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Prediction of splitting tensile strength of high-performance concrete

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Abstract

Splitting tensile strength (STS) is one of the concrete mechanical properties that are used in structural design. It can be related to numerous parameters, which include compressive strength, water/binder (W/B) ratio and concrete age. Until now, most researchers estimated the STS directly from compressive strength data. This paper suggests formulae that relate STS with that of compressive strength, W/B ratio and concrete age. The predicted STS can be obtained accurately using these formulae. It is proposed that the equation with the concrete age (t) parameter be used in predicting the STS of high-performance concrete (HPC). © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: High-performance concrete; Compressive strength; Splitting tensile strength; Water/binder ratio

1. Introduction

Recently, research works and usage of high-performance concrete (HPC) have increased tremendously. HPC is a relatively new product and its characteristics differ from that of normal concrete. According to Zia et al. [1], HPC is defined as concrete, which meets special performance and uniformity requirements that cannot always be achieved routinely by using conventional materials and normal mixing, placing and curing practices. The requirements may involve enhancements of characteristics such as placement and compaction without segregation, long-term mechanical properties, early-age strength, volume stability or service live in severe environments. Kuennen [2] states that HPC provides enhanced mechanical properties in pre-cast concrete structural elements including higher tensile and compressive strengths and enhanced modulus of elasticity (stiffness).

Compressive and tensile strengths are both required in the design of structures. Tensile strength is important for nonreinforced concrete structures such as dam under earthquake excitations. Other structures such as pavement slabs and airfield runway, which are designed based on bending strength, are subjected to tensile forces. Therefore, in the design of these structures, tensile strength value is more important than the compressive strength.

Many researchers have suggested tensile strength formulae for HPC or high-strength concrete, the majority of which involved the square root function, similar to that proposed by ACI [3]. De Larrard and Malier [4] found that the calculated STS obtained from the French regulations are in good agreement with experimental data. Kim et al. [5] found that the ACI model overestimates STS for concrete with compressive strength <20 MPa and underestimates the value for concrete with compressive strength >30 MPa.

Tensile strength can be related to compressive strength, water/binder (W/B) ratio and concrete age. This paper describes research work that was initiated to determine relationships between tensile strength with these parameters. The approach is an empirical one, using experimental data that were obtained from previous researchers [6–19]. The compressive and tensile strength data that were used in this analysis do not take into account the type of admixtures used such silica fume, fly ash, rice husk ash or blast furnace slag. The main parameters that were taken into consideration are those which fulfill the requirements of HPC, namely the compressive strength and workability.

2. Compressive strength

Compressive strength depends mainly on the W/B ratio [20]. It is also affected by the quality of the constitu-

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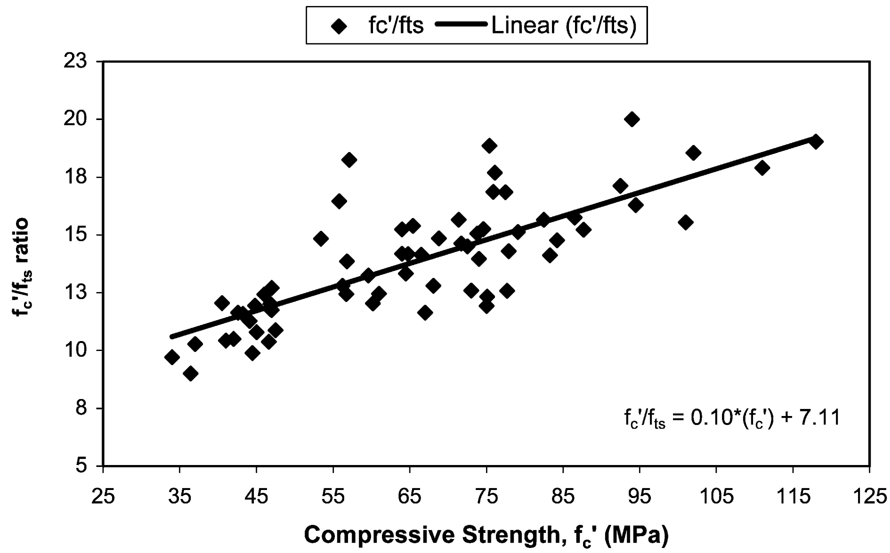

Fig. 1. Linear regression of f_c'/f_{ts} ratio and compressive strength.

ent materials, mixing and curing methods. The lower the value of W/B ratio, the higher is the strength of concrete. Aitcin and Mehta [21] classified high-strength concrete or HPC as concrete of strength greater than 40 MPa at 28 days.

