situ concrete structures. However, most authors study ultrasonic pulse velocity (UPV) [4–22] and ultrasonic pulse amplitude (UPA) [23–25] separately. The relationship between the UPV and UPA and the combined UPV and UPA method [22] has only been investigated by a few authors. In essence, the empirical formula obtained from the UPV is really quite different from that obtained from the UPA. The empirical formula of the combined UPV and UPA method is provided by Tharmaratnam and Tan [22]. To date, however, no studies have attempted to theoretically prove these empirical formulae obtained from the experimental result regression analysis. This is notable in the underestimation or overestimation of concrete compressive strength.

This paper focuses on the theoretical elucidation of the empirical formulae for the UPV, UPA and combined methods. The first part of this paper briefly introduces the theoretical background. The later part theoretically elucidates the empirical formulae of for the UPV, UPA and combined methods. In addition, this study aims to provide the theoretical basis for future ultrasonic method application studies.

2. Theoretical background

The propagation of ultrasonic waves in a solid medium is subject to two kinds of decay, geometrical diffusion and energy dissipation. The so-called geometrical diffusion occurs when the ultrasonic energy propagates from a point

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source, and the signal strength is inversely proportional to the square of the distance from the source. The energy in the solid medium during ultrasonic propagation will decay. Absorption occurs because of the friction between particles during ultrasonic vibration in a solid medium. This leads to the energy decay because the sonic energy is transformed into heat. Scatter results from the nonuniform nature of the solid medium. Since the solid medium consists of many interfaces of different densities, the impedance on the interface suddenly changes the wave propagation. This phenomenon will also cause the energy to decay.

These energy losses, which are partially a function of the strength and integrity of the propagating material, lead to detectable changes in both the UPV and UPA, and the relationship between the UPV and UPA are now considered below.

2.1. UPV

The UPV method, also known as the transit time method, uses a detector to measure the time-of-flight it takes for an ultrasonic pulse to pass through a known thickness of solid material. The UPV can be written as:

$$V_c(x,t) = \frac{x}{t} \tag{1}$$

where $V_c(x,t)$ is the UPV in concrete, x is the propagated path length and t is the transit time.

Based on experimental results, Tharmaratram and Tan [22] gave the relationship between UPV in a concrete V_c and concrete compressive strength f_c' as:

$$f_c' = a e^{bV_c} \tag{2}$$

where a and b are parameters dependent upon the material properties. This empirical formula has been studied before [4–22]. If x is fixed in Eq. (1), then Eq. (2) can be expressed as a function of t, that is:

$$f_c'(t) = a e^{bV_c(t)} \tag{3}$$

where a and b are constants.

2.2. UPA

The UPA method uses a receiver to measure the amplitude of a reflected wave. The relative amplitude ratio, A_r, is defined as the reflected wave amplitude divided by the original amplitude.

Tharmaratnam and Tan [22] combined the effect of both the divergence and dissipation factors, which applies when an ultrasonic pulse travels through a solid medium, and then derived the ultrasonic amplitude formulae using the conservation of energy principle [3]:

$$P_x = P_o K_c K_d \left(\frac{1}{x}\right) e^{-\alpha x} \tag{4}$$

where P_x is the pulse amplitude at path length x from the source, P_o is the initial pulse amplitude at the source, K_c is the attenuation factor due to contact losses, $K_d = \pi(D_0{}^2/4\delta)$ is the geometrical divergence coefficient of the material, D_0 is the diameter of the oscillator, δ is the wavelength of the sound beam and α is the attenuation coefficient. Eq. (4) can be rewritten as:

$$\frac{P_x}{P_o} = K_c K_d \left(\frac{1}{x}\right) e^{-\alpha x} \approx K_c K_d \left(\frac{1}{x+g}\right) e^{-\alpha x} = A_r(x), \qquad x+g > x \tag{5}$$

where A_r is the amplitude ratio and $x+g>x$ is the path length from the source. If K_c, K_d and α are all constants, then A_r is only a function of the path length x.

Based on experimental results, Tharmarntnam and Tan [22] also found that the relationship between the amplitude ratio A_r and the concrete compressive strength f_c' (MPa) can be represented approximately by:

$$f_c' = c e^{dA_r} \tag{6}$$

where c and d are parameters related to the materials properties. If time t is fixed, then Eq. (6) is merely a function of path length x, i.e.,

$$f_c'(x) = c e^{dA_r(x)} \tag{7}$$

where c and d are constants.

