Pergamon

Cement and Concrete Research 32 (2002) 1095–1100

CEMENT AND CONCRETE RESEARCH

# Effect of temperature and aging on the mechanical properties of concrete Part II. Prediction model

Jin-Keun Kim[a], Sang Hun Han[b,*], Seok Kyun Park[c]

[a]Department of Civil Engineering, Korea Advanced Institute of Science and Technology, 373-1 Kusong, Yusong, Taejon 305-701, Republic of Korea
[b]Coastal and Harbour Engineering Laboratory, Korea Ocean Research and Development Institute, 1270 Sadong, Ansan, Kyunggido 425-744, Republic of Korea
[c]Department of Civil Engineering, Daejeon University, 96-3 Yongun-dong, Tong-gu, Taejon 300-716, Republic of Korea

Received 12 March 2001; accepted 17 January 2002

**Abstract**

In Part I, empirical relationships between compressive strength and splitting tensile strength or elastic modulus with temperature and aging were proposed. This paper investigates new prediction models estimating splitting tensile strength and elastic modulus without knowing compressive strength. The prediction model is suggested on the basis of the equation that was suggested to predict compressive strength. The mechanical properties calculated by the model are compared with empirical results presented in Part I. To evaluate in-place applicability of the model, the empirical data on strength and elastic modulus of concrete cured at variable temperature are compared with the values estimated using the prediction model. The prediction model properly estimates the strength and elastic modulus of Types I and V cement concretes cured at constant and variable temperature conditions. 

*Keywords:* Temperature; Aging; Compressive strength; Elastic moduli; Splitting tensile strength

## 1. Introduction

Many of the existing evaluations on the influence of temperature and aging on the mechanical properties focus primarily on compressive strength, and only a few of prediction models are available for estimating elastic modulus and splitting tensile strength [1–4]. However, it is important to estimate the elastic modulus and splitting tensile strength as well as compressive strength because stress is calculated based on elastic modulus and crack initiation is estimated based on tensile strength. For example, nuclear containment structures made of concrete must be able to control cracks to maintain good air tightness. In order to control the thermal cracking of a containment structure, the temperature distribution of the structure must be obtained to analyze the heat of hydration and the temperature stress must be calculated based on the temperature gradient and elastic modulus. Temperature stress, if greater than tensile strength, induces thermal cracking. In this process, it is not necessary to estimate the compressive strength if the elastic modulus and splitting tensile strength are calculated by the prediction model. Therefore, it is important to propose the prediction model of elastic modulus and splitting tensile strength as well as that of compressive strength with temperature and aging.

The objectives of this study are to propose a prediction model estimating the mechanical properties with temperature and aging, and to evaluate the validity of the prediction model.

## 2. Prediction model

A prediction model was proposed in Ref. [4] to estimate the compressive strength development with temperature and aging. This model reduced the shortcomings of previous models and reasonably approximated the experimental results for compressive strength. This paper investigates the effectiveness of this model as a tool for predicting

* Corresponding author. Tel.: +82-31-400-6345; fax: +82-31-408-5823.

*E-mail address*: shhan@kordi.re.kr (S.H. Han).

0008-8846/02/$ – see front matter 
PII: S0008-8846(02)00745-7

Table 1
Limiting relative strength (or limiting relative elastic modulus) ($R_u$)

| | $R_u$ | | |
|---|---|---|---|
| Temperature (°C) | Compressive strength | Elastic modulus | Splitting tensile strength |
| 10 | 1.35 | 1.14 | 1.23 |
| 23 | 1.16 | 1.05 | 1.09 |
| 35 | 1.08 | 1.02 | 1.04 |
| 50 | 1.02 | 1.01 | 1.02 |

All regression results are based on the experimental data of Type I cement concrete.

splitting tensile strength and elastic modulus. The following equation is the prediction model proposed in Ref. [4].