The tensile strength of concrete is much lower than the compressive strength, largely because of the ease with which cracks can propagate under tensile loads. Although tensile strength is usually not considered directly in design (normally assumed equal to zero), its value is still needed because cracking in concrete tends to be of tensile behaviour. According to Marzouk and Chen [16], concrete can be considered a brittle material, and the tensile strength of a brittle material is due to the rapid propagation of a single flaw or microcrack. High-strength concrete is, therefore, more brittle and stiffer than normal-strength concrete.

3. Splitting tensile strength (STS)

Many researchers turn to STS test because the method of testing is simple and its value is one of the mechanical properties of concrete. Wiegrink et al. [19] found that the strength development pattern for STS is similar to that of compressive strength. Mindes and Young [22] explained that the relationship between tensile and compressive strength is not a simple one. It depends on the age and strength of concrete, type of curing, aggregate type, amount of air entrainment and degree of compaction.

There are many empirical formulae for determining STS (f_t) and compressive strength (f_c') and most researchers opted the formula of the type:

$$f_t = k(f_c')^n \tag{1}$$

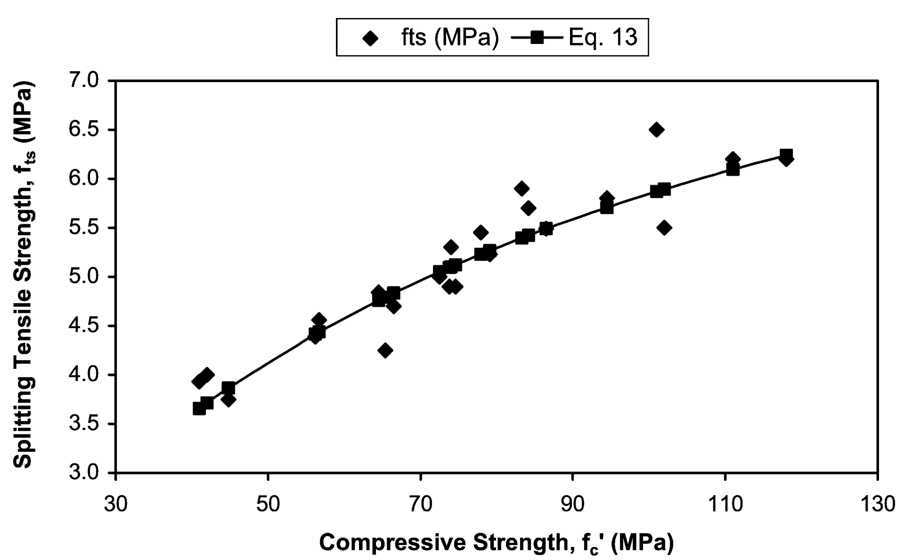

Fig. 2. Eq. (13) curve with experimental data.