2.3. The relationship between the UPV and UPA

2.3.1. One-dimensional wave equation

Based on Ref. [26], the one-dimensional wave equation with three different conditions is described as follows.

1. Wave transmission along a uniform bar by neglecting the effect of transverse strain and inertia.

$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho_0}\frac{\partial^2 u}{\partial x^2} = s_1^2 \frac{\partial^2 u}{\partial x^2} \tag{8}$$

where u is particle displacement, x is the path length, t is the time, E is the Young's modulus, ρ_0 is the density of the material in its unstrained state and s_1 is the speed of the longitudinal elastic wave, i.e.,

$$s_1 = \sqrt{\frac{E}{\rho_0}} \tag{9}$$

2. Wave transmission along a uniform bar constrained to deform under conditions of plane strain:

$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho_0(1-\nu^2)}\frac{\partial^2 u}{\partial x^2} = s_2^2 \frac{\partial^2 u}{\partial x^2} \tag{10}$$

where ν is the Poisson's ratio and the longitudinal wave speed s_2 is:

$$s_2 = \sqrt{\frac{E}{\rho_0(1-\nu^2)}} = s_1\sqrt{\frac{1}{(1-\nu^2)}} \tag{11}$$

3. Wave transmission along a uniform bar constrained to have zero transverse deformation:

$$\frac{\partial^2 u}{\partial t^2} = \frac{E(1-\nu)}{\rho_0(1+\nu)(1-2\nu)}\frac{\partial^2 u}{\partial x^2} = s_3^2\frac{\partial^2 u}{\partial x^2} \tag{12}$$

where the speed of elastic longitudinal wave propagation s_3 is:

$$s_3 = \sqrt{\frac{E(1-\nu)}{\rho_0(1+\nu)(1-2\nu)}} = s_1\sqrt{\frac{(1-\nu)}{(1+\nu)(1-2\nu)}} \tag{13}$$

According to the above description, we note that, regardless of the conditions, there will be a common result. Thus, if the transmitting medium is the same material (E, ρ_0, ν are all constants), all of the longitudinal wave speeds (s_1, s_2, s_3) are constants. Usually, we use s to represent the longitudinal wave speed. It is worthy to point out that the parameters such as density, elastic constant, Poisson's ratio, wave speed, strength, etc. of the structure can be found in the relative literatures.

2.3.2. Solution to a generalization of the wave equation

Consider the telegraph equation [27]:

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{s^2}\frac{\partial^2 u}{\partial t^2} - P\frac{\partial u}{\partial t} - qu = 0 \tag{14}$$

where $u(x,t)$ is the voltage of the transmission lines, x is the distance, t is the time, s, p and q are all constants. If $p=q=0$, then Eq. (14) reduces to the wave equation:

$$\frac{\partial^2 u}{\partial t^2} = s^2\frac{\partial^2 u}{\partial x^2} \tag{15}$$

where $u(x,t)$ is the particle displacement. If either p or q is different from zero, then Eq. (14) represents a generalization of the wave equation. The term $p(\partial u/\partial t)$ in Eq. (14) is a dissipative term and corresponds to a damping force. The term qu in Eq. (14) produces dispersion in the waves.

Put

$$u(x,t) = w(t)h(x,t) \tag{16}$$

where $h(x,t)$ is the new unknown function. In order to eliminate the first derivatives of u in the new equation for h, we must determine $w(t)$. From Eq. (16), we have:

$$\frac{\partial u}{\partial t} = w\frac{\partial h}{\partial t} + \frac{\partial w}{\partial t}h \tag{17}$$

$$\frac{\partial^2 u}{\partial t^2} = w\frac{\partial^2 h}{\partial t^2} + 2\frac{\partial w}{\partial t}\frac{\partial h}{\partial t} + \frac{\partial^2 w}{\partial t^2}h \tag{18}$$

$$\frac{\partial^2 u}{\partial x^2} = w\frac{\partial^2 h}{\partial x^2} \tag{19}$$