$$\frac{S}{S_{28}} = R_u \left\{ 1 - \frac{1}{\sqrt{1+\sum_{i=1}^{n} A\left[e^{-\frac{E_0}{RT_i}e^{-\alpha t_i}} + e^{-\frac{E_0}{RT_i}e^{-\alpha t_{i-1}}}\right](t_i - t_{i-1})}} \right\} \qquad (1)$$

where $S$ is the compressive strength, $S_{28}$ is the 28-day compressive strength at each curing temperature, $R_u$ is the limiting relative compressive strength with $R_u = S_u/S_{28}$, $S_u$ is the limiting compressive strength, $A$ is a constant, $R$ is the gas constant and equal to 8.3144 J/K-mol, $T_i$ is the curing temperature at time step $i$ (K), $E_0$ is the initial apparent activation energy (J/mol), $\alpha$ is a constant, $t_{i-1}$ is the initial age of time step $i$ (days) and $t_i$ is the final age of time step $i$ (days). This equation involves five unknown parameters, $R_u$, $t_0$, $E_0$, $\alpha$ and $A$.

The apparent activation energy can be interpreted at the micro- or macrolevel. At the macrolevel, the apparent activation energy is related to the rate of increase of the compressive strength–age curve. Therefore, if the model is applied to the elastic modulus or splitting tensile strength of which the rate of increase is different from that of the compressive strength, the regression curves will have different apparent activation energy according to the mechanical properties. On the other hand, the apparent activation energy at the microlevel is a function of the cement hydration and will have a constant value for all mechanical properties. Many researches [5–7] propose that the apparent activation energy is related to the hydration process of cement. Therefore, it is assumed that the apparent activation energy is the characteristic property of

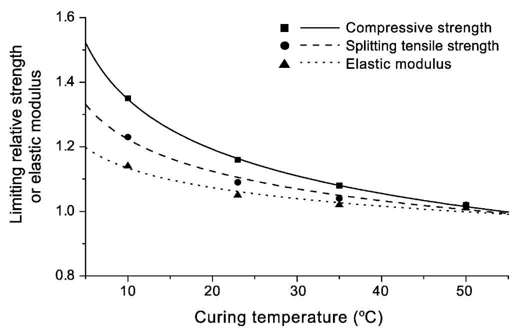
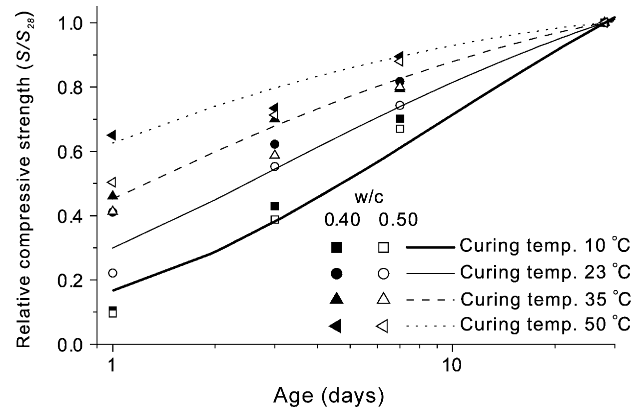
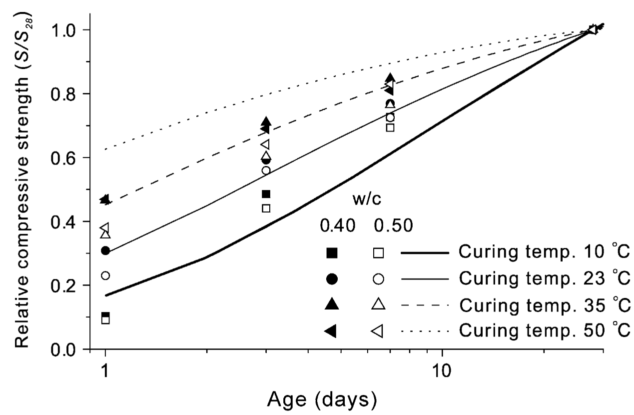



Fig. 1. Limiting relative strength or elastic modulus.