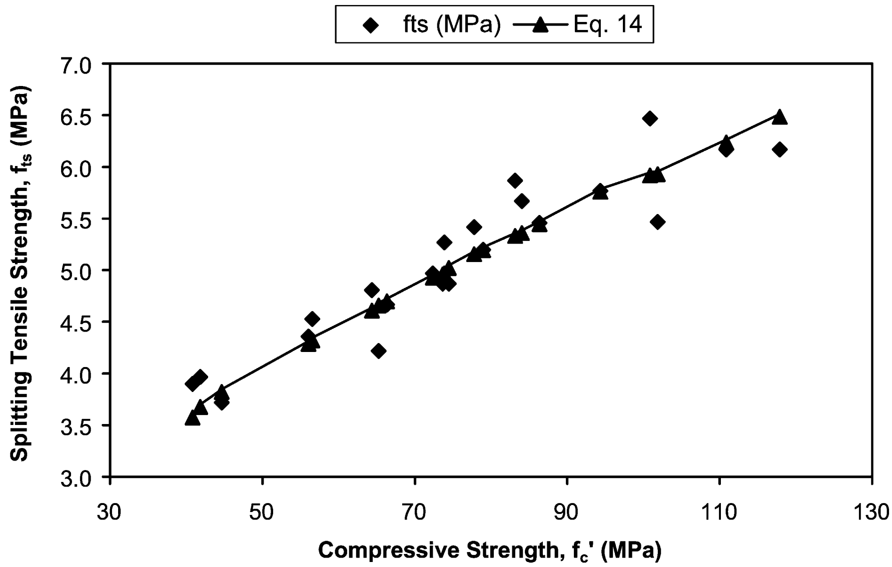

Fig. 3. Eq. (14) curve with experimental data.

where k and n are coefficients that can be obtained from regression analysis. The n value is generally between 1/2 and 3/4 (Eq. (1)).

De Larrard and Malier [4] found that the following STS equation is in good agreement with that of the French regulations:

$$f_{tj} = 0.6 + 0.06 f_{cj} \tag{2}$$

where f_{tj} and f_{cj} are average values of splitting tensile and compressive strengths, respectively, at j days (MPa) (Eq. (2)).

Other formulae for predicting tensile strength, as suggested by Iravani [23], ACI [3] and CEB-PIP [24], are given below, respectively:

$$f'_{sp} = 0.57\sqrt{f'_c}, \quad \text{for concrete strength } 50 \text{ MPa} < f'_c < 100 \text{ MPa} \tag{3}$$

$$f'_{sp} = 0.59\sqrt{f'_c}, \quad \text{for concrete strength } 21 \text{ MPa} < f'_c < 83 \text{ MPa} \tag{4}$$

$$f_t = 0.301(f'_c)^{0.67} \tag{5}$$

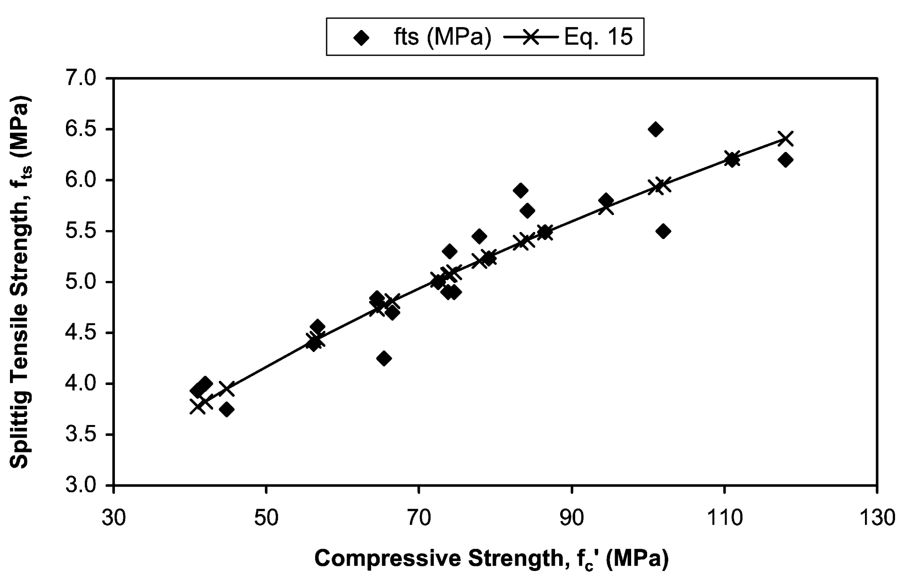

Fig. 4. Eq. (15) curve with experimental data.

The above equations (Eqs. (3)–(5)) will be used as comparison with the authors' proposed equations later.

4. Analysis and discussion

According to Neville [20], STS has a close relationship with compressive strength, but there is no direct proportionality. The ratio of the two strengths depends on the general level of the strength of concrete. In other words, as the compressive strength (f_c') increases, the tensile strength (f_t) also increases but at a decreasing rate. Marzouk and Chen [16] stated that tensile strength increases as compressive strength increases. However, the tensile strength increases at a much smaller rate when compared to the increase of compressive strength. This implies that the relationship between STS and compressive strength is nonlinear. The increases in strength for these two parameters are related to W/C, cement type and temperature of curing [9]. W/B affects both compressive and STS. Other than W/B ratio, concrete age also plays a significant role in concrete strength development. Its strength increases with age. However, the increase is not linear.

