



# Investigations on the compressive strength of silica fume concrete using statistical methods

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## Abstract

The present paper deals with a mathematical model developed using statistical methods to predict the 28-day compressive strength of silica fume concrete with water-to-cementitious material (w/cm) ratios ranging from 0.3 to 0.42 and silica fume replacement percentages from 5 to 30. Strength results of 26 concrete mixes, on more than 300 test specimens, have been analyzed for statistical modeling. The ratios of compressive strengths between silica fume and control concrete have been related to silica fume replacement percentage. The expression, being derived with strength ratios and not with absolute values of strength, is independent of the specimen parameters and is applicable to all types of specimens. On examining the validity of the model with the results of previous researchers, it was observed that for results on both cubes and cylinders, predictions were obtained within 7.5% of the experimentally obtained values. © 2002 Elsevier Science Ltd. All rights reserved.

**Keywords:** Concrete; Silica fume; Compressive strength; Modeling; Regression analysis

## 1. Introduction

The application of condensed silica fume in concrete is almost a routine one nowadays for obtaining special properties of silica fume cement (SFC) concrete. In spite of the wealth of information available in the existing literature on silica fume concrete, relationships between the compressive strength and silica fume replacement percentage that can be utilized for predicting the strength at any replacement level are quite limited. Slanicka [1] developed models for SF concrete by modification of the Abrams and Bolomey expressions. Gutierrez and Canovas [2] developed a model of the compressive strength of silica fume concrete based on the results on 15 × 30-cm cylinders, assuming that the efficiency of silica fume is constant at all replacement levels. But Ganesh Babu and Surya Prakash [3] had reported that the efficiency of silica fume depends on the levels of cement replacement. Duval and Kadri [4] proposed a model to determine the compressive strength of SF concrete based on the 28-day compressive strength of standardized mortar. All these models deal with absolute strength values and thus

are valid for a particular type of specimen. The aim of the present investigation is to overcome this inherent weakness and develop a regression model involving nondimensional parameters so that the effect of specimen shape and size can be eliminated.

## 2. Experimental program

### 2.1. Materials

The cement used was ordinary Portland cement, having a 28-day compressive strength of 54 MPa. Silica fume from Elkem Materials (Kristiansand, Norway) was used. The silica fume contained 90.9% SiO<sub>2</sub> and its BET-specific surface area was 18,000 m<sup>2</sup>/kg. Natural river sand having a fineness modulus of 2.5 was used. Crushed, angular, graded coarse aggregates of nominal maximum size 12.5 mm were used in the investigation. Potable water and high-range water-reducing admixtures (SP) were employed for mixing.

### 2.2. Experimental procedure

In an effort to determine the isolated effect of silica fume on concrete, other mix design variables like quality of

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ingredients, mix proportions including the dosage of SP, mixing procedures, curing conditions and testing procedures were kept constant. Hence, the change in concrete properties occurred primarily due to replacement of cement by silica fume. Since the SP content of all the mixes was kept constant, to minimize variations in workability, the compaction energy was varied for obtaining proper compaction. Four water-to-cementitious material (w/cm) ratios were investigated—0.3, 0.34, 0.38 and 0.42. Cement was replaced by silica fume (1:1 by mass) at 5%, 10%, 15%, 20%, 25% and even 30% (for w/cm ratios of 0.38 and 0.42) of the total binder content. The mix proportions adopted were C:FA:CA=1:1.28:2.2. The total binder content was fixed at about 500 kg/m<sup>3</sup>. The compressive strengths were measured on four different types of specimens—100- and 150-mm cubes and 100 × 200- and 150 × 300-mm cylinders. All the specimens were moist-cured under water until testing. Each strength value was the average of three specimens. For the same concrete, four different 28-day compressive strength values were determined and the database, consisting of results on 312 test specimens, was analyzed for statistical modeling.