(a) Type I cement




(b) Type V cement

Fig. 2. Compressive strength.

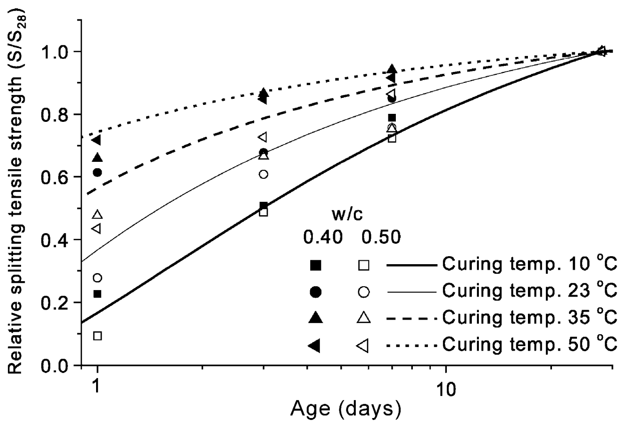
concrete and has a constant value for all mechanical properties. Therefore, the following equation of apparent activation energy suggested in Ref. [4] for estimating compressive strength can be used to predict splitting tensile strength and elastic modulus:

$$E = E_0 e^{-\alpha t} \qquad t \geq t_0 \qquad (2)$$

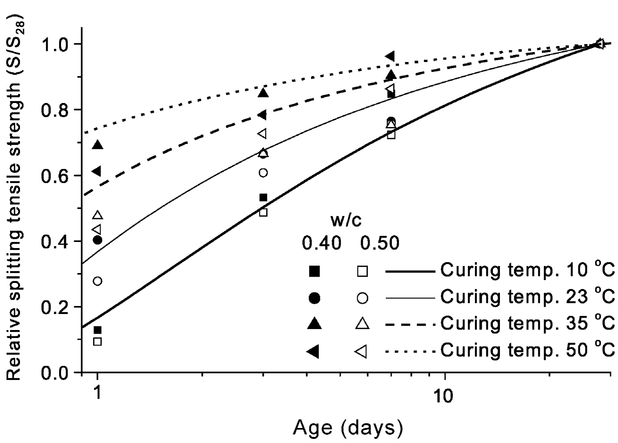
$$E_0 = 42{,}830 - 43\ T^c \quad (\text{J/mol}) \qquad (3)$$

$$\alpha = 0.00017\ T^c \qquad (4)$$

where $E_0$ is the initial apparent activation energy (J/mol), $\alpha$ is a constant and $T^c$ is the curing temperature (°C). The rates of increase of splitting tensile strength and elastic modulus are dissimilar to that of compressive strength and must be considered in Eq. (1). The difference in rates of increase can be estimated by modifying the constant $A$ with the mechanical properties. $A$ values are determined based on the regression results of experimental data of other researchers [1,4,8–10] and this paper. $A$ values of compressive strength, splitting tensile strength and elastic modulus are $1.0 \times 10^7$, $2.5 \times 10^7$ and $5.0 \times 10^7$, respectively.




(a) Type I cement




(b) Type V cement

Fig. 3. Splitting tensile strength.

The $t_0$ value of Eq. (1) is related to the setting time of concrete. Portland cement concrete remains in the plastic state for several hours before the setting time. Because the setting time is the characteristic property of concrete, compressive strength, splitting tensile strength and elastic modulus will have the same $t_0$. Therefore, the following prediction equation, which was proposed to estimate the compressive strength in Ref. [4], can be used to estimate the splitting tensile strength and elastic modulus.