In conclusion to the above statements, STS is a function of several parameters such as compressive strength, W/B ratio and concrete age and can be represented as follows:

$$f_{ts} = f(f_c') \tag{6}$$

$$f_{ts} = f(f_c', (\mathrm{W/B})) \tag{7}$$

$$f_{ts} = f(f_c', t) \tag{8}$$

From Eqs. (6)–(8), relationship between these parameters can be shown as follows:

$$f_{ts} = \frac{f_c'}{af_c' + b} \tag{9}$$

$$f_{ts} = a\sqrt{f_c'}(\mathrm{W/B})^b \tag{10}$$

$$f_{ts} = a\sqrt{f_c'}\left(\frac{t}{t_{28}}\right)^b \tag{11}$$

where f_c' is cylinder compressive strength (MPa) at 28 days, f_{ts} is STS at 28 days, W/B is W/B ratio, t is the concrete age at day of testing, t_{28} is concrete age at 28 days and a and b are coefficients, which are determined using regression analysis to the experiment data (Eqs. (9)–(11)).

Fig. 1 shows the relationship between the ratio f_c'/f_{ts} and f_c'.

Table 1
Comparison of STS (experimental and calculation)

W/B	Age of concrete (days)	Experimental		Predicted STS f_{ts} (MPa)			Experimental/predicted ratio		
		Compressive strength f_c' (MPa)	STS f_{ts} (MPa)	Eq. (13)	Eq. (14)	Eq. (15)	$f_{ts}/f_{ts(13)}$	$f_{ts}/f_{ts(14)}$	$f_{ts}/f_{ts(15)}$
0.55	28	41.0	3.9	3.66	3.61	3.78	1.075	1.090	1.040
0.44	28	42.0	4.0	3.71	3.71	3.82	1.077	1.079	1.046
0.40	28	44.8	3.8	3.87	3.85	3.95	0.970	0.973	0.950
0.40	28	56.2	4.4	4.41	4.32	4.42	0.994	1.017	0.993
0.40	28	53.4	3.6	4.29	4.21	4.31	0.839	0.856	0.835
0.38	28	56.7	4.6	4.44	4.35	4.44	1.028	1.048	1.026
0.38	28	64.5	4.8	4.76	4.64	4.74	1.018	1.043	1.021
0.36	28	66.5	4.7	4.83	4.73	4.81	0.973	0.994	0.977
0.36	28	65.4	4.3	4.79	4.69	4.77	0.887	0.906	0.891
0.35	28	73.8	4.9	5.09	4.99	5.07	0.962	0.982	0.967
0.35	28	74.0	5.3	5.10	5.00	5.08	1.039	1.060	1.044
0.34	28	72.5	5.0	5.05	4.96	5.02	0.990	1.008	0.995
0.32	28	74.6	4.9	5.12	5.05	5.10	0.957	0.970	0.962
0.30	28	77.9	5.5	5.23	5.19	5.21	1.042	1.051	1.047
0.30	28	79.1	5.2	5.27	5.22	5.25	0.993	1.001	0.997
0.30	28	84.2	5.7	5.42	5.39	5.41	1.051	1.057	1.053
0.30	28	83.3	5.9	5.40	5.36	5.38	1.094	1.100	1.096
0.29	28	75.4	4.0	5.15	5.11	5.12	0.777	0.782	0.781
0.29	28	86.5	5.5	5.49	5.48	5.49	1.000	1.002	1.000
0.28	28	102.0	5.5	5.89	5.96	5.96	0.933	0.923	0.923
0.27	28	101.0	6.5	5.87	5.95	5.93	1.108	1.093	1.096
0.25	28	111.0	6.2	6.10	6.27	6.22	1.017	0.989	0.997
0.25	28	94.5	5.8	5.71	5.79	5.74	1.016	1.002	1.011
0.22	28	118.0	6.2	6.24	6.52	6.41	0.994	0.952	0.967
Average							0.948	0.954	0.945