The substitution of Eqs. (17)–(19) into Eq. (14) yields:

$$w\frac{\partial^2 h}{\partial x^2} - \frac{1}{s^2}w\frac{\partial^2 h}{\partial t^2} - \left(\frac{2}{s^2}\frac{\partial w}{\partial t} + pw\right)\frac{\partial w}{\partial t} - \left(\frac{1}{s^2}\frac{\partial^2 w}{\partial t^2} + p\frac{\partial w}{\partial t} + qw\right)h = 0 \tag{20}$$

Thus, we choose:

$$\frac{2}{s^2}\frac{\partial w}{\partial t} + pw = 0 \tag{21}$$

Eq. (21) is an ordinary equation. It is separable and a solution:

$$w(t) = \mathrm{e}^{-\frac{ps^2 t}{2}} \tag{22}$$

Eq. (20) contains no first derivatives of h and reduces to:

$$\frac{\partial^2 h}{\partial x^2} - \frac{1}{s^2}\frac{\partial^2 h}{\partial t^2} + kh = 0 \tag{23}$$

where $k=q-(p^2s^2/4)$. Eq. (23) is known as a dispersive hyperbolic equation and has the solution:

$$h(x,t) = \mathrm{e}^{\zeta i(x-\eta t)} \tag{24}$$

where ζ and η are parameters. Substituting Eqs. (22) and (24) into Eq. (16), we have:

$$u(x,t) = \mathrm{e}^{-\frac{ps^2 t}{2}}\mathrm{e}^{\zeta i(x-\eta t)} = \mathrm{e}^{(i\zeta)x}\mathrm{e}^{-\left(\frac{ps^2}{2}+i\zeta\eta\right)t} \tag{25}$$

Based on Eq. (25), the general solution $u(x,t)$ of Eq. (14) may be written in terms of the exponential form:

$$u = (x,t) = c_5\mathrm{e}^{ax+bt} \tag{26}$$

where a, b and c_5 are all constants. In this case, our choice depends on both the initial and boundary conditions. Moreover, it must be pointed out that, if the viscosity of the material under the stress wave method is neglected, then the elastic waves are reduced into ultrasonic waves [28].

Based on the theory described above, when we want to detect a concrete structure with mass m and cross-sectional areas S_A, the compressive strength of the concrete is considered first. Thus, from Eq. (25), the wave motion acceleration is:

$$\frac{\partial^2 u}{\partial t^2} = e^{(i\zeta)x}\left[-\left(\frac{ps^2}{2}+i\zeta\eta\right)\right]^2 e^{-\left(\frac{ps^2}{2}+i\zeta\eta\right)t} = c_5 b_1^2 e^{ax+bt} = c_6 e^{ax+bt} \quad (27)$$

where c_6 is constant. Thus, the compressive strength of the concrete can be written as:

$$f_c'(x,t) = \frac{m}{S_A}\left[-\left(\frac{ps^2}{2}+i\zeta\eta\right)\right]^2 e^{(i\zeta)x} e^{-\left(\frac{ps^2}{2}+i\zeta\eta\right)t} = c_7 e^{(i\zeta)x} e^{-\left(\frac{ps^2}{2}+i\zeta\eta\right)t} = c_7 e^{ax+bt} \quad (28)$$

where c_7 is constant. We discover from Eq. (28) that:

$$f_c'(x,t) = f_c'(t) = c_7 e^{bt}, \quad \text{if } x = \text{constant}, \ t \geq 0$$
$$f_c'(x,t) = f_c'(x) = c_7 e^{ax}, \quad \text{if } t = \text{constant}, \ x \geq 0 \quad (29)$$

The concrete compressive strength prediction is usually obtained using a probabilistic method after the compressive strengths of concrete structures detected using ultrasonic apparatus were determined. Thus, the empirical formulae for both the UPV and UPA methods are all obtained from experimental results associated with statistical regression. Civanlar and Trussell [29] constructed the membership function, μ, for the fuzzy sets whose statistical data elements have a defining feature with a probability density function. They show that, for any probability density function, the method developed by Civanlar and Trussell [29] is capable of generating a membership function, in agreement with the possibility–probability consistency principle. If $f_c'(x)$, as