$$t_0 = 0.66 - 0.011\ T^c \geq 0. \tag{5}$$

In summary, three of the five unknown variables in Eq. (1) can be estimated using Eqs. (3)–(5) and $A$ value is determined (see also Eq. (2)). Lastly, $R_u$ can be obtained by analyzing the experimental results of Type I cement concretes of Part I. $R_u$ values are tabulated in Table 1. The following prediction equations of $R_u$ for Type I cement concrete are proposed based on the regression results:
for compressive strength,

$$R_u = 2.04\ T^{c^{-0.18}} \geq 1 \tag{6}$$

for splitting tensile strength,

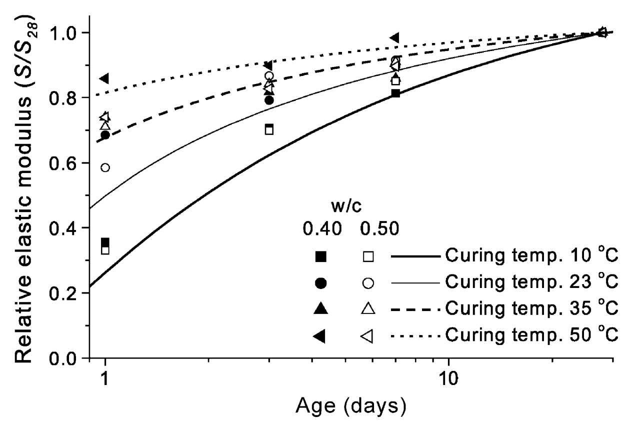
$$R_u = 1.62\ T^{c^{-0.122}} \geq 1 \tag{7}$$

for elastic modulus,

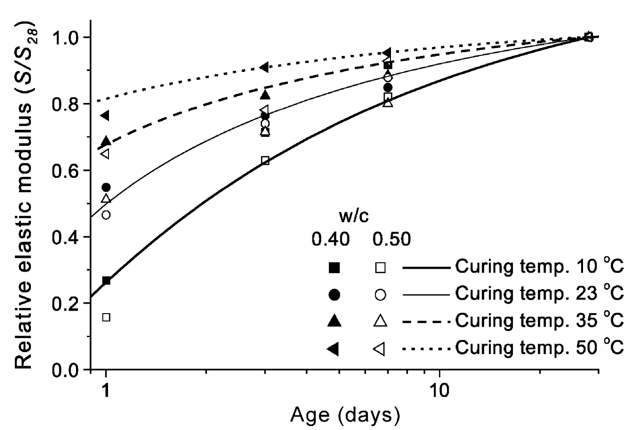
$$R_u = 1.36\ T^{c^{-0.079}} \geq 1. \tag{8}$$

Fig. 1 presents the regression results and the estimation curves. The $R_u$ associated with compressive strength is the largest of the three $R_u$ values reported in Fig. 1. Concretes cured at curing temperature larger than 50 °C have $R_u$ near 1, and the strength and elastic modulus of the concretes cured at temperature larger than 50 °C hardly increase at later ages.

Figs. 2–4 compare the prediction curves with the experimental results. The difference between calculated and experimental values with cement type and curing temperature are shown in Fig. 5. As shown in Fig. 5, the difference between calculated and experimental values is the smallest for elastic modulus and the biggest for splitting tensile strength. Also, the difference of Type I cement concrete is