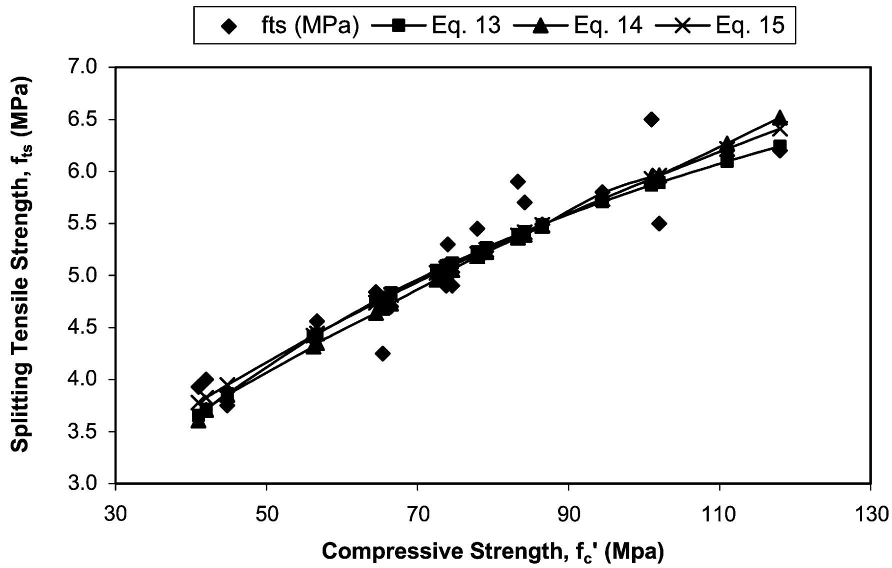

Fig. 5. Curves obtained from Eqs. (13)–(15) with experiment data.

By using the linear regression technique, the following equation is obtained (Eq. (12)):

$$\frac{f_c'}{f_{ts}} = 0.10(f_c') + 7.11 \tag{12}$$

The relation between STS and compressive strength can be determined by

$$f_{ts} = \frac{f_c'}{0.10(f_c') + 7.11} \tag{13}$$

Similarly, the following equations, which relate splitting tensile with compressive strength and W/B ratio, can be derived according to the above procedure:

$$f_{ts} = 0.54\sqrt{f_c'}(\mathrm{W/B})^{-0.07} \tag{14}$$

The relationship between tensile strength, compressive strength, and concrete age by

$$f_{ts,t} = 0.59\sqrt{f_{c,t}'}\left(\frac{t}{t_{28}}\right)^{0.04} \tag{15}$$

Table 2
Comparison of STS between experiment data, proposed equations, codes of practice and published data

W/B	Age of concrete (days)	Experimental: Compressive strength f_c' (MPa)	Experimental: STS f_{ts} (MPa)	Predicted STS f_{ts} (MPa): Eq. (13)	Eq. (14)	Eq. (15)	French code	ACI code	CEB/FIP code	Iravani [23]
0.55	28	41.0	3.9	3.66	3.61	3.78	3.06	3.78	3.62	3.65
0.44	28	42.0	4.0	3.71	3.71	3.82	3.12	3.82	3.68	3.69
0.40	28	44.8	3.6	3.87	3.85	3.95	3.29	3.95	3.85	3.82
0.40	28	56.2	4.4	4.41	4.32	4.42	3.97	4.42	4.48	4.27
0.38	28	56.7	4.6	4.44	4.35	4.44	4.00	4.44	4.50	4.29
0.38	28	64.5	4.8	4.76	4.64	4.74	4.47	4.74	4.91	4.58
0.36	28	66.5	4.7	4.83	4.73	4.81	4.59	4.81	5.01	4.65
0.36	28	65.4	4.3	4.79	4.69	4.77	4.52	4.77	4.95	4.61
0.35	28	73.8	4.9	5.09	4.99	5.07	5.03	5.07	5.37	4.90
0.35	28	74.0	5.3	5.10	5.00	5.08	5.04	5.08	5.38	4.90
0.34	28	72.5	5.0	5.05	4.96	5.02	4.95	5.02	5.31	4.85
0.32	28	74.6	4.9	5.12	5.05	5.10	5.08	5.10	5.41	4.92
0.30	28	77.9	5.5	5.23	5.19	5.21	5.27	5.21	5.57	5.03
0.30	28	79.1	5.2	5.27	5.22	5.25	5.35	5.25	5.63	5.07
0.30	28	84.2	5.7	5.42	5.39	5.41	5.65	5.41	5.87	5.23
0.30	28	83.3	5.9	5.40	5.36	5.38	5.60	5.38	5.83	5.20
0.29	28	86.5	5.5	5.49	5.48	5.49	5.79	5.49	5.98	5.30
0.28	28	102.0	5.5	5.89	5.96	5.96	6.72	5.96	6.67	5.76
0.27	28	101.0	6.5	5.87	5.95	5.93	6.66	5.93	6.63	5.73
0.25	28	111.0	6.2	6.10	6.27	6.22	7.26	6.22	7.06	6.01
0.25	28	94.5	5.8	5.71	5.79	5.74	6.27	5.74	6.34	5.54
0.22	28	118.0	6.2	6.24	6.52	6.41	7.68	6.41	7.36	6.19