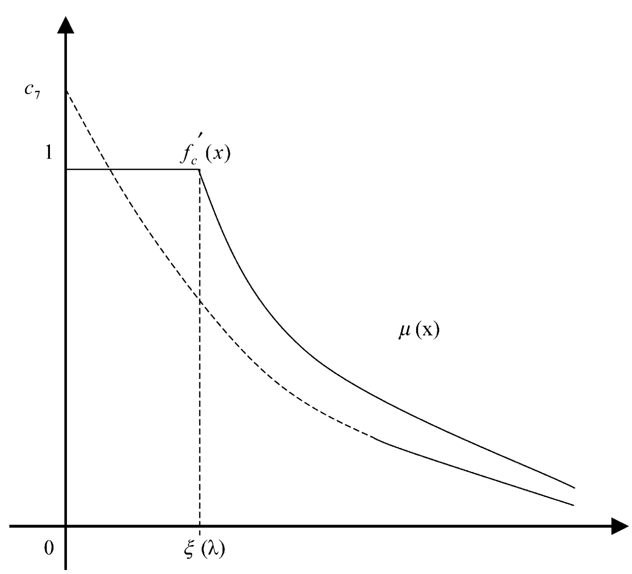

Fig. 1. A membership function corresponding to the compressive strength of concrete with exponential density.

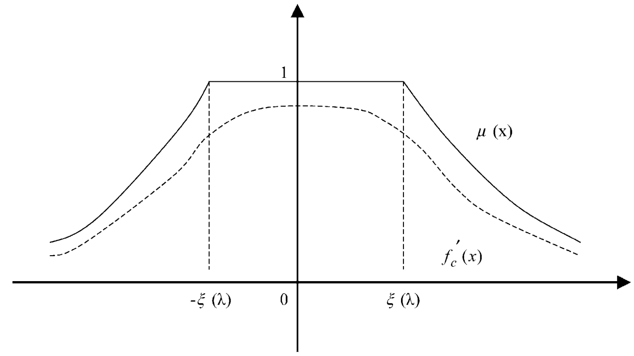

Fig. 2. A membership function corresponding to the compressive strength of concrete with standard normal density.

described in Eq. (29) that is about the UPA method, is from the probability density function, then the membership function, μ(x), is determined by the parameter ξ as demonstrated in Fig. 1. The membership function, μ(x), derived from the exponential density satisfies the consistency principle for confidence level $\lambda \geq 0.50$. If $f_c'(x)$, as denoted in Eq. (29), is from the probability density function, then the optimal membership function as demonstrated in Fig. 2 is determined by a single scalar ξ, which is a function of the confi dence level λ. Hence, Eq. (29) can be approximated by:

$$f_c'(x) = c e^{dA_r(x)} \quad (30)$$

Similarly, Eq. (29) can be approximately expressed by:

$$f_c'(t) = a e^{bV_c(t)} \quad (31)$$

The UPV and UPA methods are, respectively, functions of t and x. Based on Eqs. (30) and (31), the right term with exponential form e^{ax+bt} in Eq. (28) can be written as $e^{aV_c(t)+bA_r(x)}$ Using the separation of variables method, Eq. (28) is changed as:

$$f_c'(x,t) = Ae^{[BV_c(t)+CA_r(x)]} = ae^{bV_c(t)} ce^{dA_r(x)} = f_c'(t) f_c'(x) \quad (32)$$

where A, B and C are constants dependent on the material properties. Based on the separation of variables method, we have Eqs. (30) and (31).

Table 1
Comparison of ultrasonic pulse velocity and pulse amplitude with the combined method

Description	$f_c'(t) = ae^{bV_c(t)}$	$f_c'(x) = ce^{dA_r(x)}$	$f_c'(x,t) = Ae^{[BV_c(t)+CA_r(x)]}$
Cube	150 mm	150 mm	150 mm
Constants	$a = 0.23$	$c = 0.502$	$A = 0.52$
	$b = 1.72$	$d = 2.47$	$B = 1.09$
			$C = 1.13$
Error average	2.36	2.40	2.23
Reliability β	0.064	0.061	0.368
Failure probability P_f	47.05	47.13	34.8