### 3. Analysis of results and statistical modeling

The compressive strengths of concrete, as obtained on the different specimens, and the strength ratios of silica

fume and control (reference concrete with 0% silica fume) concrete are presented in Table 1. Since the same concrete was used to prepare four different types of specimens, the results obtained on all the specimens exhibit a common trend as expected. If absolutely identical methods of preparation and testing conditions can be ensured, then the ratio of the strengths of two concretes measured on different specimens should be identical and independent of the specimen parameters. Hence, for the same SF and same control concretes, the ratios of strengths measured on different specimens should be similar. But absolutely identical conditions of sample preparation and testing are virtually impossible to achieve in practice. Hence, the ratios of strengths for the same concretes are observed to be somewhat different for different specimens. These ratios can be utilized for the development of generalized expressions which, being free from the influence of specimen parameters, can be used for the strength prediction of any type of specimen. The absolute values of the compressive strength of concrete depend on a host of parameters. It is impossible to define a model that can cater to all these variables simultaneously and can make accurate predictions under all conditions. But if the strength of SF concrete is defined with respect to the control concrete and expressed in the form of ratios, then the uncertainties involved in the predictions are considerably reduced. Moreover, for a particular set of concretes, the changes in the prevailing

Table 1  
The 28-day strengths of silica fume and control concrete and their corresponding ratios

w/cm ratio	SF%	100 mm cubes		150 mm cubes		100 × 200 mm cylinders		150 × 300 mm cylinders	
		$f$	$f_{SF}/f_C$	$f$	$f_{SF}/f_C$	$f$	$f_{SF}/f_C$	$f$	$f_{SF}/f_C$
0.30	0	74.7	1.0	67.4	1.0	58.2	1.0	58.7	1.0
	5	80.0	1.07	75.4	1.12	63.9	1.10	68.5	1.17
	10	88.3	1.18	79.1	1.17	64.3	1.11	69.8	1.19
	15	99.0	1.33	84.2	1.25	67.7	1.16	70.6	1.20
	20	99.7	1.34	87.6	1.30	72.2	1.24	74.3	1.27
	25	97.7	1.31	82.8	1.23	70.9	1.22	73.6	1.25
0.34	0	69.0	1.0	59.3	1.0	49.7	1.0	48.9	1.0
	5	73.7	1.07	64.3	1.09	58.8	1.18	60.6	1.24
	10	79.7	1.16	72.0	1.22	61.1	1.23	63.6	1.30
	15	88.3	1.28	76.3	1.29	65.4	1.32	67.0	1.37
	20	90.0	1.30	81.3	1.37	66.6	1.34	69.2	1.42
	25	85.0	1.23	78.5	1.33	64.1	1.29	66.7	1.37
0.38	0	58.0	1.0	54.5	1.0	40.1	1.0	43.0	1.0
	5	66.0	1.14	60.2	1.10	46.7	1.16	46.8	1.09
	10	69.7	1.20	66.5	1.22	50.9	1.27	51.9	1.21
	15	70.3	1.21	67.6	1.24	56.0	1.40	60.7	1.41
	20	72.7	1.25	69.3	1.27	59.4	1.48	64.3	1.50
	25	79.0	1.36	71.4	1.31	63.2	1.58	64.9	1.51
0.42	30	75.7	1.31	67.4	1.24	53.3	1.33	61.3	1.43
	0	55.0	1.0	48.3	1.0	36.5	1.0	39.1	1.0
	5	58.3	1.06	50.8	1.05	39.7	1.09	44.3	1.14
	10	59.3	1.08	57.5	1.19	43.5	1.19	46.2	1.18
	15	65.7	1.19	60.7	1.26	50.1	1.37	52.8	1.35
	20	66.3	1.21	62.7	1.30	52.8	1.45	55.6	1.43
	25	68.0	1.24	63.7	1.32	55.4	1.52	57.7	1.48
	30	60.3	1.10	51.1	1.06	40.3	1.11	48.3	1.24

$f$  represents the strength of concrete;  $f_C$  refers to the strength of control concrete;  $f_{SF}$  refers to the strength of silica fume concrete.

Table 2

Comparison between the results obtained by previous researchers and predictions obtained by the present model

Researchers and type of specimen used	w/cm ratio	Silica fume replacement percentage	Compressive strength (MPa)		Percentage variation
			Experimentally obtained	Predicted by the present model	
Results obtained by Hooton on 150 × 300-mm cylinders	0.35	0	55.6	—	—
		10	70.7	66.9	5.37
		15	75.2	71.8	4.52
		20	74.2	74.5	0.00
Results obtained by Khedr and Abou-Zeid on 150-mm cubes	0.3	0	62.5	—	—
		5	72.5	68.7	5.24
		10	81.0	75.1	7.28
		15	86.25	80.7	6.43
		20	83.75	83.8	0.00
		25	77.5	82.9	−6.97