(a) Type I cement

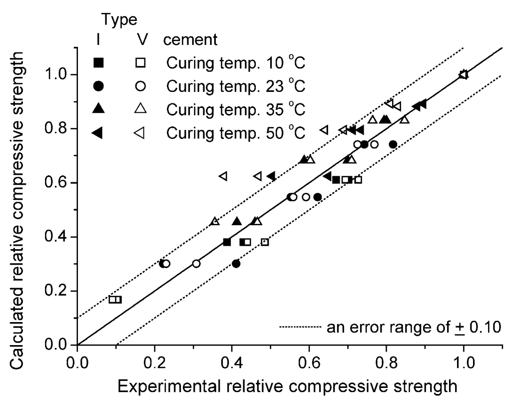



(b) Type V cement

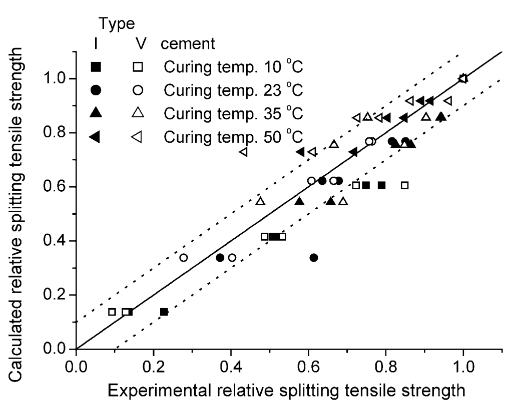
Fig. 4. Elastic modulus.

smaller than that of Type V cement concrete at early ages. This is because the general equations of Eqs. (3)–(8) are based on the experimental results of Type I cement concrete. This points to the need of many experimental tests according to cement type prior to proposing general equations of apparent activation energy, $t_0$ and $R_u$.

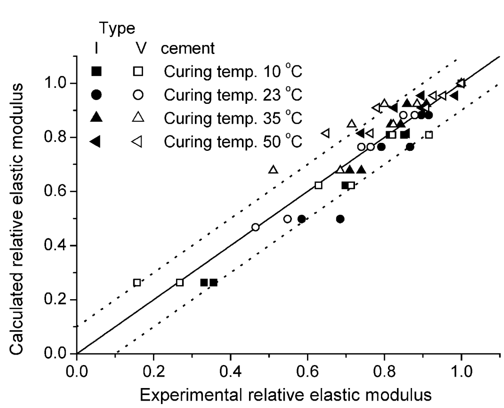
Comparisons of calculated and experimental values show that the difference of two values for elastic modulus is the smallest of mechanical properties. This can be explained with the concept of new prediction model. Cement hydration occurs all over the concrete and apparent activation energy is related to this hydration reaction. Therefore, the concept of new prediction model including apparent activation energy is linked to the mean property of concrete specimen. As the elastic modulus is calculated based on the secant modulus of stress–strain curve, the property is a mean value of concrete. However, the strength is determined by the propagation of the local crack of concrete. Therefore, the new prediction model having similar concept to elastic modulus estimates the elastic modulus accurately.




**(a) Compressive strength**

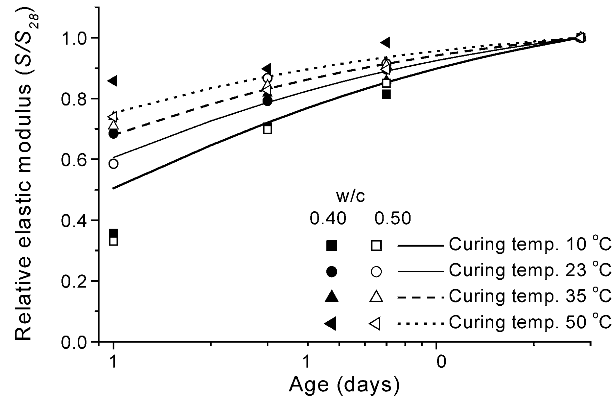



**(b) Splitting tensile strength**

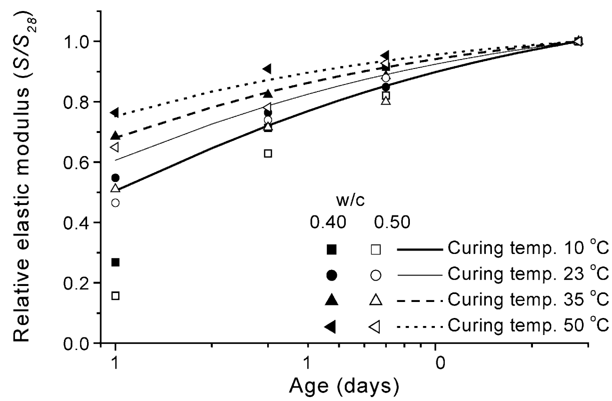



**(c) Elastic modulus**

Fig. 5. Comparison of experimental and calculated values.