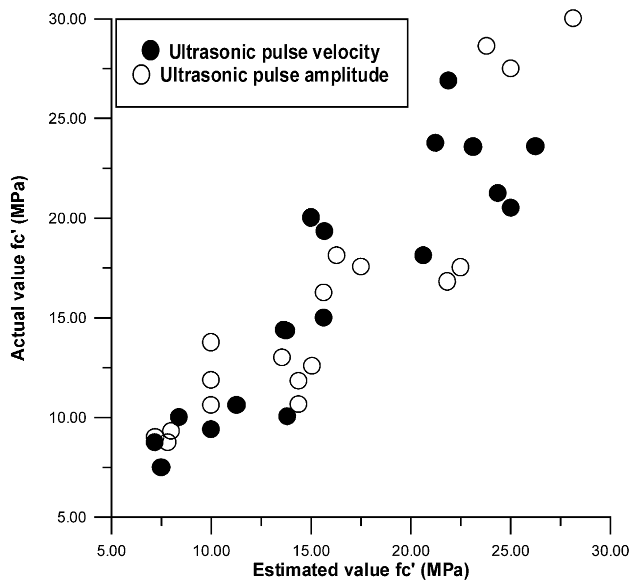

Fig. 3. Relationship between actual and estimated values for the ultrasonic pulse velocity and amplitude methods.

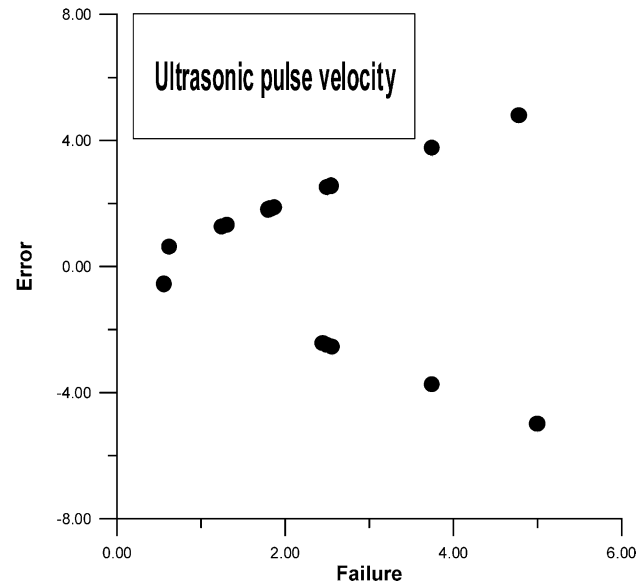

Fig. 5. Relationship between the error and failure values for the ultrasonic pulse velocity method.

2.3.3. Illustrative examples

In order to emphasize the predicted compressive strength of concrete, f_c', obtained from the combined cor relation of UPV and UPA, better than those obtained from either the UPV or UPA, the experimental results obtained by Tharmarntnam and Tan [22] were adopted for illustration. The constants a, b, c, d, A, B and C dependent on the material properties are also listed in Table 1. After the material property constants were determined, we may theoretically calculate the values of f_c' with respect to the experimental result of f_c' for comparing the accuracy of three methods mentioned above. Figs. 3 and 4 show the comparison among the calculated values using the three methods and experimental values obtained by Tharmarntnam and Tan [22] for concrete with 150-mm cubes. Through the analysis, the calculated value errors using the UPV, UPA and combined methods respective of the experimental values were 16%, 18% and 13%, respectively. To emphasize and represent the three methods for calculating the compressive strength of concrete f_c' is very important. The reliability analysis concept must be introduced. Based on the concrete compressive strength obtained by experimental results and in order to obtain the reliability index of f_c', the experimental and calculated

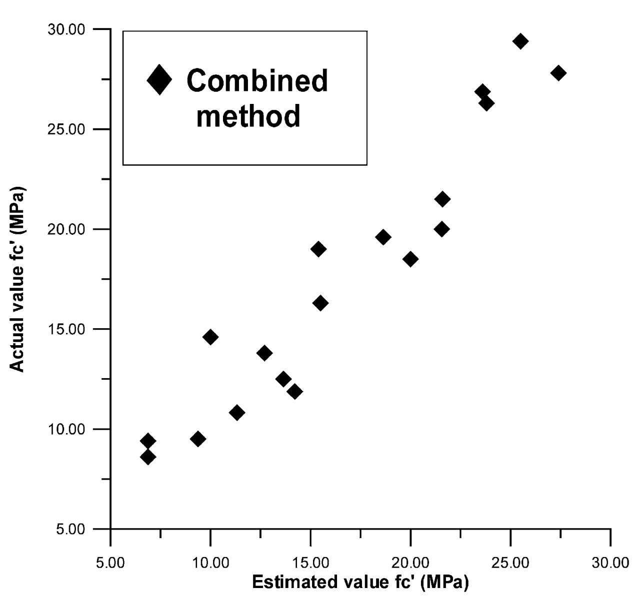

Fig. 4. Relationship between actual and estimated values for the combined method.