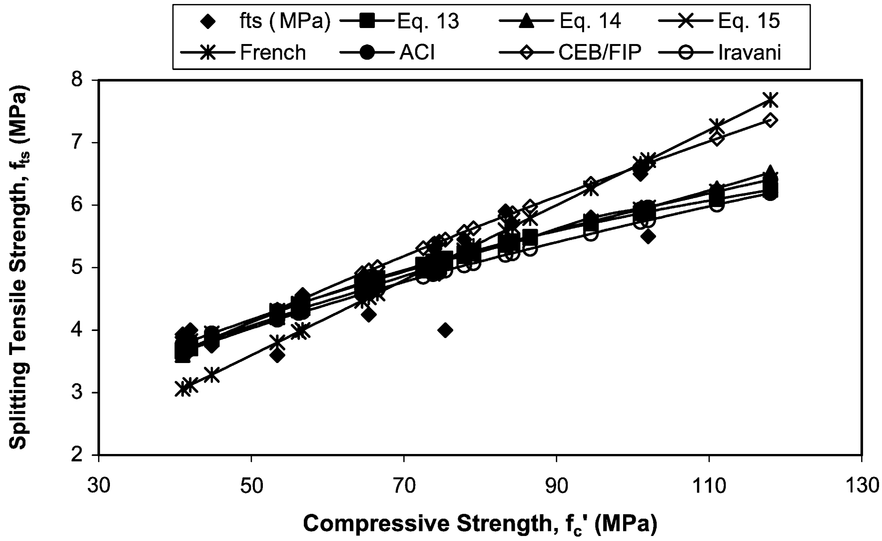

Fig. 6. Relationship derived from Eqs. (13)–(15), published data and established codes of practice.

where $f_{ts,t}$ is STS at the age of t days, $f'_{c,t}$ is compressive strength at t days and t is age of concrete at t days.

Figs. 2–4 show the predicted curves derived from Eqs. (13)–(15) plotted together with actual experimental data. From these graphs, it can be seen that the relationship between the parameters is, indeed, nonlinear. This demonstrates that the proposed formulae are reasonably true and accurate. It also imply that the STS increases with increasing compressive strength at a decreasing rate, similar to that suggested by Neville [20] as well as Marzouk and Chen [16]. The STS increases slower than the compressive strength so that the ratio f_{ts}/f_c' decreases with time. This