(a) Type I cement




(b) Type V cement

Fig. 6. Elastic modulus by CEB-FIP model.

As previously stated, there are few prediction models estimating elastic modulus or splitting tensile strength with temperature and aging. Only CEB-FIP 1990 model [11] of several model codes proposes an equation estimating elastic modulus with temperature and aging. This paper compares the experimental result with the calculated value by the CEB-FIP 1990 model. The following equations are the prediction model proposed by CEB-FIP 1990 model (Eqs. (9) and (10)):

$$E_{ci}(t_T) = \sqrt{\mathrm{e}^{0.25\left(1-\left(\frac{28}{t_T}\right)^{0.5}\right)}}\,E_{ci,t=28,T^{c}=20} \tag{9}$$

$$t_T = \sum_{i=1}^{n} \Delta t_i \exp\left[13.65 - \frac{4000}{273 + T^c(\Delta t_i)}\right] \quad (10)$$

where $E_{ci}(t_T)$ is the elastic modulus, $E_{ci,t=28,T^c=20}$ is the elastic modulus at 28 days and 20 °C, $\Delta t_i$ is the age at curing temperature $T^c$ and $T^c$ is the curing temperature during $\Delta t_i$. If $E_{ci}(t_T)$ is divided with $E_{ci,t=28,T^c}$, $E_{ci}(t_T)$ can be presented with the ratio of $E_{ci,t=28,T^c}$. Fig. 6 compares the experimental results with the calculated relative elastic modulus. As shown in Fig. 6, the difference between the experimental and calculated values becomes greater at early ages and decreases with aging. Based on comparison between Figs. 4 and 6, it can be concluded that the proposed equation estimates the variation of elastic modulus with curing temperature and aging more accurately than CEB-FIP 1990 model.

## 3. Application of prediction model into variable curing temperature

Previous experimental specimens were cured at constant curing temperatures. However, temperature variation by the heat of hydration in mass concrete or the change of external environment varies with aging. Therefore, it is necessary to evaluate the validity of new prediction model for variable curing temperature condition. This study performed its experiments using concretes cured at variable curing temperature. This paper compares the experimental results with the calculated values by the proposed model.

Variable curing temperature is shown in Fig. 7. The temperature curve of Fig. 7. is the result of the hydration heat analysis of the tendon gallery of containment building for nuclear power plant and the analysis is based on the program that has been developed in KAIST [12]. Specimens were cured in the chamber to the test ages of 1, 3, 7 and 28 days. Mixture proportion with water/cement ratio of 0.40 was used. Further details are presented in Table 3 of Part I. As stated in

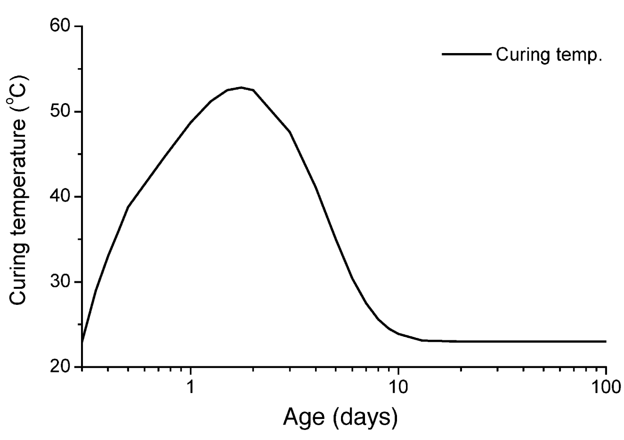



Fig. 7. Variable curing temperature.