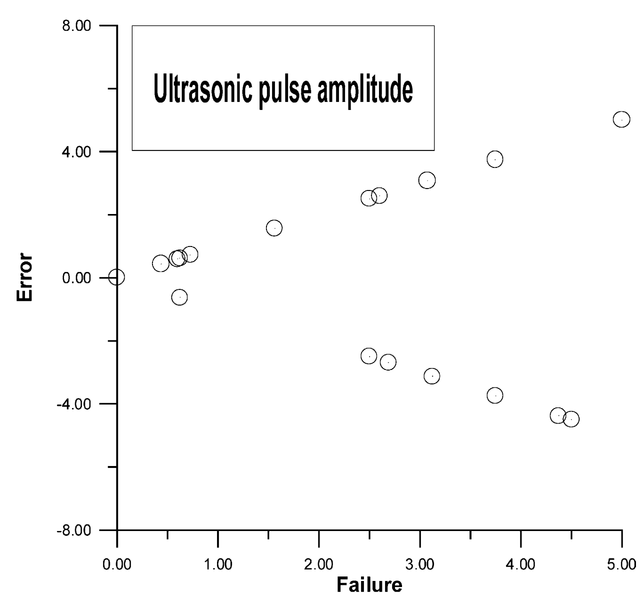

Fig. 6. Relationship between the error and failure values for the ultrasonic amplitude method.

values are S and R, respectively. Thus, the failure state of the three methods can be expressed as:

$$F = R - S \tag{33}$$

where F is the structural effect function. Failure occurs when F is less than or equal to zero. After calculation, a set of data with 20 specimens can be obtained using the three methods. The error and failure values of f_c' obtained from the three methods are plotted in Figs. 5–7. From the distribution of these figures, we observe the results as described in the following: (a) If the point distribution is more concentrated, then the reliability of the error values calculated from the three methods is more liable. (b) If the point distribution is close to coordinate (0,0), then the accuracy of the calculated results obtained from the three methods is very good, i.e., both the average error and failure values are low. (c) Under the crossed line located at the zero value of the longitudinal axis, the lesser this point number is the greater the unsatisfactory strength. (d) A linear relationship in the distribution represents the error and failure values tendency obtained from the three methods. After the comparison among Figs. 5–7, the distribution of Fig. 5 is definitely the best. Moreover, the reliability index [30] can be represented as

$$\beta = \frac{\bar{F}}{\sigma_F} \tag{34}$$

where $\bar{F}$ is the average of the failure values and σ_F is the standard deviation of the failure value. Based on the reliability index, the failure probability can be calculated according to:

$$P_f = 1 - \Phi(\beta) \tag{35}$$

where $\Phi(\beta)$ is the standard normal distribution function.

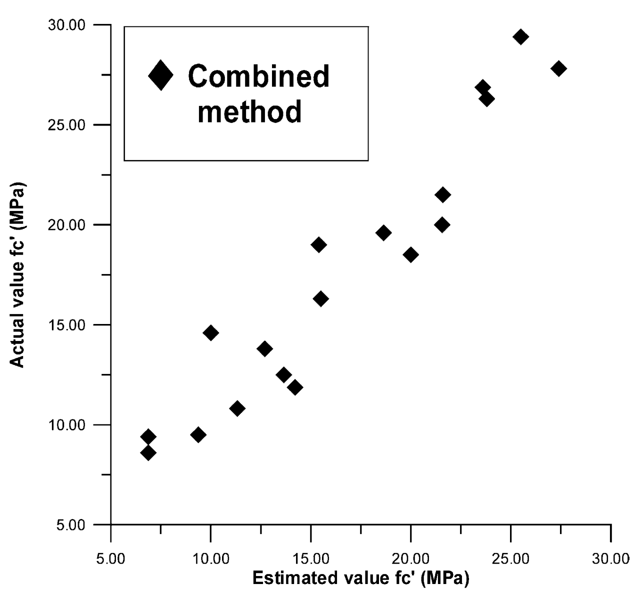

Fig. 7. Relationship between the error and failure values for the combined method.