conditions for the SF and control concretes are generally not very significant. The primary factor that governs the compressive strength of concrete is the w/cm ratio. Again, at a particular w/cm ratio, the percentage of cement replaced by silica fume also affects the compressive strength. As per the classical formulation of Abrams' law, there exists an inverse relationship between the compressive strength and water-to-cement ratio of concrete. But the relationship of compressive strength with silica fume replacement percentage is yet to be well established. In situations where there is no prior relationship known between variables, a scatter diagram is prepared and the information portrayed in the diagram is used in the search for an appropriate mathematical model. The scatter diagrams for all the four different specimens exhibited a nonlinear relationship. Slanicka [1] reported that the contribution of SF to the strength of concrete is nonlinear and the increase of SF content does not necessarily lead to a proportional effect on the concrete strength. From the scatter diagrams, it was observed that a third degree polynomial might be appropriate for the test results. Accordingly, statistical regression analysis was performed for this model using the method of least squares and the unknown parameters were determined. The validity of the model was investigated by examining relevant statistical coefficients [5,6].

The relationship between the 28-day strength ratio of SF and control concrete and silica fume replacement percentage based on the results of the present investigation, for w/cm ratios ranging from 0.3 to 0.42, has been obtained as:

$$\frac{f_{SF}}{f_C} = 1.0063 + 0.0159(\text{SF}\%) + 0.0007(\text{SF}\%)^2 - 0.00003(\text{SF}\%)^3 \quad (1)$$

where SF% refers to the silica fume replacement percentage and  $f_{SF}$  and  $f_C$  denote the strengths of the silica fume and control concretes, respectively. The values of the standard error of estimate ( $s$ ) and correlation coefficient ( $r$ ) have been obtained as 0.079 and 0.842, respectively.

### 3.1. Checking the validity of the model with results of previous researchers

In order to check whether the proposed model is independent of specimen parameters, strength results of SF concretes obtained by previous researchers on different types of specimens have been considered. A comparison between the experimental results obtained by Hooton [7] on cylinders and Khedr and Abou-Zeid [8] on cubes and those predicted by the present model (Eq. (1)) is presented in Table 2 and the variation lies within 7.5%. A comparison of the experimental values of compressive strength [7,8] and the corresponding values predicted by the present model is presented in Fig. 1.

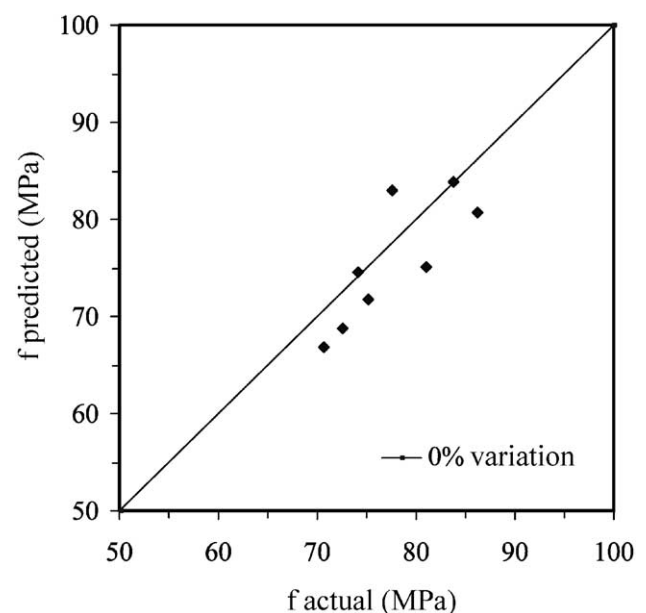


Fig. 1. Comparison between experimental and predicted values of compressive strength.

#### 4. Conclusions

Extensive experimentation was performed to determine the isolated effect of silica fume on the properties of concrete over a wide range of w/cm ratios varying from 0.3 to 0.42 and silica fume replacement percentages ranging from 5 to 30. On the basis of regression analysis of a large number of experimental results, a statistical model has been developed, which can serve as a useful tool for optimizing and predicting the strengths of silica fume concretes over a wide range of replacement percentages and w/cm ratios ranging from 0.3 to 0.42. This model, involving nondimensional variables, is independent of the specimen parameters. The validity of the model has been verified with the results obtained by different researchers on different types of specimens.

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