Table 3
Estimation of STS at different ages

	Experimental			
Age of concrete (days)	Compressive strength $f'_{c,t}$ (MPa)	STS $f_{ts,t}$ (MPa)	Prediction of STS (Eq. (15)) $f_{ts,t}$ (MPa)	Experimental/ predicted ratio
3	19.0	2.3	2.35	0.98
	20.0	2.4	2.41	0.99
	28.6	2.9	2.89	0.99
	28.9	3.0	2.90	1.02
	36.1	3.3	3.24	1.03
	44.5	3.7	3.60	1.03
Average				*1.01*
7	23.0	2.5	2.68	0.93
	33.0	3.5	3.21	1.09
	40.5	3.2	3.55	0.91
	48.2	4.1	3.88	1.06
	58.0	4.2	4.25	0.98
	69.0	4.5	4.64	0.98
Average				*0.99*
14	36.0	3.3	3.44	0.95
	42.8	3.9	3.75	1.03
	59.3	4.0	4.42	0.91
	63.3	4.5	4.57	0.99
	67.7	4.7	4.72	1.00
	77.0	5.1	5.04	1.01
Average				*0.98*
56	49.9	4.3	4.29	1.00
	53.1	5.4	4.42	1.22
	61.7	4.6	4.76	0.97
	73.2	5.1	5.19	0.99
	86.5	5.5	5.64	0.98
	92.3	5.9	5.83	1.01
Average				*1.03*

confirms the fact that high-strength concrete is more brittle than normal concrete.

Table 1 presents the results of the STS based on calculations derived from Eqs. (13)–(15) as well as those from experiment. The last three columns of the table present the ratio of experimental/predicted data of STS. This ratio ranged between 0.835 and 1.108, with an average value of 0.945 and 0.954. These values are close to unity, with a correction factor of 5%. These results indicate that the accuracy of Eqs. (13)–(15) are high. The degree of accuracy in descending order can be achieved using Eqs. (13)–(15) (average ratio of experimental and predicted data based on these equations are 0.948, 0.954 and 0.945, respectively).

Fig. 5 shows the curves derived from Eqs. (13)–(15) plotted together with the experimental data. The curves almost aligned with one another. The graph shows that the three parameters (f_c', W/B, and t), which are used in the three equations, are significant variables in determining STS. To test the accuracy of these equations, results obtained from them are compared with those recommended by the French regulations as that mentioned by De Larrard and Malier [4], ACI [3] and CEB-PIP [24] codes as well as that proposed by Iravani [23]. These are presented in Fig. 6 and Table 2.

From Table 2 and Fig. 6, it is apparent that with the exception of results obtained from the French code, for compressive strength lower than 50 MPa, all the predicted values are more or less similar, implying that any one of the proposed equations is suitable to be used. For concrete of 50–90 MPa, with the exception of the French [4] and CEB-PIP [24] codes, all the other data lie close to one another. This means that all the proposed formulae are satisfactory in estimating STS. For strengths greater than 90 MPa, Eqs. (13)–(15), ACI [3] and Iravani [23] provide good correlation compared to the experimental data, but a large divergence was observed when the French and CEB-PIP codes were used, probably due to the linear behaviour of these equations.

To determine the STS for concrete of Grade 50–120 MPa, Eqs. (13)–(15) are accurate enough to be used. These equations are strongly supported by those derived from the ACI Code [3] and Iravani's [23]. Fig. 6 shows that curves obtained from Eq. (15) and that using the ACI formula fell on the same line because both equations are almost similar (refer to Eqs. (4) and (15)). If the t value of Eq. (15) is taken as 28, then the term $(t/t_{28})^{0.04}$ becomes unity. This means that Eq. (15) is exactly the same as that of ACI when the age of concrete is 28 days.

Eq. (15) contains the time variable (t), where t is the concrete age. Compared to Eqs. (13) and (14), the advantage of employing this equation is that it can be used for any concrete at any age. As seen in Table 3, data obtained from experiment with those obtained from this equation for various concrete ages show good correlation and high accuracy. The average value of experimental/predicted ratio of this property is close to unity. Based on this result, it is proposed that Eq. (15) be used to estimate the STS of HPC.

5. Conclusions

(1) To estimate the STS of HPC, three equations based on compressive strength (f_c'), W/B ratio and concrete age as given in Eqs. (13)–(15) are sufficiently accurate.

(2) Eqs. (13)–(15) give good estimation of STS. The average value of the experimental/predicted ratio of this property is close to unity.

(3) Compared to the other two equations, the advantage of employing Eq. (15) is that it can be used for any concrete at any age. The STS values obtained from experiment compared well with that estimated from this equation, with the average ratio of the experimental/predicted data being close to unity. Based on this result, it is proposed that Eq. (15) be used to estimate the STS of HPC.

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