Using Eqs. (34) and (35) to evaluate the f_c' of the three methods, these results are listed in Table 1. From Table 1, we see that the combined method is the best. The failure probability of f_c' obtained from the combined method can be reduced to 13%.

3. Discussion

The purpose of this study was to theoretically elucidate the empirical formula of the ultrasonic testing method used for predicting the compressive strength of concrete. Strictly speaking, ultrasonic propagation should usually be modeled by a kind of three-dimensional wave equation. However, since the ultrasonic pulse receiver or detector is only concerned with the first arriving signal of the longitudinal wave (P-wave) and subsequent signals are ignored, the simplified one-dimensional wave equation used in this paper is perfectly adequate. The propagation speed s is independent of $\partial u/\partial t$, or the local velocity, or speed shift, of the elements transmitting the wave. The wave speed s depends only on the elastic properties of the transmitting medium, its density and Poisson's ratio. In this paper, all of the s values were assumed to be constant. The failure probability of the three methods was evaluated using the reliability index. The result of the combined method is really better than the results obtained from either the UPV or the UPA method. However, the failure probability of the combined method is still 34.8%. It is obvious that the constants of the three methods are not accurate and are inclined to be too large.

Generally speaking, in order to apply the regression method to predict the compressive strength of concrete structures, as many specimens as possible must be examined. From the determined, concrete structure compressive strength system, the probability distribution suited for a specimen can be found. In this situation, regardless of the number of specimens, the statistical method is still effective. The important problem is the reliability of the estimated result. Of course, the greater the number of specimens, the greater the reliability. In other respects, under uncertain probability distribution conditions, a quantitative conclusion with respect to specimens cannot be made. For determining data suited to the probability distribution, a suitable testing method should be adopted. Under this situation, the number of specimens ranged from 100 to 200 for the statistical analysis. However, if we depend only on the statistical method for predicting the compressive strength of concrete, structures using the ultrasonic testing method is not enough. In essence, choosing a suitable probability distribution should be based on the developed mechanism and concrete structure process subjected to loads.

This study could not rigorously prove the empirical formulae by mathematical means. However, this theoretical elucidation may provide a theoretical concept why the empirical formulae can be used to predict the compressive strength of concrete. More research is needed to prove the empirical formulae for the ultrasonic method for predicting the compressive strength of concrete. In addition, there is a critical need for in-situ research on practical concrete structures so that predicted values of compressive strength of concrete can produce exact results.

4. Conclusion

A theoretical elucidation on the empirical formulae for ultrasonic testing methods in concrete structures has been presented. The ultrasonic empirical formulae testing method can be theoretically elucidated from a generalization of the one-dimensional wave equation under conditions in which the viscosity of the concrete material is neglected. The approximate solution for a generalization of the one-dimensional wave equation, provided that the viscosity of concrete material is ignored, may be expressed as an exponential empirical formulae of the ultrasonic method through the concept of membership functions in fuzzy set theory. Both the UPV and UPA methods are a special case of the combined method. The predicted results of using the combined method are better than those obtained from either the UPV or UPA method. However, the slightly comprehensive ultrasonic testing method associated with piezoelectric transducers suggests further research with large groups of professional engineers.

References

[1] V.M. Malhotra, In Situ/Nondestructive Testing of Concrete, SP-82, vol. 1, American Concrete Institute, Detroit, 1984, pp. 1–16.

[2] L. Filipczynski, Z. Pawlowski, J. Wehr, Ultrasonic Methods of Testing Materials, Butterworths, London, 1966.

[3] J. Krautkramer, H. Krautkramer, Ultrasonic Testing of Material, Springer-Verlag, New York, 1983.

[4] V.R. Sturrup, F.J. Vecchio, J.A. Sauer, Dispersion of ultrasonic pulse velocity in cylindrical roads, J. Acoust. Soc. Am. 27 (1955) 550–555.

[5] R.H. Elvery, Non-destructive testing of concrete and its relationship its specifications Concrete, 5 (1971) 137–141.

[6] D.N. Newokoye, Prediction and assessment of concrete properties from pulse–velocity tests, Mag. Concr. Res. 25 (1973) 39–46.