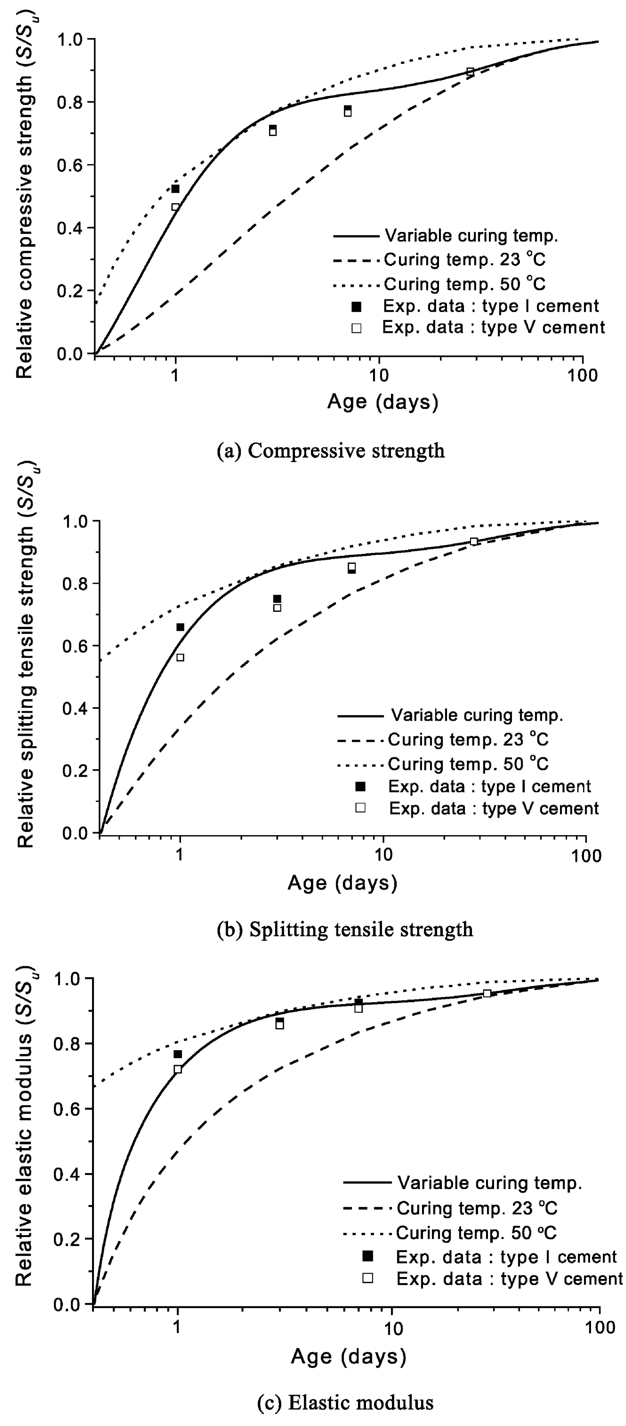



Fig. 8. Mechanical properties with variable curing temperature.

Part I, compressive strength was tested according to ASTM C 39, splitting tensile strength according to ASTM C 496 and elastic modulus according to ASTM C 469. Table 2 shows the experimental results.

As $E_0$ and $\alpha$ of Eq. (1) are constant with aging, Eqs. (3) and (4) can be used to estimate $E_0$ and $\alpha$ of concrete cured at variable curing temperature. However, $R_u$ is a function of age and temperature in variable curing temperature and Eq. (6) cannot estimate $R_u$ of concrete cured at variable

Table 2
Experimental results

| | Compressive strength (MPa) | | | | Splitting tensile strength (MPa) | | | | Elastic modulus (GPa) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *w/c* = 0.40 | 1 day | 3 days | 7 days | 28 days | 1 day | 3 days | 7 days | 28 days | 1 day | 3 days | 7 days | 28 days |
| Type I | 32.2 | 43.5 | 47.4 | 54.6 | 3.7 | 4.2 | 4.8 | 5.3 | 26.0 | 29.3 | 31.3 | 32.3 |
| Type V | 26.0 | 39.3 | 42.8 | 50.1 | 3.1 | 4.0 | 4.8 | 5.3 | 25.8 | 30.6 | 32.5 | 34.1 |

curing temperature. In this case, it is necessary to modify Eq. (1) as follows [4]:

$$\frac{S}{S_u} = \left\{ 1 - \frac{1}{\sqrt{1 + \sum_{i=1}^{n} A\left[ e^{-\frac{E_0}{RT_i}} e^{-\alpha_i t_i} + e^{-\frac{E_0}{RT_i}} e^{-\alpha_i t_{i-1}} \right](t_i - t_{i-1})}} \right\} \quad (11)$$

If $S/S_u$ calculated by Eq. (11) is divided by $S/S_u$ of 28 days, $S/S_{28}$ can be obtained. In Fig. 8, the solid line is the predicted compressive strength curve of concrete cured at variable curing temperature and the dotted lines are the predicted compressive strength curves of concretes cured at 23 and 50 °C. As shown in Fig. 8, the new prediction model reasonably estimates the mechanical properties of concrete cured at variable curing temperature. Therefore, it is safe to assume that the new prediction model can be used for estimating the mechanical properties of in-place concrete.

## 4. Conclusions

This study has proposed a new prediction model for estimating mechanical properties according to temperature and aging and evaluated the validity of the prediction model.

The new prediction model using the same apparent activation energy in all mechanical properties reasonably estimates the strength and elastic modulus of Types I and V cement concretes tested in Part I. In addition, the new prediction model gives more accurate estimation values of elastic modulus than CEB-FIP 1990 model.

To evaluate in-place applicability of the model, concrete cured at variable temperature condition was tested. The comparison between experimental and calculated values showed that the new prediction model properly estimates the variation of mechanical properties of concrete cured at variable curing temperature condition.

## References

[1] G. Chengju, Maturity of concrete: method for predicting early-stage strength, ACI Mater. J. 86 (4) (1989) 341–353.
[2] R.C. Tank, N.J. Carino, Rate constant functions for strength development of concrete, ACI Mater. J. 88 (1) (1991) 74–83.
[3] N.J. Gardner, Effect of temperature on the early-age properties of type I, type III, and type I/fly ash concrete, ACI Mater. J. 87 (1) (1990) 68–78.
[4] J.K. Kim, S.H. Han, Estimation of compressive strength by a new apparent activation energy function, Cem. Concr. Res. 31 (2) (2001) 217–255.
[5] K.O. Kjellsen, R.J. Detwiler, Later ages strength prediction by a modified maturity method, ACI Mater. J. 90 (3) (1993) 220–227.
[6] G. Chanvillard, L. D'Aloia, Concrete strength estimation at early ages: modification of the method of equivalent age, ACI Mater. J. 94 (6) (1997) 520–530.
[7] H. Kada-Benameur, E. Wirquin, B. Duthoit, Determination of apparent activation energy of concrete by isothermal calorimetry, Cem. Concr. Res. 30 (2) (2000) 301–305.
[8] P. Klieger, Effect of mixing and curing temperature on concrete strength, ACI J. 54 (1) (1958) 1063–1074.
[9] J.K. Kim, Y.H. Moon, S.H. Eo, Compressive strength development of concrete with different curing time and temperature, Cem. Concr. Res. 28 (12) (1998) 1761–1773.
[10] J.J. Brooks, A.F. Al-Kaisi, Early strength development of Portland and slag cement concretes cured at elevated temperatures, ACI Mater. J. 87 (5) (1990) 503–506.
[11] CEB-FIP MODEL CODE 1990, Thomas Telford, U.K., (1993).
[12] J.K. Kim, K.H. Kim, J.K. Yang, Thermal analysis of hydration heat in concrete structures with pipe-cooling system, Comput. Struct. 79 (2001) 163–171.