[7] British Standards Institution, Recommendations for non-destructive methods of test for concrete: Part 5. Measurement of the velocity of ultrasonic pulses in concrete 1974, 1–20.

[8] H.W. Chung, An appraisal of the ultrasonic pulse technique for detecting voids in concrete, Concrete 11 (1978) 25–28.

[9] H.W. Chung, Effects of embedded steel bare upon ultrasonic testing of concrete, Mag. Concr. Res. 30 (1978) 19–25.

[10] J.H. Bungey, The validity of ultrasonic pulse velocity testing of in-place concrete for strength, N.D.T. Int. (1980) 296–300.

[11] H.W. Chung, K.S. Law, Diagnosing in situ concrete by ultrasonic pulse technique, Concr. Int. (1983) 42–49.

[12] J. Bungey, Effects of steel on ultrasonic measurements for concrete members, Int. Assoc. Bridge Struct. Eng. Colloq. (1983) 93–100.

[13] V.R. Sturrup, F.J. Vecchio, H. Caratin, Pulse velocity as a measure of concrete compressive strength. ACI, SP82-11, International Conference on In Situ/Nondestructive Testing of Concrete, Publication SP82-11, American Concrete Institute, Detroit, 1984, pp. 201–227.

[14] J.H. Bungey, The influence of reinforcement on ultrasonic pulse velocity testing, in: V.M. Malhotra (Ed.), International Conference on In Situ/Nondestructive Testing of Concrete, Publication SP82-12, American Concrete Institute, Detroit, 1984, pp. 229–246.

[15] R.N. Swamy, A.H. Al-Hamed, The use of pulse velocity measurements to estimate strength of air-dried cubes and hence in situ strength of concrete. ACI, SP 82-13, in: V.M. Malhotra (Ed.), International Conference on In Situ/ Nondestructive Testing of Concrete, Publication SP82-13, American Concrete Institute, Detroit, 1984, pp. 247–276.

[16] S. Popovics, J.L. Rose, J.S. Popovics, The behavior of ultrasonic pulses in concrete, Cem. Concr. Res. 20 (1990) 259–270.

[17] S. Popovics, J.S. Popovics, Effect of stresses on the ultrasonic pulse velocity in concrete, Mater. Struct. 24 (1991) 15–23.

[18] Y. Berthaud, Damage measurements in concrete via an ultrasonic technique: Part I. Experiment, Cem. Concr. Res. 21 (1991) 73–82.

[19] Y. Berthaud, Damage measurements in concrete via an ultrasonic technique: Part I. Experiment, Cem. Concr. Res. 21 (1991) 219–228.

[20] M. Hassan, O. Burdet, R. Favre, Ultrasonic measurements and static load tests in bridge evaluation, NDT&E Int. 28 (1995) 331–337.

[21] A. Samarin, P. Meynink, Use of combined ultrasonic and rebound hammer method for determining strength of concrete structural members, Concr. Int. (1981) 25–29.

[22] K. Tharmaratnam, B.S. Tan, Attenuation of ultrasonic pulse in cement mortar, Cem. Concr. Res. 20 (1990) 335–345.

[23] L.I. Knab, G.V. Biessing, J.R. Clifton, Laboratory evaluation of ultrasonic for crack detection in concrete, ACI Mater. J. 80 (1983) l7–27.

[24] F.M. Burdekin, D.G. John, P.A. Payne, T.A. Smith, P.A. Gaydecki, Non-destructive methods for field inspection of embedded or encased high strength steel rods and cables, NCHRP Project l0-30(3) Final Report, Phase 2, 1991.

[25] W. Suaris, V. Fernando, Ultrasonic pulse attenuation as a measure of damage growth during cycle. Loading of concrete, ACI Mater. J. 84 (1987) 185–193.

[26] W. Johnson, Impact strength of material, Crane, Edward Arnold, London, 1972.

[27] C.R. Chester, Technique in Partial Differential Equation, McGraw-Hill, New York, 1980.

[28] A. Bedford, D.S. Drumheller, Elastic Wave Propagation, Wiley, New York, 1994.

[29] M.R. Civanlar, H.J. Trussell, Constructing membership functions using statistical data, Fuzzy Sets Syst. 18 (1986) 1–13.

[30] G.C. Hart, Uncertainty Analysis, Loads, and Safety in Structural Engineering, Prentice-Hall, New Jersey, 1982.